MODERN
BUSINESS ARITHMETIC

COMPLETE COURSE

FINNEY
AND
BROWN

GIFT OF
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MODERN BUSINESS ARITHMETIC

COMPLETE COURSE

BY

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UNIV. OF
CALIFORNIA

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PREFACE

This book provides a year's work in the arithmetic of modern business. All topics which have no reason for inclusion except tradition have been omitted. Topics which have recently acquired importance because of changes that have been brought about in the organization and conduct of business are included. Among the topics so included are Contract Purchases and Installment Payments, Depreciation, Advertising, Insolvency and Bankruptcy, Comparative Statistics to Promote Buying and Selling Efficiency, Profit and Loss by Departments, Finding the Profit and Loss on Each Sale, Factory Costs, Tabulations for the Sales Manager, and the Income Tax.

The illustrations and model forms were chosen with a view to acquainting the student with actual business conditions. The problems were selected from those which actually arise in important business activities, and the methods of solution are those which are used in business practice. Many of the drill exercises are put into ruled forms such as will be met in the business world; and the student is given practice in the art of presenting business statistics and other material in a neat, concise, and attractive manner. The traditional sequence of topics has been abandoned in some cases in favor of a grouping of topics in which there is a relation of business experience.

The necessity for a high degree of accuracy and facility in arithmetical computation has been fully recognized. Abundant material for practice and drill in the fundamental processes is provided. The student is required to check his results at every point in order to secure accuracy; and numerous exercises to be finished within a time limit are included to improve speed in

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computation. Since business expediency demands that many problems shall be solved orally, a large amount of oral work has been included.

From the point of view of both material and method this book is based not on theory but on actual business conditions and practices. The material has been successfully subjected to the test of preparing hundreds of students to meet the exacting demands of the business world. The ruled forms and the tabulated business statistics so extensively used in the book have demonstrated their effectiveness in developing rapidity, accuracy, and neatness. Many men who are specialists in various business and industrial activities were consulted, and the statements of business customs as well as the tabulation of materials and the arrangement of problems are based on their advice.

The authors wish to acknowledge their indebtedness to the numerous teachers, business men, and professional men who have so kindly aided them by offering valuable suggestions and discriminating criticisms, and by furnishing materials. They acknowledge especial indebtedness to: Mr. Seymour Walton, Certified Public Accountant, Dean of the Walton School of Commerce, Chicago, who read several of the chapters, the subject matter of which borders on accountancy; Professor Norris A. Brisco, head of the School of Commerce and of the Departments of Political Economy and Sociology in the University of Iowa and Editor of the Efficiency Society Journal; Mr. Stanley C. Crafts, Auditor of Customs, Port of Chicago; Mr. T. H. Fuller, Auditor for Carson, Pirie, Scott & Co., Chicago; Mr. H. A. Brinkman, Cashier of the Harris Trust and Savings Bank, Chicago; and Mr. H. V. Church, Principal of the Cicero Township High School, Berwyn, Ill.

HARRY ANSON FINNEY.
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INTRODUCTION

To the Student

If you expect to succeed in the business world, you should cultivate accuracy, neatness, and speed in all computations. A high degree of accuracy is indispensable. Be very careful about all of your work, and use adequate means of checking your results.

Take pride in the appearance of your work. Your papers from day to day should be prepared with ink, because that is the way business records are kept. When ruling is to be done, make the lines fine. Make your figures small and similar to those in the following model:

1 2 3 4 5 6 7 8 9 0

Work as rapidly as you can without detriment to your accuracy. The material in this book has been prepared with the special purpose of making students accurate, careful, and rapid business workers.

FUNDAMENTAL PROCESSES

CHAPTER I

ADDITION

1. Drill Tables. The following table contains the forty-five combinations of two numbers, each of which is less than ten. Practice until you can state these forty-five sums, without error, in less than twenty-five seconds. Do not repeat the numbers to be added. State *results only*.

2	6	4	6	1	7	4	7	2	2	3	3	1	2	1
9	8	4	9	2	7	6	9	5	4	5	6	1	8	5

7	1	8	5	4	8	2	3	4	5	2	1	5	9	4
8	4	8	9	5	9	3	4	9	5	7	3	6	9	7

1	6	3	6	1	3	1	5	2	3	5	1	2	4	3
9	7	3	6	7	8	8	7	2	9	8	6	6	8	7

The following group of thirty-six combinations contains all the inversions possible, omitting the pairs of equal numbers. Practice until you can state the sums in any order without hesitation.

4	7	9	2	6	3	6	8	7	4	7	8
3	5	5	1	5	1	1	5	4	1	6	3

7	4	7	8	9	8	9	9	5	6	9	8
1	2	2	4	3	6	7	2	2	4	4	7

9	8	9	3	9	8	6	5	7	5	5	6
8	1	6	2	1	2	3	4	3	1	3	2

Following are the eighty-one combinations, including inversions, of two numbers, each of which is less than ten. Practice on this exercise until you can state the sums in less than a minute.

6	1	3	4	4	5	2	4	6	5	1	3	3	5	9	7	5
1	3	2	4	6	4	7	3	2	9	1	8	3	2	6	8	8

2	1	1	8	4	8	6	4	8	9	5	3	7	8	4	9	3
1	8	9	6	5	5	7	7	9	8	6	9	7	8	8	4	7

7	2	9	1	3	9	8	9	1	6	9	2	4	5	8	6	6
6	9	1	2	4	5	7	3	7	5	2	6	9	7	2	8	9

4	2	7	6	2	8	8	7	3	2	1	2	9	1	7	7	5
1	3	9	6	4	1	3	1	6	2	5	8	7	6	3	4	5

6	8	4	2	5	6	9	5	1	7	3	7	3
3	4	2	5	1	4	9	3	4	5	5	2	1

The following exercise contains the nine digits in groups of three. Practice on these combinations until you can state the sums in less than two and a half minutes.

8	8	7	5	8	4	5	8	9	6	7	5	4	4	6	8	5	4
6	8	3	5	3	2	5	6	7	5	5	3	3	4	3	2	4	3
2	2	3	2	2	2	3	3	2	3	3	2	3	3	3	2	2	2

3	5	3	6	6	4	5	7	6	9	7	8	6	7	9	8	6	6
2	2	3	3	6	4	4	7	4	8	2	7	2	3	2	7	6	5
2	2	2	2	2	2	4	2	3	2	2	2	2	2	2	1	3	4

7	8	9	8	9	8	7	7	9	9	6	8	7	9	8	9	5	7
7	8	6	5	4	8	4	5	4	3	5	3	6	9	4	8	5	4
3	3	2	3	2	1	2	2	3	2	2	3	2	1	2	3	4	4

8	8	9	9	9	7	5	9	9	8	9	7	8	8	9	6	7	6
4	6	9	3	6	4	3	6	8	8	6	7	5	7	5	4	7	6
4	1	2	3	1	3	3	3	1	4	4	1	4	6	2	2	4	5

6	6	7	8	9	9	9	8	7	8	9	9	5	7	9	9	8	9
4	6	7	7	7	7	5	5	6	8	7	8	4	6	8	8	5	9
4	4	6	4	5	7	3	5	5	7	6	5	3	3	6	8	2	8

7	9	8	9	9	9	7	7	8	8	9	8	9	9	9	8	7	8
5	6	6	9	5	9	6	5	4	6	9	8	5	7	4	7	6	7
5	5	5	5	5	7	6	4	3	4	4	6	4	1	4	3	4	4

8	9	9	8	9	6	9	9	7	8	9	9
7	9	8	6	7	5	6	9	7	8	8	7
5	6	7	6	4	5	6	3	5	5	4	3

Oral Work

Add the following. In this, and in all other work in addition, name *the sums only*. For example, in adding 3, 4, 7, 9, say 7, 14, 23.

1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**	**9.**	**10.**	**11.**	**12.**	**13.**	**14.**	**15.**	**16.**
3	5	7	2	9	1	6	8	5	4	9	3	5	5	7	9
7	3	9	1	5	3	7	5	6	9	8	9	3	9	3	8
2	8	3	5	7	1	4	9	6	4	6	7	9	4	9	3
6	3	8	1	4	2	7	8	9	6	7	8	7	8	2	7
6	3	7	5	2	9	5	1	7	4	8	4	1	9	8	5
7	8	5	3	9	1	4	7	6	4	3	9	8	7	9	9
8	4	9	5	7	3	5	8	5	9	9	6	9	8	9	8
9	2	5	3	7	4	8	5	9	6	6	1	6	3	5	6

Name the sums in each of the following. The first two numbers in each example should be thought of as one number. Thus, in the first example, think 10, 17.

1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**	**9.**	**10.**	**11.**	**12.**	**13.**	**14.**
7	5	9	1	2	5	8	2	4	6	3	7	5	9
2	4	1	6	7	4	1	7	3	2	8	4	3	9
8	4	6	9	2	9	5	7	3	8	5	3	7	5

15.	**16.**	**17.**	**18.**	**19.**	**20.**	**21.**	**22.**	**23.**	**24.**	**25.**	**26.**	**27.**	**28.**
8	5	3	7	2	5	6	9	6	8	5	7	8	4
6	5	3	8	4	7	4	7	8	5	7	4	8	5
4	6	2	8	3	5	6	5	7	3	6	9	3	6

29.	**30.**	**31.**	**32.**	**33.**	**34.**	**35.**	**36.**	**37.**	**38.**	**39.**	**40.**	**41.**	**42.**
7	7	4	9	5	7	4	6	8	3	6	5	7	9
6	7	3	6	9	5	8	3	6	7	4	5	7	3
7	4	8	5	3	7	3	8	5	4	2	6	9	4

2. Adding numbers by grouping will increase your speed. Most rapid computers use group addition. For example,

$$\begin{array}{r}\left.\begin{array}{r}8\\3\end{array}\right\}11\\\left.\begin{array}{r}4\\\underline{2}\end{array}\right\}6\\17\end{array}$$

Use this method to find the sums in the following exercise. This work should be done orally.

1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**	**9.**	**10.**	**11.**	**12.**	**13.**	**14.**
6	5	8	4	7	4	8	5	2	6	3	5	4	6
3	2	2	3	4	3	6	7	3	7	7	8	9	6
5	7	3	4	8	5	3	7	5	4	9	4	7	3
4	3	6	2	3	5	7	6	4	8	4	7	3	5

15.	**16.**	**17.**	**18.**	**19.**	**20.**	**21.**	**22.**	**23.**	**24.**	**25.**	**26.**	**27.**	**28.**
3	7	4	2	5	3	6	2	4	6	8	9	7	5
3	6	4	8	5	3	7	6	4	8	5	7	3	6
7	5	3	6	5	8	4	5	6	4	7	3	8	2
7	6	3	4	1	7	4	7	4	8	9	5	3	7

29.	**30.**	**31.**	**32.**	**33.**	**34.**	**35.**	**36.**	**37.**	**38.**	**39.**	**40.**	**41.**	**42.**
3	2	6	8	6	4	2	4	6	4	8	5	2	3
3	6	2	5	8	9	5	3	7	5	3	5	7	9
9	5	3	6	8	9	5	2	6	2	8	5	3	9
5	2	7	4	2	7	3	7	9	4	2	7	4	6

43.	**44.**	**45.**	**46.**	**47.**	**48.**	**49.**	**50.**	**51.**	**52.**	**53.**
4	8	9	5	3	2	6	8	4	3	6
3	6	8	4	2	6	7	4	3	6	8
7	4	6	3	7	8	5	3	6	7	9
4	7	5	2	8	6	4	8	3	7	9
2	6	4	7	9	6	4	2	6	7	5
7	4	8	3	9	5	9	3	6	7	5

54.	**55.**	**56.**	**57.**	**58.**	**59.**	**60.**	**61.**	**62.**	**63.**
7	3	2	8	5	3	7	3	7	5
5	3	6	8	7	4	2	6	8	3
4	5	6	4	7	3	8	4	2	8
5	2	3	5	7	8	5	3	6	7
4	7	6	4	8	9	2	6	3	7
2	3	6	7	8	4	2	5	6	3

Find the sums in examples 1–14 by the following method:

43
24 Think 43, 63, 67.
67

1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**	**9.**	**10.**	**11.**	**12.**	**13.**	**14.**
24	46	37	53	58	67	48	39	68	73	66	81	93	65
33	52	44	63	47	59	73	58	42	86	59	78	38	75

3. Recording Addition by Columns. For convenience in checking, the total of each column may be recorded separately. This will enable you to resume the work where it was discontinued if you are interrupted after having added one or more columns. Three slightly different methods are in use.

a.	*b.*	*c.*
369	369	369
487	487	487
352	352	352
896	896	896
294	294	294
28	20	**28**
37	37	**39**
20	28	**23**
2398	2398	

Illustrations (*a*) and (*b*) show the method of adding each column *without carrying* the tens. The column totals must be added to obtain the final result. Illustration (*c*) shows the method of adding to each column the amount carried from the column at the right.

Written Work

Make the figures plain and easily legible and place units of the same order in the same vertical column.

Copy and find the sum of:

1.	**2.**	**3.**	**4.**	**5.**
5823	84239	387935	9328577	38421
4923	42937	512937	4912374	31238
9382	31629	491327	5823746	312
5128	49238	395825	4923748	273845
3258	83153	416239	7239482	39547
7362	42841	482423	1423849	315288
8127	52384	923857	8237421	4234
4239	94856	412831	9423748	21582

6.	**7.**	**8.**	**9.**	**10.**	**11.**
27	371	419	372	97	996
83	927	82	496	36	37
95	482	92	18	547	321
84	413	412	739	969	493
82	958	892	847	38	782
15	285	9216	96	437	946
81	381	594	382	692	531
48	294	5	75	99	28
79	148	592	989	831	327

Each student should prepare original examples for addition, following the instruction of the teacher. Exchange papers, criticize the form, and find the sums.

The ability to give and to take dictation of numbers should be developed. Numbers may be dictated by the teacher, or by members of the class.

Written Work

The following table shows the weekly sales report made by a number of salesmen during a certain week.

Find the total sales made by each salesman during the week, and enter these totals in the proper places at the bottom of the table.

Find the total sales made each day, and enter these totals in the column at the right.

Find the total sales made by all salesmen during the week.

The grand total should be found in two ways: by adding the totals at the foot of the blank, and those at the right. If these two grand totals agree, you may assume that the additions are correct; if they do not agree, you should find the error.

DAY	J.C OLSEN		D.R.BACON		F.G.BATES		D.O.ESTEY		TOTAL	
MONDAY	216	45	538	74	816	98	319	47		
TUESDAY	358	93	619	84	725	48	629	47		
WEDNESDAY	412	57	593	27	639	74	725	84		
THURSDAY	385	13	634	96	724	47	594	72		
FRIDAY	518	92	718	96	639	37	584	38		
SATURDAY	639	47	639	62	729	47	846	94		
TOTAL										

4. Courtis Standards. S. A. Courtis has determined certain standards of achievement in addition, subtraction, multiplication, division, and the copying of figures.

His test in addition includes twenty-four examples similar to the two which follow.

127	996
375	320
953	778
333	886
325	913
911	164
554	897
167	972
554	119

Of the thousands of students examined by means of the Courtis tests about 5% are able to obtain the correct result to twenty-four such examples in addition in eight minutes or less.

NOTE. Information in regard to the Courtis Standards and Tests may be obtained by addressing S. A. Courtis, Detroit, Michigan.

5. How to Rule a Blank Form. Ruling the forms for your written work will furnish you excellent practice in the use of pen, ink, and ruler. This practice will be valuable as a preparation for bookkeeping in school or in a business office. Follow the instructions given below:

a. Make the columns just wide enough for the figures, — about seven figures to an inch. Remember that the total may have one or two more figures than any of the addends (numbers added).

b. Make the horizontal lines parallel and about $\frac{1}{4}$ of an inch apart.

c. Make the spaces for headings and totals $\frac{3}{8}$ of an inch deep.

d. Try to place the form as nearly as possible in the middle of the paper.

6. How to Enter Statistics. Much of the clerk's work in a business office consists of entering statistics on ruled forms.

a. Always write figures on the line.

b. Point off figures in groups of three.

c. Make small figures; they look better and are more easily read than large ones.

d. Keep the figures close to the right side of the column.

e. Keep the columns vertical: units above units, tens above tens, etc.

f. Write the figures at equal distances from each other.

Written Work

1. The following table shows the delivery records of a store running five wagons. A record of each day's deliveries is kept and the week's totals are found. If the summary shows that any one wagon is required to make an unreasonably large number of deliveries, a change in the routes may be necessary. By finding the total number of deliveries made each week, the average cost per delivery may be computed.

DAY	WAGON NO 1	WAGON NO 2	WAGON NO 3	WAGON NO 4	WAGON NO 5	TOTAL
MONDAY	243	319	273	143	241	
TUESDAY	216	326	391	178	216	
WEDNESDAY	257	287	294	121	322	
THURSDAY	301	291	341	147	296	
FRIDAY	294	304	303	218	213	
SATURDAY	318	361	420	223	301	
TOTAL						G.T.

Rule a blank form similar to this model; copy the statistics, and find:

a. The total number of deliveries made during the week by each wagon.

b. The total number of deliveries made each day.

c. The total number of deliveries for the week.

2. Rule a blank form similar to the first of the tables on the opposite page, copy the statistics, and find the totals indicated.

3. Referring to the second of the tables on the opposite page, what was the value of each crop in each of the geographical divisions, and in the entire United States?

Rule a blank in ink and indicate these values as totals.

WEEKLY SALES REPORT BY DEPARTMENTS

DEPT. No.	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY	TOTAL
1	$932.87	$823.75	$239.75	$724.16	$295.16	$840.19	
2	829.47	584.92	395.58	645.97	396.74	492.10	
3	723.85	486.47	523.78	419.52	296.15	492.58	
4	836.47	594.38	723.91	391.16	749.15	581.07	
5	486.74	432.95	312.85	492.43	723.85	239.57	
6	728.74	483.96	472.58	439.17	396.14	211.64	
7	385.75	539.59	542.87	734.67	723.96	447.92	
8	648.57	823.85	514.29	439.76	824.57	238.11	
9	923.37	239.73	629.85	329.54	385.47	294.10	
10	473.48	865.94	518.59	172.39	749.80	663.19	
Total							Grand Total

VALUE OF THE WHEAT, CORN, AND OAT CROPS IN THE VARIOUS STATES

(In Thousands of Dollars)

STATE AND DIVISION	CORN	WHEAT	OATS
Maine	480	72	2,347
New Hampshire	794		225
Vermont	1,296	24	1,589
Massachusetts	1,629		128
Rhode Island	401		26
Connecticut	2,310		166
New York	13,834	5,306	15,420
New Jersey	7,054	1,433	814
Pennsylvania	38,797	21,204	14,915
N. Atlantic			
Delaware	3,381	1,864	55
Maryland	13,450	8,536	608
Virginia	33,739	8,682	2,020
West Virginia	15,928	3,412	1,461
N. Carolina	42,418	5,907	2,352
S. Carolina	29,136	865	4,598
Georgia	45,864	1,498	4,921
Florida	6,727		518
S. Atlantic			
Ohio	78,484	9,565	30,782
Indiana	83,733	9,374	23,940
Illinois	174,791	8,641	54,818
Michigan	31,492	6,720	17,103
Wisconsin	29,714	2,958	27,119
N. C. E. Miss. R.			
Minnesota	28,925	48,938	31,962
Iowa	151,207	10,023	58,811
Missouri	112,196	21,375	12,994
North Dakota	3,766	99,236	20,948
South Dakota	28,248	36,008	13,098
Nebraska	67,568	37,985	16,653
Kansas	69,690	68,295	19,264
N. C. W. Miss. R.			
Kentucky	60,192	6,791	1,775
Tennessee	53,862	7,077	2,632
Alabama	42,802	359	3,224
Mississippi	40,356	93	1,180
Louisiana	22,093		361
Texas	98,112	10,253	13,390
Oklahoma	41,770	15,072	7,988
Arkansas	33,828	884	1,741
S. Central			
Montana	428	12,381	7,997
Wyoming	236	1,745	3,171
Colorado	4,368	8,006	4,717
New Mexico	1,562	1,109	828
Arizona	528	778	188
Utah	202	4,544	2,069
Nevada	29	1,137	208
Idaho	276	9,613	5,956
Washington	651	36,535	5,476
Oregon	472	15,132	5,623
California	1,635	5,850	4,290
Far Western			
United States			

NOTE. Different classes, or different groups of students of the same class, should be held responsible for computing the desired sums for different geographical divisions.

Checking Addition by Casting Out Nines

7. Excess of Nines. The remainder after dividing any number by nine is called the *Excess of Nines.* If the number 47 is divided by 9, the excess of nines is 2. The excess of nines in 30 is 3, because when 30 is divided by 9, there is a remainder of 3. When we "cast out nines" from a number, we divide the number by 9, and indicate the excess.

What is the excess of nines in 17? 14? 39? 45? 117? 23?

8. Method of Checking. If two numbers, each exactly divisible by 9, are added, their sum also is divisible by 9. If one number with an excess of 3 is added to a number with an excess of 2, the sum will have an excess of 5. *The excess of nines in a sum is equal to the excess in the sum of the excesses of the numbers added.*

Examples. Add 30 and 19; check the result by casting out nines.

Solution: $30 + 19 = 49$.
The excess of nines in the sum, 49, is 4.
30 has an excess of 3;
19 has an excess of 1.
The sum of the excesses of the numbers added is 4, therefore the addition checks.

For convenience, the work may be arranged as follows:

	Numbers Added	Excesses
a.	30	3
	19	1
Sum	49	4
Excess of sum	4	

	Numbers Added	Excesses
b.	23	5
	13	4
Sum	36	9 or 0
Excess of sum	0	

	Numbers Added	Excesses
c.	49	4
	34	7
Sum	83	11 or 2
Excess of sum	2	

9. Short Method of Finding the Excess. The excess of nines in any number may be found by the following method :

Add the digits composing the number. If the sum is composed of two or more digits add them. Continue this procedure until a result of one digit is secured.

Examples. 1. What is the excess of nines in 25 ?

Solution : $2 + 5 = 7$, the excess.

2. What is the excess of nines in 328 ?

Solution : $3 + 2 + 8 = 13$

$1 + 3 = 4$, the excess.

The addition of digits to find the excess will be further simplified by observing the following suggestions :

a. Ignore 9's, and combinations which add to 9 or multiples of 9.

Example. What is the excess of nines in 9457 ?

Solution : Ignore 9 and $5 + 4$; the excess is 7.

b. When the addition results in a number of two digits, add the digits and proceed as before.

Example. What is the excess of nines in 7528 ?

Solution : $7 + 5 = 12$; $1 + 2 = 3$;

$3 + 2 + 8 = 13$; $1 + 3 = 4$, the excess.

10. Limitation of the Method. Casting out nines is not an absolute check for any process. It will not disclose an error of 9 or any multiple of nine, neither will it disclose an interchange of digits, such as 763 for 673.

Written Work

1. Add and check :

Numbers to be Added	Excesses
213	?
412	?
617	?
362	?
145	?
215	?
829	?
657	?
923	?
?	?

2. Rule a suitable blank form for the following : At the right of each column of figures rule a narrow column in which to indicate the excesses.

Find the totals as indicated ; check all additions by casting out nines.

Value of Farm Lands, Buildings, and Implements in the United States

Division	Land	Ex.	Buildings	Ex.	Implements	Ex.	Total	Ex.
New England .	382,134,424		336,410,384		50,798,826			
Middle Atlantic	1,462,321,005		980,628,098		167,480,384			
E. N. Central .	7,231,699,114		1,642,292,480		268,806,550			
W. N. Central .	10,052,560,913		1,562,104,957		368,635,544			
South Atlantic .	1,883,349,675		603,086,799		98,230,147			
E. S. Central .	1,326,826,864		411,570,975		75,339,333			
W. S. Central .	2,716,098,530		412,498,352		119,720,377			
Mountain . . .	1,174,370,096		145,026,777		49,429,975			
Pacific	2,246,313,548		231,832,706		66,408,647			
U. S. Totals .							Grand Total	

3. The following table shows a method used by department stores to determine the cost of the purchases made for various departments during the month. For example, Invoice No. 1, purchased on January 2, included goods for the six departments of the store. Invoice No. 2 included goods for only three departments.

The total amount of the invoice may be found by horizontal addition.

The total purchases for each department during the month may be found by vertical addition.

Rule a blank suitable for this material, enter the data, and find:

a. The total of each invoice.

b. The total purchases for each department during the month.

c. The total purchases for all departments during the month.

Check all additions by casting out nines.

INVOICE NO.	DATE		DEPT. 1		DEPT. 2		DEPT. 3		DEPT. 4		DEPT. 5		DEPT. 6		TOTAL	
1	Jan.	2	129	30	340	60	290	39	360	89	240	30	376	80		
2	Jan.	3	276	90			347	80					174	44		
3	Jan.	3			295	67	298	87	350	27	289	67	287	57		
4	Jan.	5	246	32	238	47	350	27	347	25	250	68	276	57		
5	Jan.	6	231	53	350	21			378	89			190	23		
6	Jan.	8	212	23	325	67	345	67	367	78	310	57	185	67		
7	Jan.	8	227	85			346	78	357	89	298	67				
8	Jan.	10											170	67		
9	Jan.	11	160	87	325	67	287	67	367	57	287	00	189	00		
10	Jan.	15	170	97	341	28	321	67	378	98	287	89				
11	Jan.	15	180	89			333	33			278	90	234	86		
12	Jan.	15			345	67	345	67	328	98	300	00	350	90		
13	Jan.	16	203	25	356	78			378	68						
14	Jan.	17	204	67			367	78	376	56	267	87	347	52		
15	Jan.	19			290	87	356	87			278	65	378	67		
16	Jan.	23			297	76	347	89	398	67	256	28	250	00		
17	Jan.	24	234	56					388	88						
18	Jan.	25	245	67	287	98	367	34			256	78	234	56		
19	Jan.	28			229	39	323	23	376	29	234	56	167	89		
20	Jan.	31	236	78	310	21	289	98	387	85	245	67	245	67		
Total																

CHAPTER II

SUBTRACTION

11. Drill Tables. Daily practice on the following combinations will increase your speed and accuracy in the process of subtraction.

Drill on these combinations until you can state all of the results in less than thirty seconds.

9	8	7	9	6	7	4	7	3	6	8	7	8	4	5	8
2	1	5	3	1	2	4	4	3	6	4	3	8	2	1	3

2	7	9	5	9	5	6	5	9	4	4	9	8	2	6	8
1	7	4	5	5	4	3	2	7	4	3	8	5	2	4	6

5	8	4	8	6	7	7	9	8	1	3	3	5	7	9	6
3	8	1	6	2	1	6	1	2	1	2	1	5	7	6	5

Drill on the following until you can state all of the results in less than thirty seconds.

16	14	15	18	13	17	12	11	16	11	12	17	11
8	6	8	9	4	9	5	9	7	6	6	8	4

13	12	15	16	11	12	12	12	11	13	15	11	12
5	3	6	9	3	3	8	7	7	9	7	8	9

11	12	13	14	14	13	14	15	14	11	13
5	4	7	8	7	6	5	9	9	2	8

When performing subtraction, one is frequently obliged to "borrow" as in the following.

Copy these examples, and make the subtractions.

1.	2.	3.	4.	5.	6.
479	536	4279	14973	3084	12379
284	198	3682	8297	1596	3084

7.	8.	9.	10.	11.
42937	946320	37649	947302	794680
39048	298452	20863	842943	409694

12. Checking Subtraction. Subtraction may be checked by either of the following methods:

(*a*) Difference + Subtrahend = Minuend.

(*b*) Minuend − Difference = Subtrahend.

Example. Subtract 21 from 78.

SOLUTION.	78 Minuend	CHECK. *a.*	57 Difference	*b.*	78 Minuend
	21 Subtrahend		21 Subtrahend		57 Difference
	57 Difference		78 Minuend		21 Subtrahend

Written Work

Perform the subtractions indicated; check the first five by method (*a*), and the last five by method (*b*),

1.	2.	3.	4.	5.
489	7932	82578	63217	49216
392	494	58214	41283	21953

6.	7.	8.	9.	10.
58310	40513	53821	79608	184732
49168	39458	41927	20973	75806

11. The following data show the sales in the various departments of a large store for the month of August, 1914, and the month of August, 1915.

Rule a blank similar to the following:

COMPARATIVE SALES RECORD

DEPARTMENT NO.	AUGUST, 1914	AUGUST, 1915	INCREASE OR DECREASE
1	$ 5,483.85	$ 5,834.75	
2	7,239.74	8,238.74	
3	15,493.67	14,835.92	
4	9,834.56	10,385.47	
5	23,842.12	22,147.94	
6	15,858.43	16,614.34	
7	6,395.75	6,114.32	
8	19,432.86	22,324.73	
9	9,445.87	10,302.95	
10	43,221.42	46,932.56	
11	29,496.56	31,416.47	
12	18,853.16	17,542.85	
13	22,542.19	25,436.83	
14	25,746.91	24,427.85	
15	18,422.16	21,541.86	
Total			

Enter the sales in the proper column of the blank.

Find the increase or decrease in the amount of the sales in each department for August, 1915, over the sales in the same department for August, 1914.

Enter increases in black ink; decreases in red ink.

Find the net increase for the entire store.

12. The following shows the total population of the United States for the years indicated.

1790 — 3,929,214; 1800 — 5,308,483; 1810 — 7,239,881;
1820 — 9,638,453; 1830 — 12,866,020; 1840 — 17,069,453;
1850 — 23,191,876; 1860 — 31,443,321; 1870 — 38,558,371;
1880 — 50,155,783; 1890 — 62,947,714; 1900 — 75,994,575;
1910 — 91,972,266.

Rule a blank form to show the year the census was taken, the population each census year, and the increase in population for each interval of 10 years.

See how simple you can make this blank.

13. Rule a form, enter the following statistics, find the gross profit of each department, and the gross profit of the entire store.

GROSS PROFIT OF A DEPARTMENT STORE

DEPT. NO.	SALES	COST OF GOODS SOLD	GROSS PROFIT
1	$5,629.80	$4,984.37	
2	7,358.92	6,295.16	
3	4,916.09	4,192.86	
4	7,329.16	6,593.54	
5	10,609.15	9,835.17	
6	6,123.18	5,902.52	
7	7,212.47	6,275.43	
8	9,475.37	8,594.48	
9	4,238.16	3,725.91	
10	5,824.78	5,014.78	
Total			

14. Rule a suitable form, enter the following statistics, and find:

a. The gross profit by departments.

b. The gross profit of the entire store (two ways).

c. The net profit or net loss by departments; enter net profit in black, net loss in red.

d. Net profit or net loss of the entire store (two ways).

DEPT. NO.	SALES	COST OF GOODS SOLD	GROSS PROFIT	EXPENSES	NET PROFIT OR LOSS
1	$7816.40	$6715.32		$423.85	
2	9317.42	8492.17		530.08	
3	6842.56	5914.72		731.96	
4	7319.62	6593.57		671.42	
5	8295.17	7942.17		456.72	
6	5732.88	5101.59		322.75	
7	9514.82	8899.55		693.15	
8	2289.74	1856.80		216.77	
9	5793.66	5135.60		788.51	
10	9559.38	8625.50		523.80	

13. Subtracting by Adding Complements. A series of additions and subtractions may be performed by the method of adding complements.

The difference between any number and the next higher power of 10 is called the *complement* of the number. Thus, the complement of 7 is 3; the complement of 6 is 4; the complement of 83 is 17.

If, instead of subtracting a number less than ten from a given number, its complement be added, the result will be 10 too large. Thus, $13 - 6 = 7$ or $13 + 4 = 17$ (a result 10 too large).

If, instead of subtracting *two* numbers less than ten from a given number, their complements are added, the result will be 20 too large; if the complements of three such numbers are added, the result will be 30 too large, etc.

Examples. 1. $29 - 6 - 7 = ?$

SOLUTION. $29 + 4 + 3 = 36$. Since two complements were added, the result is 20 too large. Therefore, subtract 20, leaving 16.

2. $38 - 4 - 7 - 8 = ?$

SOLUTION. $38 + 6 + 3 + 2 = 49$. Since three complements were added, subtract 30, leaving 19.

3. $46 - 8 + 4 - 7 = ?$

SOLUTION. $46 + 2 + 4 + 3 = 55$. Since two complements were added, 20 must be subtracted, leaving 35.

The practical value of this method will be shown by solving the following examples.

1. $36 - 24 + 19 - 12 + 21 - 13 - 6 = ?$

$$\begin{array}{r} 36 \\ -\,24 \\ 19 \\ -\,12 \\ 21 \\ -\,13 \\ -\quad 6 \\ \hline 21 \end{array}$$

SOLUTION. By combining as indicated in the units' column, beginning at the top (complements are marked with an *) we have

$$6 + 6^{*} + 9 + 8^{*} + 1 + 7^{*} + 4^{*} = 41$$

Since four complements were added, the result is 40 too large. Therefore write 1 and drop the 4.

By combining as indicated in the tens' column we have

$$3 + 8^{*} + 1 + 9^{*} + 2 + 9^{*} = 32$$

Since three complements were added, the result is 30 too large. Write 2 and drop the 3.

Result, 21.

2. $985 + 234 - 126 - 34 - 125 - 174 + 386 = ?$

$$\begin{array}{r} 985 \\ 234 \\ -126 \\ -\ \ 34 \\ -125 \\ -174 \\ 386 \\ \hline 1146 \end{array}$$

Solution. Combining in the units' column, beginning at the top we have

$$5 + 4 + 4^* + 6^* + 5^* + 6^* + 6 = 36$$

Write the 6. Since four complements were added, the result is 40 too large. Therefore we must drop the 3 in 36, and also subtract 1 from the 8 at the top of the tens' column.

Combining in the tens' column we have

$$7 + 3 + 8^* + 7^* + 8^* + 3^* + 8 = 44$$

Since four complements were added, we must deduct 40, leaving 4 which is written in the tens' column of the result.

Combining in the hundreds' column we have

$$9 + 2 + 9^* + 9^* + 9^* + 3 = 41$$

Three complements were added, the result is therefore 30 too large. Subtract 30 and write 11.

Result, 1146.

Oral Work

Find the results by the addition of complements :

1. $46 - 4 - 7 =$

2. $72 - 8 - 6 - 3 =$

3. $39 - 7 + 3 - 9 =$

4, $48 - 17 - 6 =$

5. $81 - 23 - 42 =$

6. $49 - 22 + 15 - 11 =$

7. $85 - 26 - 35 + 62 - 16 =$

8. $643 - 289 + 364 =$

9. $781 - 247 + 64 =$

10. $1046 - 987 + 649 =$

11. $943 - 876 + 629 =$

12. $1349 - 268 + 421 =$

13. $1264 - 34 + 1321 =$

14. $1789 - 347 + 736 =$

15.	**16.**	**17.**
2346	1932	923,578
+ 1267	+ 9845	− 5,284
− 321	− 932	+ 28,956
+ 6964	+ 2122	− 123,749
− 1235	− 2375	+ 83,219
+ 367	− 23	− 259,734
− 2960	+ 692	+ 125,982
− 985	− 1243	− 429,764

Written Work

1. The following model shows the ruling of the ledger in which banks keep accounts with their depositors. Deposits are added to the balance of the previous day, and checks are subtracted, to find the new balance to the credit of the depositor's account.

Find the daily balances:

WILLIAM HATFIELD

DATE	DEPOSITS	CHECKS	BALANCE
June 1			$328.57
	$ 85.68	$ 14.25	?
2	125.80	65.90	
		12.73	?
3	245.85	127.89	
		114.56	
		35.87	?
4	319.45	38.51	
	26.82	102.50	?
5		95.90	
		141.66	
		95.68	?
6	450.00	132.88	
		6.20	?
8	139.75	216.45	
		39.46	
		19.99	
		213.55	
		66.82	?

2. The following blank shows a convenient method of keeping a record of cash received and paid by a small business.

Rule a blank similar to the model and enter the statistics.

DATE	CASH RECEIVED		CASH PAID			DAILY CASH
	Cash Sales	On Account	Purchases	On Account	Expenses	Balance
Feb. 1						$216.27
1	$215.78	$124.35	$ 80.75	$ 68.55	$ 8.75	?
2	294.80	95.88	157.47	113.25	3.73	?
3	188.47	148.23	95.48	316.57	27.49	?
4	218.43	49.47	17.49	42.39	9.12	?
5	388.92	112.67	221.81	76.29	75.84	?
6	478.07	290.04	88.12	448.14	6.90	?
	?	?	?	?	?	?

a. Find the daily cash balances.

To the balance of the preceding day add the receipts from cash sales and receipts on account, and subtract the various amounts listed under "Cash Paid."

Thus, $216.27 + 215.78 + 124.35 – 80.75 – 68.55 – 8.75 = ?

b. Find the totals for each column.

3. Add upward; subtract across:

a. 7,463 – 2,847 =
5,928 – 3,804 =
9,604 – 2,870 =
3,962 – 1,436 =
9,287 – 5,426 =
– =

b. 174,638 – 94,273 =
38,270 – 27,409 =
70,563 – 2,879 =
924,360 – 14,287 =
7,503 – 2,769 =
– =

CHAPTER III

MULTIPLICATION

ACCURACY and speed in multiplication depend largely upon a thorough mastery of the multiplication tables. The student should thoroughly review the tables previously learned and should continue with daily drills on combinations up to 25 times 25.

14. Drill Tables.

Multiply across; add upward:

1.	2.	3.	4.
74 × 436 =	83 × 423 =	25 × 624 =	35 × 624 =
74 × 523 =	83 × 157 =	25 × 726 =	35 × 706 =
74 × 287 =	83 × 284 =	25 × 37 =	35 × 753 =
74 × 492 =	83 × 307 =	25 × 869 =	35 × 496 =
74 × ? =	83 × 596 =	25 × 493 =	35 × 548 =
	83 × ? =	25 × 468 =	35 × 784 =
		25 × ? =	35 × ? =

Oral Work

Use 2, 3, 4, 5, 6, 7, 8, and 9 as multipliers. Name the results for each column in less than 20 seconds.

1.	2.	3.	4.	5.
3	18	5	4	9
5	10	8	9	11
7	4	12	15	16
14	7	15	20	30
11	9	7	17	25
6	15	30	8	14
8	16	22	13	17
9	20	16	18	11
15	12	25	22	16

Written Work

1. A factory made an investigation of the number of articles of a certain kind manufactured by each of its employees. It found that:

9 men produced 46 articles each.
9 men produced 48 articles each.
11 men produced 53 articles each.
15 men produced 55 articles each.
18 men produced 59 articles each.
23 men produced 61 articles each.
25 men produced 62 articles each.
25 men produced 63 articles each.
21 men produced 64 articles each.
17 men produced 65 articles each.
17 men produced 66 articles each.
14 men produced 68 articles each.
12 men produced 69 articles each.
8 men produced 70 articles each.

Rule a form with a heading similar to the following:

Production Record

Number of Men	Number of Articles Made by Each	Total

Enter the statistics, find the number of articles produced by each group of employees, and the total number of articles produced by all of the employees.

2. Nine workmen were employed in the manufacture of different articles, and were paid a certain number of cents for each piece completed. Complete the following table, finding the wages earned by each workman.

DAILY PIECEWORK LABOR COST

WORKMAN No.	NUMBER OF ARTICLES MADE	WAGE RATE PER PIECE	WAGES EARNED
1	17	$ 0.27	
2	19	.22	
3	38	.16	
4	79	.06	
5	32	.15	
6	28	.18	
7	64	.07	
8	81	.05	
9	29	.21	

15. Checking Multiplication. Multiplication may be checked by several methods. The following methods are commonly used.

a. Repeating the multiplication and assuming that if the same product is obtained the work is correct. This is not a reliable check because an error may be repeated.

b. Dividing the product by the multiplier to obtain the multiplicand, or by the multiplicand to obtain the multiplier.

c. Casting out nines.

16. Casting Out Nines. The method is as follows: *Find the excess of nines in the multiplicand and in the multiplier. Find the product of these excesses. Find the excess of nines in this product. It should equal the excess of nines in the result.*

Example. Multiply 23 by 16; check by casting out nines.

SOLUTION. 23 5 = the excess of nines in 23.
16 7 = the excess of nines in 16.
368 35 = the product of these excesses.
CHECK. 8 = the excess in this product.
The excess of nines in 368 is also 8. The multiplication, therefore, checks.

Without performing the multiplications determine the probable correctness of the following products, by means of casting out nines:

1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**
25	82	36	286	4172	344	733
38	35	87	37	39	281	492
950	2870	3132	10,382	162,698	96,664	362,636

8.	9.	10.
398	43,962	34,276
241	47,835	21,578
95,918	2,102,922,270	738,627,528

Written Work

Multiply and check by casting out nines :

11.	12.	13.	14.	15.	16.
347	279	627	132,879	63,154	78,293,567
861	439	123	642,378	9,837	20,417,839

17. The following table gives information regarding the corn crop in the United States in a recent year.

CORN PRODUCTION IN THE UNITED STATES

STATES	NUMBER OF THOUSANDS OF ACRES	AVERAGE YIELD PER ACRE	AVERAGE PRICE PER BUSHEL IN CENTS
North Atlantic			
Maine	16	40.0	75
New Hampshire	23	46.0	75
Vermont	45	40.0	75
Massachusetts	47	45.0	77
Rhode Island	11	41.5	86
Connecticut	60	50.0	77
New York	512	38.6	70
New Jersey	273	38.0	68
Pennsylvania	1,499	42.5	63
South Atlantic			
Delaware	195	34.0	51
Maryland	670	36.5	55
Virginia	1,980	24.0	71
West Virginia	725	33.8	65
North Carolina	2,808	18.2	83
South Carolina	1,915	17.9	85
Georgia	3,910	13.8	85
Florida	655	13.0	79
North Central, East of Miss.			
Ohio	4,075	42.8	45
Indiana	4,947	40.3	42
Illinois	10,658	40.0	41
Michigan	1,625	34.0	57
Wisconsin	1,632	35.7	51

Corn Production in the United States — *Continued*

States	Number of Thousands of Acres	Average Yield per Acre	Average Price per Bushel in Cents
North Central, West of Miss.			
Minnesota	2,266	34.5	37
Iowa	10,047	43.0	37
Missouri	7,622	32.0	46
North Dakota	328	26.7	43
South Dakota	2,495	30.6	37
Nebraska	7,609	24.0	31
Kansas	7,575	23.0	40
South Central			
Kentucky	3,600	30.4	55
Tennessee	3,332	26.5	61
Alabama	3,150	17.2	79
Mississippi	3,106	18.3	71
Louisiana	1,805	18.0	68
Texas	7,300	21.0	64
Oklahoma	5,448	18.7	41
Arkansas	2,475	20.4	67
Far Western			
Montana	24	25.5	70
Wyoming	16	23.0	64
Colorado	420	20.8	50
New Mexico	93	22.4	75
Arizona	19	33.0	100
Utah	9	30.0	75
Nevada	1	30.0	97
Idaho	12	32.8	70
Washington	31	27.3	77
Oregon	20	31.5	75
California	52	37.0	81

Prepare a blank similar to the model.

States	Acres		Average Yield per Acre		Total Bushels Produced		Average Price per Bushel		Total Crop Value		
		Ex.		Ex.		Ex.		Ex.	$	¢	Ex.

a. Find the total number of bushels of corn produced in each state.

b. Find the total value of the corn crop of each state.

Different classes or various groups of a class should make the required computations for assigned geographical divisions of the country. In checking the results, the "excesses" should be placed in the columns marked "Ex."

Many interesting comparisons may be made from the data of the preceding table. For illustration: Name the five states which produced the most corn and compare the yields in these states. What geographical section of the country produced the largest corn crop? What relation, if any, is there between the size of the corn crop in the various states and the average price per bushel in the various states?

SHORT METHODS OF MULTIPLICATION

These short methods will be found to be very practical. Master two or three of them thoroughly before taking up others. Use those that you have mastered whenever you have opportunity to do so. It is not necessary that all of these short methods be mastered before the succeeding chapters are studied.

17. To multiply by 10, 100, 1000, 10000, etc.

a. When the multiplicand is an integer. *Annex to the multiplicand as many zeros as there are zeros in the multiplier.*

Thus, to multiply an integer by 10, annex one zero; to multiply by 100, annex two zeros.

Examples. 1. $37 \times 10 = 370$. 2. $29 \times 100 = 2900$.

b. When the multiplicand is a decimal fraction. *Move the decimal point as many places to the right as there are zeros in the multiplier.*

It may be necessary to annex zeros in order to move the decimal point the desired number of places.

Examples. 1. Multiply .1357 by 100.

SOLUTION. Move the decimal point two places to the right, 13.57.

2. Multiply 32.46 by 1000.

SOLUTION. In order to move the decimal point three places to the right, it is necessary to annex one zero, giving, as a result, 32,460.

Oral Work

Multiply as indicated :

1. 37,946 × 100.
2. 5293 × 10,000.
3. 639 × 100,000.
4. 120 × 1000.
5. .376 × 1000.
6. 1.349 × 1000.
7. 27.9637 × 100.
8. .000932 ÷ 10.
9. .00873 × 1000.
10. .7032 × 1000.
11. 3.69 × 1000.
12. .0027 × 100,000.
13. 1427.834 × 10,000.
14. 625.086 × 1000.

18. To multiply numbers ending with zeros.

a. When both numbers are integers. *Multiply the numbers represented by the significant figures. To the product thus obtained, annex as many zeros as there are final zeros in both the multiplicand and multiplier.*

Example. Multiply 3400 by 1200.

SOLUTION. 34 × 12 = 408. Annexing four zeros, we obtain the product 4,080,000.

Perform the following multiplications. Whenever possible, do the work orally.

1. 169 × 300.
2. 210 × 300.
3. 4567 × 700.
4. 1390 × 1200.
5. 2300 × 1500.
6. 19,000 × 16.
7. 3194 × 23,000.
8. 420 × 3400.

b. When one of the numbers is a decimal fraction. *Multiply the numbers represented by the significant figures. Move the decimal point as many places to the right in the product as there are final zeros in the integer.* (This may necessitate annexing zeros.)

Examples.

1. Multiply .486 by 300.

SOLUTION. 3 × .486 = 1.458.
Move the decimal point two places to the right, the result is 145.8.

2. Multiply 3.2 by 400.

SOLUTION. $4 \times 3.2 = 12.8$.
Move the decimal point two places to the right, the result is 1280.

Written Work

Perform the following multiplications:

1. .47 × 200. **2.** 3.786 × 4000.
3. 17.682 × 500. **4.** .0746 × 3000.
5. .072 × 6000. **6.** .382 × 1200.
7. .0837 × 14000. **8.** .042 × 170.
9. .0036 × 1500. **10.** 4.26 × 7000.

19. To multiply by 9, 99, 999, etc.

a. To multiply by 9. *Annex one zero to the number to be multiplied, thus multiplying it by* 10; *from this result subtract the number to be multiplied.*

Example. Multiply 346 by 9.

SOLUTION.

$$\begin{array}{r} 3460 \\ 346 \\ \hline 3114 \end{array}$$

b. To multiply by 99. *Annex two zeros to the number to be multiplied, thus multiplying it by* 100; *from this result subtract the number to be multiplied.*

Example. Multiply 293 by 99.

SOLUTION.

$$\begin{array}{r} 29300 \\ 293 \\ \hline 29007 \end{array}$$

Written Work

Multiply each of the following numbers by 9, 99, and 999:

1. 632. **2.** 748. **3.** 135. **4.** 737. **5.** 427. **6.** 166.

20. To multiply by numbers slightly smaller than 10, 100, 1000, 10,000, etc.

A modification of the short method explained in the preceding section may be used to multiply by numbers slightly smaller than 10, 100, 1000, 10,000, etc.

To multiply by 98. *Annex two zeros to the number to be multiplied and from this result subtract twice the number to be multiplied.*

How can the short method be used if you are to multiply by 97, 96, 95, or 998?

Written Work

Apply short methods to the following examples:

1. 675 × 96.
2. 350 × 95.
3. 535 × 91.
4. 687 × 97.
5. 94 × 3.4.
6. 995 × 82.
7. 634 × 994.
8. .48 × 997.
9. 23 × 99.
10. 48 × 997.
11. 12 × 988.
12. 47 × 998.

21. To multiply by 11.

a. When the multiplicand contains two digits.

Place between these two digits, their sum.

Example. $34 \times 11 = 374$.

When the sum of the two digits is 10 or more, 1 must be carried to the digit at the left.

Example. $68 \times 11 = 748$.

Written Work

Multiply each of the following numbers by 11:

1. 27.
2. 63.
3. 93.
4. 74.
5. 26.
6. 35.
7. 22.
8. 87.
9. 28.
10. 46.
11. 75.
12. 96.
13. 37.
14. 36.
15. 57.
16. 85.
17. 98.
18. 72.

b. When the multiplicand contains three or more digits:

The units' digit of the multiplicand is the units' digit of the product; the sum of the units' and tens' digits is the tens' digit of the product; the sum of the tens' and hundreds' digits is the hundreds' digit of the product, etc.

Whenever the sum of two digits is ten or more, 1 must be carried.

Examples. 1. Multiply 793 by 11.

SOLUTION. 3 (the units' digit of the multiplicand) becomes the units' digit of the product.

$$\begin{array}{rl} 9+3=12 & \text{(carry the 1)} \\ 1+7+9=17 & \text{(carry the 1)} \\ 1+7=8 & \\ \hline 8723 & \end{array}$$

2. Multiply 52,635 by 11.

SOLUTION. 52,635 × 11 = 578,985.

Written Work

Multiply each of the following by 11:

1. 363.
2. 271.
3. 823.
4. 456.
5. 3742.
6. 876,394.
7. 3,578,962.
8. 34,579.
9. 263,789.
10. 123,496,287.

22. To multiply by 111.

362,941 × 111 = ?

When the multiplication is performed in the customary manner, the multiplicand is repeated as follows:

$$\begin{array}{r} 362941 \\ 362941 \\ \underline{362941} \end{array}$$

When the short method is applied, *the units' digit of the multiplicand is the units' digit of the product; the sum of the units' and tens' digits of the multiplicand is the tens' digit of the product; the sum of the units', tens', and hundreds' digits of the multiplicand is the hundreds' digit of the product; the sum of the tens', hundreds', and thousands' digits of the multiplicand is the thousands' digit of the product, etc.*

The excess above 10 is always to be carried to the next sum.

Written Work

Without recopying, write the products obtained by multiplying each of the following by 111:

1. 729,361.
2. 124,396.
3. 1,793,862.
4. 5,374.
5. 235,692.
6. 8,354,927.

23. To multiply two numbers ending in 5.

a. When the sum of the digits at the left of the 5's is an even number.

Multiply the digits at the left of the 5's; to this product add one half the sum of these digits; to this result annex 25.

Examples. 1. Multiply 65 by 25.

SOLUTION. $2 \times 6 = 12$, the product of the digits at the left of the 5's.
$\frac{1}{2}$ of $(2 + 6) = \underline{4}$
16
1625, result obtained by annexing 25.

2. Multiply 625 by 445.

SOLUTION. $62 \times 44 = 2728$
$\frac{1}{2}$ of $(62 + 44) = \underline{53}$
2781
278125, result obtained by annexing 25.

Written Work

Multiply as indicated:

1. 35×75.
2. 95×75.
3. 35×55.
4. 25×45.
5. 325×45.
6. 725×65.
7. 835×175.
8. 145×165.
9. 195×115.
10. 225×185.

b. When the sum of the digits at the left of the 5's is an odd number.

Multiply the digits at the left of the 5's; to this product add one half the sum of these digits, dropping the ½; *to this result annex* 75.

Examples. 1. Multiply 75 by 45.

Solution. $4 \times 7 = 28$, the product of the digits at the left of the 5's.

$$\begin{array}{rr} \tfrac{1}{2} \text{ of } (4+7) = & \underline{5\tfrac{1}{2}} \\ & 33\tfrac{1}{2} \end{array}$$

3375, the result obtained by dropping the fraction ½, and annexing 75.

2. Multiply 325 by 475.

Solution.

$$\begin{array}{rl} 32 \times 47 = & 1504 \\ \tfrac{1}{2} \text{ of } (32+47) = & \underline{\quad 39\tfrac{1}{2}}, \\ & 1543, \text{ adding and dropping the } \tfrac{1}{2}. \\ & 154{,}375, \text{ result secured by annexing } 75. \end{array}$$

Written Work

Multiply as indicated:

1. 75 × 65.
2. 125 × 135.
3. 95 × 85.
4. 145 × 175.
5. 225 × 75.
6. 165 × 135.
7. 145 × 215.
8. 435 × 125.

24. To multiply two numbers, when certain digits of the multiplier are contained an integral number of times in other digits of the multiplier.

Multiply 224 by 279.

Since 9 is contained 3 times in 27, first multiply 224 by 9, then multiply this product by 3.

Thus,

$$\begin{array}{rl} 224 & \\ \underline{279} & \\ 2016 & = 9 \times 224 \\ \underline{6048} & = 3 \times 2016 \\ 62496 & \end{array}$$

When using this method, be careful to place the product of the second multiplication in the proper position.

Example. 1. Multiply 341 by 618.

$$\begin{array}{rl} 341 & \\ \underline{618} & \\ 2046 & = 6 \times 341 \\ \underline{6138} & = 3 \times 2046 \\ 210738 & \end{array}$$

Solution. Since 18 is a multiple of 6, multiply first by 6. Place the right-hand figure of the product, 2046, directly under the 6 of the multiplier. The product of 18 × 341 can now be obtained by multiplying 2046 by 3. The right-hand figure of this product, 6138, is placed directly under the 8 of the multiplier.

Written Work

Multiply the following, stating by what numbers you multiplied in order to take advantage of the short method:

1. 468 × 243. **2.** 1,235 × 981. **3.** 719 × 427.
4. 687 × 654. **5.** 739 × 848. **6.** 7,362 × 1,248.
7. 1,247 × 1,864. **8.** 146,387 × 315. **9.** 1,235 × 819.

25. The **supplement** of a number is the difference between the number and the next lower power of 10. The supplement of 15 is 5; the supplement of 134 is 34; the supplement of 1042 is 42.

26. To multiply two numbers each of which is a little larger than 100.

To either of the numbers add the supplement of the other; to this sum annex the product of the supplements.

Example. Multiply 131 by 103.

SOLUTION. 131 + 3 = 134
(or 103 + 31 = 134).
Annex 93 (31 × 3).
The result is 13,493.

The same rule may be applied to numbers a little larger than 1000.

Example. Multiply 1,062 by 1,006.

$$\begin{array}{rr} 1062 + 6 = & 1068 \\ 62 \times 6 = & \underline{\quad 372} \\ & 1068372 \end{array}$$

SOLUTION.

NOTE. When the supplements are based on 100 and the product of the supplements is a number of only one digit, a zero must be put in tens' place. For example: 102 × 103 = 10,506. Similarly, when the supplements are based on 1000 and the product of the supplements is a number of less than three digits, zeros must be put in the proper places.

Written Work

1. 104 × 120. **2.** 127 × 102. **3.** 113 × 106.
4. 114 × 105. **5.** 114 × 106. **6.** 109 × 106.
7. 126 × 107. **8.** 109 × 111. **9.** 1,007 × 1,003.

10. $1{,}009 \times 1{,}012$.

11. $1{,}214 \times 1{,}006$.

12. $1{,}206 \times 1{,}012$.

13. $1{,}112 \times 1{,}006$.

14. $1{,}416 \times 1{,}009$.

15. $1{,}374 \times 1{,}005$.

16. $1{,}674 \times 1{,}012$.

27. The **complement** of a number is the difference between the next higher power of 10 and the number. The complement of 92 is 8; the complement of 89 is 11; the complement of 996 is 4.

28. To multiply two numbers both slightly less than 100.

From either number subtract the complement of the other number. To this result annex the product of the complements.

Example. Multiply 96 by 93.

SOLUTION. $96 - 7 = 89$ (or $93 - 4 = 89$).
$4 \times 7 = 28$ (product of the complements).

Annex 28 to 89 and the result is 8928.

NOTE. When the complements are based on 100 and the product of the complements is less than 10, a zero must be put in tens' place. For illustration: $98 \times 97 = 9506$. Similarly, the tens' and hundreds' places must be filled when the complements are based on 1000.

Written Work

Multiply as indicated. Perform orally as much of this work as possible.

1. 98×95.

2. 97×84.

3. 87×91.

4. 95×80.

5. 93×85.

6. 87×97.

7. 88×87.

8. 91×92.

9. 86×94.

The same method may be applied to numbers slightly less than 1000.

Example. Multiply 996 by 987.

SOLUTION. $996 - 13$ (the complement of 987) $= 983$.
$4 \times 13 = 52$ (the product of the complements).
Since the complements are based on 1000 and the product of the complements is a number of only two digits, a zero must be put in hundreds' place. The result is 983,052.

Multiply:

Written Work

1. 994	**2.** 996	**3.** 983	**4.** 987
992	984	991	992
5. 991	**6.** 983	**7.** 978	**8.** 981
981	982	983	988

29. To multiply any two numbers in the *teens*.

To either of the numbers add the units' digit of the other number and annex a zero.

To this result add the product of the units' digits.

Example. Multiply 15 by 17.

Solution. 15 + 7 = 22; annex a zero, 220
35
255

Oral Work

Multiply:

1. 17	**2.** 13	**3.** 16	**4.** 17	**5.** 14	**6.** 19
12	12	18	13	13	15
7. 13	**8.** 17	**9.** 18	**10.** 13	**11.** 17	**12.** 15
16	19	12	19	16	17

CHAPTER IV

DIVISION

Oral Work

30. Short Division.

1. Divide by 2 : 16, 28, 76, 248, 368, 926, 1,054.

2. Divide by 3 : 27, 36, 57, 75, 417, 732, 873.

3. Divide by 4 : 32, 48, 64, 72, 96, 196, 384, 748.

4. Divide by 5 : 75, 95, 145, 545, 725, 965, 1,025, 1,370.

5. Divide by 6 : 72, 96, 126, 366, 528, 732, 1,044.

6. Divide by 7 : 56, 91, 147, 203, 476, 924, 1,575.

7. Divide by 8 : 72, 96, 144, 360, 424, 792, 1,240.

8. Divide by 9 : 54, 98, 171, 243, 378, 567, 981.

State the remainder when each of the following numbers is divided by 3, 4, 5, 7, 8, and 9.

9.	**10.**	**11.**	**12.**	**13.**
473	9,846	2,638	4,020	19,206
692	3,723	7,284	7,302	47,308
876	9,264	3,047	7,960	23,074
479	3,749	2,636	8,403	95,306

14. How many pounds of meat can be bought for $1.26 at 18¢ per pound?

15. How many weeks in 91 days? 175 days? 266 days?

16. The dividend is 176 and the quotient is 8. What is the divisor?

17. The divisor is 9, the quotient is 7, and the remainder is 3. What is the dividend?

18. Six dozen oranges were bought for $1.62. What was the price per dozen?

19. How many yards in 186 feet? 924 feet? 5280 feet?

20. A grocer buys 9 dozen eggs for $2.07. What is the cost per dozen?

21. If you are to discharge a debt of $1.14 in 6 equal weekly payments, how much must you pay each week?

31. Long Division.

Examples. 1. Divide 4,625 by 37. 2. Divide 47,285 by 327.

SOLUTION.

$$\begin{array}{rr}
 & 125 \\
37 & \overline{)4625} \\
 & \underline{37} \\
 & 92 \\
 & \underline{74} \\
 & 185 \\
 & \underline{185}
\end{array}$$

SOLUTION.

$$\begin{array}{rr}
 & 144\frac{197}{327} \\
327 & \overline{)47285} \\
 & \underline{327} \\
 & 1458 \\
 & \underline{1308} \\
 & 1505 \\
 & \underline{1308} \\
 & 197
\end{array}$$

(The quotient should be written above the dividend in long division.)

32. Checks. Division may be checked by several methods. The method stated below is easily understood and readily applied.

Multiply the quotient by the divisor and add the remainder to this product to obtain the dividend.

Written Work

Perform the following divisions and check each result:

1. 1,758 ÷ 144.

2. 15,762 ÷ 37.

3. 73,627 ÷ 425.

4. 78,264 ÷ 738.

5. 508,573 ÷ 97.

6. 66,816 ÷ 928.

7. 41,228 ÷ 44.

8. 2,235,812 ÷ 284.

9. Divide the sum of 478, 392, 648, 971, and 1483 by 27.

10. Divide the sum of 3024, 4763, 8297, 9468, and 293 by 38.

11. Complete:

$$\begin{array}{r}
37 \times 29 = \phantom{\underline{\qquad\qquad}} \\
42 \times 29 = \phantom{\underline{\qquad\qquad}} \\
36 \times 29 = \phantom{\underline{\qquad\qquad}} \\
94 \times 29 = \phantom{\underline{\qquad\qquad}} \\
402 \times 29 = \underline{\qquad\qquad} \\
\text{Total } = \phantom{\underline{\qquad\qquad}}
\end{array}$$

12. The total circulation of a certain daily newspaper for 23 consecutive days was 524,423 copies. What was the average daily circulation?

13. At $2.18 a yard, how many yards of cloth can be bought for $37.06?

14. The dividend is 14,286, the quotient is 18, and the remainder is 12. What is the divisor?

15. The product of two numbers is 135,468. One of the numbers is 426. What is the other?

16. In a factory 437 men produce 5681 articles in a week. What is the average weekly production for each employee?

17. At the rate of 43 miles an hour, how long will it take a train to run 473 miles?

18. Into how many states as large as Rhode Island (1067 sq. mi.) could Texas (262,398 sq. mi.) be divided?

19. The expenses for 5 months for a family of 5 amounted to $258 for food, $170 for rent, $216 for clothes, $147 for amusements and other expenses. What was the average expense a month for the family and the average per month for each person?

33. Pointing Off the Quotient. When it is necessary to continue a division to tenths or hundredths, place a decimal point to the right of the units' figure of the dividend and the quotient, annex zeros to the dividend, and continue the process.

Example. Divide 2916 by 26.

SOLUTION.

$$\begin{array}{r|l} & 112.1 \\ \hline \underline{26} & 2916.0 \end{array}$$

Written Work

The following table gives the area of each of the states in square miles, and also the population of each state.

Review the instructions given on page 9 for ruling a blank form and for entering statistics.

Prepare a form similar to the following:

State and Division	Land Area in Square Miles	Population 1910	Population per Square Mile
New England			
Maine	29,895	742,371	
New Hampshire	9,031	430,572	
Vermont	9,124	355,956	
Massachusetts	8,039	3,366,416	
Rhode Island	1,067	542,610	
Connecticut	4,820	1,114,756	
Middle Atlantic			
New York	47,654	9,113,614	
New Jersey	7,514	2,537,167	
Pennsylvania	44,832	7,665,111	
East North Central			
Ohio	40,740	4,767,121	
Indiana	36,045	2,700,876	
Illinois	56,043	5,638,591	
Michigan	57,480	2,810,173	
Wisconsin	55,256	2,333,860	
West North Central			
Minnesota	80,858	2,075,708	
Iowa	55,586	2,224,771	
Missouri	68,727	3,293,335	
North Dakota	70,183	577,056	
South Dakota	76,868	583,888	
Nebraska	76,808	1,192,214	
Kansas	81,774	1,690,949	
South Atlantic			
Delaware	1,965	202,322	
Maryland	9,941	1,295,346	
D. C.	60	331,069	
Virginia	40,262	2,061,612	
West Virginia	24,022	1,221,119	
North Carolina	48,740	2,206,287	
South Carolina	30,495	1,515,400	
Georgia	58,725	2,609,121	
Florida	54,861	752,619	
East South Central			
Kentucky	40,181	2,289,905	
Tennessee	41,687	2,184,789	
Alabama	51,279	2,138,093	
Mississippi	46,362	1,797,114	

State and Division	Land Area in Square Miles	Population 1910	Population per Square Mile
West South Central			
Arkansas	52,525	1,574,449	
Louisiana	45,409	1,678,339	
Oklahoma	69,414	1,657,155	
Texas	262,398	3,896,542	
Mountain			
Montana	146,201	376,053	
Idaho	83,354	325,594	
Wyoming	97,594	145,965	
Colorado	103,658	799,024	
New Mexico	122,503	327,301	
Arizona	113,810	204,354	
Utah	82,184	373,351	
Nevada	109,821	81,875	
Pacific			
Washington	66,836	1,141,990	
Oregon	95,607	672,765	
California	155,652	2,377,549	

Note. The class may be divided into groups and each group made responsible for a geographical division.

Complete as much of the table as the teacher assigns, showing the population per square mile of each state, of each section, and of the United States.

Carry results to one decimal place.

Exercises for Drill

I. ESTIMATED TIME 7 MINUTES

1. Add:

a.	*b.*	*c.*	*d.*	*e.*	*f.*	*g.*	*h.*
27	14	29	73	29	83	72	94
36	33	43	99	48	92	86	55
92	27	38	26	73	17	93	76
23	19	64	10	27	64	14	87
49	86	27	18	65	20	27	31
85	35	85	35	42	78	38	49

2. Multiply:

a.	*b.*	*c.*
4736	9463	7427
3	5	3

3. Subtract:

a.	*b.*	*c.*	*d.*
72049	4706	32964	92807
32608	2985	7193	1968

4. Add across and down:

a.	*b.*
7 + 3 + 9 + 2 =	24 + 7 + 6 =
5 + 4 + 8 + 3 =	13 + 5 + 3 =
4 + 5 + 1 + 9 =	17 + 4 + 7 =
7 + 6 + 5 + 1 =	9 + 15 + 3 =
1 + 9 + 3 + 4 =	18 + 9 + 5 =
8 + 1 + 2 + 5 =	13 + 2 + 1 =
3 + 2 + 7 + 8 =	7 + 19 + 8 =
9 + 7 + 4 + 6 =	15 + 3 + 9 =
+ + + =	+ + =

II. Estimated Time 9 Minutes

1. Add across and down:

741 + 437 + 563 + 234
526 + 318 + 725 + 238
382 + 746 + 128 + 407
387 + 209 + 504 + 916
328 + 460 + 937 + 659
473 + 295 + 243 + 175
+ + +

2. Multiply:

a.	*b.*	*c.*	*d.*
7460	9047	739	7036
38	29	86	43

3.

143 × 6 =
427 × 5 =
793 × 8 =
429 × 7 =
384 × 3 =
827 × 6 =
538 × 9 =

4. Divide:

a.
326 |6846

b.
22 | 6996

III. ESTIMATED TIME 8 MINUTES

Add across and down :

7	9	7	3	5	8	1	8
4	2	5	9	2	4	7	3
3	8	3	6	4	5	3	6
6	4	7	7	8	9	8	9
2	3	9	4	7	7	6	7
9	5	8	8	9	6	4	2
8	7	6	5	3	3	5	4
4	6	4	1	1	4	9	8
7	1	9	2	2	1	2	5
6	9	2	8	6	2	7	1

Subtract :

a.
74036
21809

b.
92407
38795

c.
769082
432767

Multiply :

a.
3742
37

b.
9427
45

c.
3897
28

Divide :

a.
6|748632

b.
8|938424

IV. ESTIMATED TIME 8 MINUTES

1. Add:

a.	*b.*	*c.*	*d.*	*e.*	*f.*	*g.*	*h.*
37	24	33	51	26	72	43	18
24	13	15	32	39	46	75	37
36	43	31	44	72	92	76	11
15	25	17	65	43	54	28	26
13	17	65	23	86	28	35	34
11	38	22	25	54	63	54	82

2. Subtract:

a.	*b.*	*c.*	*d.*
4372	7246	9264	3164
2439	3817	5728	1476

3. Add from left to right:

a.	*b.*
$7 + 3 + 2 + 4 =$	$6 + 9 + 4 + 8 =$
$3 + 7 + 5 + 6 =$	$13 + 17 + 19 + 12 =$
$4 + 8 + 9 + 1 =$	$15 + 13 + 12 + 16 =$
$3 + 7 + 2 + 4 =$	$5 + 7 + 8 + 6 =$
$8 + 3 + 7 + 5 =$	$13 + 27 + 32 + 17 =$
$9 + 3 + 2 + 8 =$	$48 + 27 + 3 + 2 =$
$5 + 7 + 3 + 8 =$	$7 + 9 + 2 + 3 =$

4. Multiply:

a.	*b.*	*c.*	*d.*
3724	4923	6728	3107
2	3	2	3

V. ESTIMATED TIME 12 MINUTES

1. Divide:

a.	*b.*	*c.*
$5\underline{)742385}$	$8\underline{)943824}$	$9\underline{)736848}$

2. Multiply:

a.	*b.*	*c.*
1437	1943	1048
246	632	453

3. Subtract:

a.	*b.*	*c.*	*d.*
472,387	940,360	472,863	796,048
29,389	397,685	30,940	397,059

4. Add across and down:

$$\begin{array}{ccccccccl}
943 & + & 790 & + & 408 & + & 941 & = & \\
276 & + & 463 & + & 312 & + & 806 & = & \\
841 & + & 287 & + & 637 & + & 420 & = & \\
732 & + & 934 & + & 924 & + & 387 & = & \\
\underline{286} & + & \underline{742} & + & \underline{836} & + & \underline{209} & = & \underline{\qquad} \\
 & + & & + & & + & & = &
\end{array}$$

5. Divide:

a.	*b.*
$423\overline{)13536}$	$91\overline{)46228}$

Computing Machines of various kinds are generally used by banks and other business houses. Machines on which addition, subtraction, multiplication, and division may be performed are quite common.

CHAPTER V

AVERAGE

34. Simple Average. A boy walked 9 miles on Monday, 5 miles on Tuesday, and 10 miles on Wednesday. If he had divided the trip into three equal distances, how many miles would he have walked each day? In other words, what was the average distance walked each day?

A man owned a five-acre field. One acre yielded 71 bushels of potatoes, another 77 bushels, a third 85 bushels, the fourth 93 bushels, and the fifth 64 bushels. What was the total crop? What was the average crop per acre?

What two processes are usually involved in computing an average?

Written Work

1. The noon temperatures in a certain city for a week were as follows: Monday, 62°; Tuesday, 70°; Wednesday, 74°; Thursday, 68°; Friday, 62°; Saturday, 70°; Sunday, 76°. What was the average noon temperature for the week?

2. A newsdealer sold papers as indicated in the table. Find the average number of each sold.

	MON.	TUES.	WED.	THURS.	FRI.	SAT.	AVG.
Sentinel . . .	39	47	63	82	74	67	
Argus	13	19	27	31	29	26	
Tribune . . .	39	42	48	51	39	41	
Recorder . . .	28	43	56	29	37	29	

3. The following table gives the prices received by farmers in certain states for the butter sold during the months of a recent year. Find the average price received in each state during the year.

BUTTER PRICES IN CERTAIN STATES

	CENTS PER POUND												AVERAGE PRICE BY STATES
	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	
Massachusetts	37	38	36	35	33	34	31	33	34	34	32	35	
Rhode Island	35	39	39	34	34	36	32	32	35	34	34	34	
Connecticut	36	39	38	35	34	35	33	34	34	36	34	36	
New York	34	35	32	31	31	30	27	29	29	30	32	35	
New Jersey	37	40	35	34	34	34	32	32	33	32	34	36	
Pennsylvania	33	35	33	31	31	29	26	27	28	30	32	34	
Maryland	28	29	29	28	25	25	25	25	26	28	29	28	
Virginia	25	26	26	26	25	23	21	22	22	24	26	26	
West Virginia	26	26	26	26	26	22	21	21	22	24	25	27	
Georgia	25	25	28	24	24	24	24	24	24	25	25	26	
Ohio	27	28	27	25	25	24	22	23	24	25	27	29	
Indiana	25	26	25	24	24	22	21	22	22	24	25	27	
Illinois	27	28	26	25	25	24	24	23	24	26	26	28	
Michigan	30	31	28	27	27	25	23	23	24	25	27	29	
Wisconsin	33	34	28	28	29	26	25	25	26	27	28	31	
Minnesota	31	32	29	27	27	27	24	24	25	26	28	30	
Iowa	29	30	27	26	26	25	24	24	24	25	27	29	
Missouri	23	23	23	23	23	22	21	21	21	22	23	24	
Nebraska	26	26	24	24	23	22	21	21	22	23	25	27	
Kansas	26	26	25	24	24	22	21	22	22	24	25	26	
Kentucky	21	22	21	21	21	21	19	19	18	20	20	23	
Tennessee	21	22	21	20	20	19	18	18	18	19	20	22	
Louisiana	28	30	28	27	28	27	26	27	27	27	28	30	
Texas	22	25	23	22	21	21	21	20	21	23	23	24	
Oklahoma	27	25	23	22	22	22	20	20	19	23	24	25	
Montana	36	37	35	33	31	31	30	29	31	31	32	35	
Colorado	33	33	30	30	28	28	26	26	28	28	32	31	
Utah	33	31	30	29	31	30	29	27	28	30	32	32	
Idaho	35	33	32	32	31	28	28	27	29	31	32	34	
Washington	36	37	32	32	30	28	28	30	30	32	33	35	
Oregon	34	35	33	32	31	26	28	28	30	30	35	36	
California	34	36	34	32	30	29	29	31	31	33	34	36	

4. The following table gives the acreage planted to certain crops in the United States, and the crops raised. Compute the average yield per acre. Carry your results to two decimal places. Show the statistics on a ruled form.

ACREAGE AND PRODUCTION OF CROPS

CROP	1911			1912		
	1000 Acres	1000 Bushels	Bushels per Acre	1000 Acres	1000 Bushels	Bushels per Acre
Corn . .	105,825	2,531,488		107,083	3,124,746	
Wheat . .	49,543	621,338		45,814	730,267	
Oats . .	37,763	922,298		37,917	1,418,337	
Rye . . .	2,127	33,119		2,117	35,664	
Potatoes .	3,619	292,737		3,711	420,647	

5. The following table shows the number of cases of eggs shipped to seven leading markets in the United States.

RECEIPTS OF EGGS AT SEVEN LEADING MARKETS IN THE UNITED STATES 1906–1912

YEAR	BOSTON	CHICAGO	CINCINNATI	MILWAUKEE	NEW YORK	ST. LOUIS	SAN FRANCISCO	TOTAL
1906	1,709,531	3,583,878	484,208	187,561	3,981,013	1,023,125	137,074	
1907	1,594,576	4,780,356	588,636	176,826	4,262,153	1,288,977	379,429	
1908	1,436,786	4,569,014	441,072	207,558	3,703,990	1,439,868	347,436	
1909	1,417,397	4,557,906	519,652	160,418	3,903,867	1,395,987	340,185	
1910	1,431,686	4,844,045	511,519	169,448	4,380,777	1,375,638	469,698	
Av. 1906–1910								
1911	1,441,748	4,707,335	605,131	175,270	5,021,757	1,736,915	587,115	
1912	1,580,106	4,556,643	668,942	136,621	4,723,558	1,391,611	638,920	
Av. 1911–1912								

Find (1) the total receipts of eggs each year, (2) the average receipts at each city for the periods indicated.

6. A subscription was taken to secure funds to purchase a gift.

3 men gave $1.00 each
1 man gave .75
2 men gave 1.50 each
1 man gave .60
4 men gave .90 each

Find the average amount given.

7. The following table shows the number of men employed and the total weekly wages in each of the four departments of a factory. Find the average wage in each department and the average wage for the four departments.

Department	Number of Men Employed	Total Wage	Average Wage
A	47	$ 960	
B	18	290	
C	62	1054	
D	26	318	

Oral Review

1. What is the difference in the meaning of the following statements?

a. Each of the 300 employees in our factory earns $2.50 per day.

b. The average daily wages of employees in our factory is $2.50 per day.

2. What would you have to know and how would you proceed to find:

a. The average weight of twenty boxes?

b. The average value of a herd of cattle?

c. The average number of miles a train traveled per hour, going from Chicago to St. Louis?

d. The average age of the students in your class?

e. The average daily sales of a clerk?

3. What could you find if you were told:

a. The average value of farm land per acre in your state, and the number of acres of farm land?

b. The average daily sales of a clerk during the twenty-five week days of June?

c. The average monthly grocery bill of your family?

4. If you were told:

a. The daily circulation of the *Evening Herald* for the twenty-six week days of July, and the average daily circulation of the *Evening Transcript* for the same month, how could you compare their circulation?

b. The average daily wages of A, who worked 265 days last year and the average daily wages of B, who worked 303 days, how could you find the yearly wages of each, and which one earned the larger amount?

c. The difference between the average daily outputs of two shoe factories, how could you find the difference in their production for a year of 300 working days?

5. What would you have to know and what would you do to find the average daily speed of an ocean liner on a given voyage?

6. Several boys worked for a farmer picking strawberries. One earned $5.00 more than the average of the other boys' earnings. What else must you know and what would you do to find the total earnings of all the boys? What else would you have to know and how would you find the average wage cost of picking a quart of strawberries?

Written Work

1. A merchant kept a record of the deliveries of goods made to his customers for a week. The record follows:

DAY	DELIVERIES
Monday	213
Tuesday	187
Wednesday	208
Thursday	221
Friday	168
Saturday	251

a. What was the average number of deliveries per day?

b. The expense of running the delivery wagons for a week, including care of horses, interest on the money invested in horses and wagons, repairs and wages, was $64.75. What was the average cost per delivery?

2. Five clerks in a store sold the following amounts of goods during a month:

A.	$1246.50
B.	1076.85
C.	944.90
D.	1388.20
E.	1109.75

a. What was the average amount of sales per clerk?

b. Which clerks sold more than the average? Which clerks sold less than the average?

3. The distance from Chicago to Aurora, Illinois, via the C. B. & Q. Railroad, is 37.4 miles. Train No. 55 makes no stops between these stations. It leaves Chicago at 6:10 P.M. and arrives at Aurora at 7:10 P.M. How many miles an hour does this train travel? What fractional part of a mile does it run in one minute?

4. The single fare between these two stations is 74 cents. Ten-trip tickets may be purchased for $6.25. What is the saving in fare per trip?

5. A 25-trip ticket may be purchased for $13.00. What is the average cost per trip?

6. The following table shows the number and value of pianos and organs manufactured in the United States in 1904 and 1909.

PIANOS	1909	1904
Number	261,197	374,154
Value	$41,476,479	$59,501,225
ORGANS		
Number	113,065	64,111
Value	$4,162,053	$2,595,429

What was the average value of a piano manufactured in each of the years?

What was the average value of an organ manufactured in each of the years?

7. In 1900 the population of the United States was, in round numbers, 77 million. The combined daily circulation of all daily newspapers was about 15 million copies; an average of 1 copy of a daily paper to every ------- persons. In 1910 the population had increased to 93 million, and the circulation of daily newpapers to 24 million, or an average of one copy for every -------- persons.

8. During the year 1913 the United States Congress appropriated $1,098,678,788 for the expenses of the government. The last preceding census showed a population of 93,402,151. What was the average governmental expenditure per person on this basis?

CHAPTER VI

FACTORS AND MULTIPLES

35. Terms. An **integer** is a number of whole units.

The **factors** of a number are the integers which, multiplied together, produce the given number. Thus, the factors of 15 are 3 and 5; the factors of 18 are 3 and 6, or 2 and 9.

A **factor** of a number is a divisor of that number.

A number which is not exactly divisible by any other number (except 1) is called a **prime number**. Thus, 1, 3, 5, 7, 11, and 13 are prime numbers.

Numbers are said to be **prime to each other** when they have no common factor except 1. Thus, 10 and 27 are **prime to each other**, although neither is a prime number.

36. Test of Divisibility of Numbers. A number is divisible by:

a. Two, if it ends with 0, 2, 4, 6, or 8.

b. Three, if the sum of its digits is divisible by 3.

c. Four, if the number expressed by its last two digits is divisible by 4.

d. Five, if it ends in 0 or 5.

e. Six, if it is divisible by both 2 and 3.

f. Eight, if the number expressed by its last three digits is divisible by 8.

g. Nine, if the sum of its digits is divisible by 9.

h. Ten, if its right-hand digit is zero.

i. Eleven, if the difference between the sums of the numbers represented by the odd and even orders of digits is divisible by 11. Thus, 16,280 is divisible by 11, since $(8+6)-(0+2+1)$ is divisible by 11.

(There is no simple method of testing divisibility by 7.)

37. Factoring is the process of separating a number into its factors.

Oral Work

1. Learn the prime numbers from 1 to 100 so that you can recognize them at sight.

2. Apply the tests of divisibility to the following.

Find the *prime* factors of :

28
42
72
36
98
160
320
48
360
280
728
640
386
31
100
478
96
84
92
81
76
84
90
360
760
720
37
145
390
625

3. What numbers between 161 and 200 are divisible by 9 ?

4. What numbers between 746 and 800 are divisible by 6 ?

5. Name the factors of 36 which are not prime to each other.

38. Cancellation is the process of shortening certain computations involving division by removing or canceling equal factors from both dividend and divisor.

Example. Divide the product of 4, 9, 8, 36, 24, and 7 by the product of 18, 2, 8, 3, 14, and 4.

SOLUTION.

$$\frac{\cancel{4}\times\cancel{9}\times\cancel{8}\times\overset{18}{\cancel{36}}\times\overset{\overset{2}{\cancel{12}}}{\cancel{24}}\times\cancel{7}}{\underset{\cancel{2}}{\cancel{18}}\times\cancel{2}\times\cancel{8}\times\cancel{3}\times\underset{\cancel{2}}{\cancel{14}}\times\cancel{4}}=36.$$

In all computations involving only multiplication and division, cancellation should be used when possible. Indicate the multiplication and division as in the illustration above, then cancel the common factors.

Written Work

(Use cancellation when possible.)

1. $\dfrac{24\times 36\times 15}{16\times 5\times 9}$.

2. $\dfrac{4\times 37\times 16\times 5\times 60}{48\times 32\times 74}$.

3. $\dfrac{48\times 32\times 100\times 360}{16\times 50\times 72}$.

4. $\dfrac{27\times 64\times 96\times 38}{19\times 16\times 9\times 2}$.

5. $\dfrac{130 \times 14 \times 18 \times 121 \times 15}{7 \times 27 \times 13 \times 11}$.

6. $\dfrac{144 \times 32 \times 63 \times 7}{16 \times 9 \times 28}$.

7. $\dfrac{1728 \times 360 \times 100 \times 32 \times 3}{18 \times 144 \times 64 \times 75}$.

8. $\dfrac{21 \times 72 \times 160 \times 340 \times 27}{180 \times 36 \times 35}$.

39. Greatest Common Divisor. An integer that is a factor of two or more numbers is called a **common divisor**, or a **common factor** of those numbers.

The **greatest common divisor** of two or more numbers is the greatest factor common to the numbers. "Greatest common divisor" is usually expressed as g. c. d.

Example. Find the greatest common divisor of 12, 20, and 36.

SOLUTION.

$$12 = 2 \times 2 \times 3$$
$$20 = 2 \times 2 \times 5$$
$$36 = 2 \times 2 \times 3 \times 3.$$

The factor 2 occurs twice in all the numbers and none of the other factors occurs in all the numbers, hence, 4 is the g. c. d. of 12, 20, and 36.

To find the g. c. d. of two or more numbers, *separate the numbers into prime factors and find the product of the prime factors common to the numbers.*

Written Work

Find the g. c. d. of:

1. 12, 18, 24.
2. 24, 60, 72.
3. 15, 20, 30.
4. 60, 90, 100.
5. 84, 32, 60.
6. 60, 96, 120.
7. 18, 32, 48.
8. 27, 36, 45.
9. 360, 120, 40.
10. 121, 88, 242.
11. Find the g. c. d. of 8 ft. and 12 ft.
12. Find the g. c. d. of $48 and $60.

40. Least Common Multiple. A **multiple** of a number is an integral number of times that number. Thus, 28 is a multiple of 7. 60 is a multiple of 12.

A **common multiple** of two or more numbers is a number that is a multiple of each of them. It is therefore divisible by each of them.

The **least common multiple** of two or more numbers is the least number that is a multiple of each of them.

Thus, 60 is the least common multiple of 12, 15, and 30.

Example. Find the least common multiple (l. c. m.) of 18, 20, and 24.

SOLUTION.

$$\begin{aligned} 18 &= 2 \times 3 \times 3 \\ 20 &= 2 \times 2 \times 5 \\ 24 &= 2 \times 2 \times 2 \times 3. \\ \text{l. c. m.} &= 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360. \end{aligned}$$

To find the least common multiple of two or more numbers, *separate each number into its prime factors. Find the product of these factors, using each factor the greatest number of times it occurs in any one of the given numbers.*

Written Work

Find the l. c. m. of the following:

1. 12, 15.
2. 8, 12.
3. 6, 15.
4. 7, 8, 12.
5. 8, 9, 12, 15.
6. 24, 36, 60.
7. 8, 12, 16.
8. 36, 24, 75.
9. 360, 345.
10. 75, 130, 190.
11. 425, 345, 336.
12. 360, 240, 420.

Find the g. c. d. and the l. c. m. of the following:

13. 60, 80, 95.
14. 36, 75, 48.
15. 480, 360, 120.

CHAPTER VII

COMMON FRACTIONS

41. Terms. A **fraction** is one or more equal parts of a unit.

A **common fraction** is usually expressed by writing one figure above and one below a short line; thus, $\frac{2}{3}$.

The **numerator** of a fraction is the number which shows how many of the equal parts of the unit are taken. It is written above the line.

The **denominator** of a fraction is the number which shows into how many equal parts the unit is divided. It is written below the line.

The numerator and the denominator are called the **terms** of the fraction. Thus, 3 and 4 are the terms of the fraction $\frac{3}{4}$.

A common fraction may be either proper or improper.

A **proper fraction** is one whose numerator is less than its denominator, as $\frac{3}{4}$.

An **improper fraction** is one whose numerator is equal to or greater than its denominator, as $\frac{4}{4}$ or $\frac{5}{4}$.

A **mixed number** consists of a whole number and a fraction, as $5\frac{2}{3}$.

Oral Work

Which is greater, $\frac{1}{4}$ or $\frac{1}{7}$? $\frac{1}{9}$ or $\frac{1}{10}$? $\frac{1}{20}$ or $\frac{1}{12}$?

How is the value of a fraction affected by increasing the numerator, the denominator remaining the same?

REDUCTION OF FRACTIONS

42. Reducing Fractions to Lower Terms. When the numerator and the denominator contain one or more common factors, the fraction may be reduced to a fraction of equivalent value expressed in lower terms.

Thus, $\frac{10}{12}$ may be reduced to the equivalent fraction $\frac{5}{6}$, by dividing both terms by 2. Similarly, $\frac{18}{24} = \frac{3}{4}$; $\frac{7}{21} = \frac{1}{3}$.

State a rule for reducing fractions to lower terms.

43. Reducing Fractions to Higher Terms. Fractions may be raised to equivalent fractions in higher terms, *by multiplying both the numerator and the denominator by the same number.*

Example. Express $\frac{3}{5}$ as an equivalent fraction whose denominator is 60.

SOLUTION. The given denominator, 5, must be multiplied by 12 to obtain the desired denominator, 60. Therefore, multiply both terms of the fraction by 12, and obtain the equivalent fraction $\frac{36}{60}$.

Multiplying or dividing both terms of a fraction by the same number does not change the value of the fraction.

44. Reducing Improper Fractions to Mixed Numbers. An improper fraction may be reduced to a mixed number. The following example shows the method:

Example. Reduce $\frac{19}{4}$ to a mixed number.

SOLUTION. $19 \div 4 = 4\frac{3}{4}$.

State a rule for changing an improper fraction to a mixed number.

45. Reducing Mixed Numbers to Improper Fractions. A mixed number may be reduced to an improper fraction. The following example shows the method.

Example. Reduce $13\frac{3}{7}$ to an improper fraction.

SOLUTION. $13 \times 7 = 91$. $91 + 3 = 94$, the numerator of the improper fraction.

The denominator of the fraction in the mixed number is retained as the denominator of the improper fraction.

$$13\frac{3}{7} = \frac{94}{7}.$$

Oral Work

1. Reduce to lowest terms: $\frac{4}{12}$, $\frac{3}{18}$, $\frac{5}{10}$, $\frac{6}{24}$, $\frac{18}{30}$, $\frac{24}{28}$, $\frac{15}{30}$, $\frac{72}{84}$.

2. Express each of the following fractions with the denominator 16: $\frac{5}{8}$, $\frac{3}{4}$, $\frac{1}{8}$, $\frac{4}{8}$, $\frac{9}{4}$, $\frac{7}{8}$, $\frac{5}{2}$, $\frac{5}{1}$, 3.

3. Express with the denominator 36: $\frac{4}{9}$, $\frac{5}{6}$, $\frac{2}{3}$, $\frac{7}{12}$, $\frac{4}{12}$, $\frac{1}{3}$, $\frac{8}{9}$, $\frac{7}{3}$, $\frac{8}{4}$, $\frac{1}{9}$, $\frac{7}{2}$.

4. Reduce to twenty-fourths: $\frac{5}{6}$, $\frac{7}{12}$, $\frac{2}{3}$, $\frac{3}{2}$, $\frac{7}{6}$, $\frac{1}{12}$, 2.

5. Reduce to lowest terms: $\frac{8}{12}$, $\frac{9}{15}$, $\frac{4}{16}$, $\frac{8}{10}$, $\frac{9}{12}$, $\frac{14}{16}$.

6. Reduce to fiftieths: $\frac{2}{5}$, $\frac{9}{10}$, $\frac{7}{25}$, $\frac{19}{10}$, $\frac{5}{2}$, $\frac{7}{10}$.

7. Reduce to improper fractions: $3\frac{1}{5}$, $5\frac{1}{3}$, $7\frac{1}{4}$, $8\frac{2}{3}$, $4\frac{3}{5}$, $7\frac{2}{7}$, $9\frac{4}{5}$, $8\frac{1}{4}$.

8. Reduce the following fractions to equivalent fractions in lowest terms: $\frac{9}{24}$, $\frac{16}{68}$, $\frac{20}{85}$, $\frac{18}{144}$, $\frac{28}{210}$, $\frac{27}{63}$, $\frac{30}{150}$, $\frac{21}{15}$, $\frac{72}{180}$, $\frac{24}{36}$, $\frac{8}{12}$, $\frac{18}{21}$.

9. Change the following fractions as indicated:

$\frac{3}{4}$ to an equivalent fraction whose denominator is 20.
$\frac{7}{8}$ to an equivalent fraction whose denominator is 64.
$\frac{4}{9}$ to an equivalent fraction whose denominator is 63.
$\frac{2}{18}$ to an equivalent fraction whose denominator is 72.
$\frac{6}{11}$ to an equivalent fraction whose denominator is 121.
$\frac{5}{14}$ to an equivalent fraction whose denominator is 84.

10. Change the following improper fractions to mixed numbers: $\frac{8}{3}$, $\frac{17}{6}$, $\frac{39}{7}$, $\frac{25}{9}$, $\frac{31}{2}$, $\frac{84}{11}$, $\frac{488}{15}$, $\frac{300}{29}$, $\frac{341}{17}$, $\frac{83}{13}$, $\frac{216}{13}$, $\frac{235}{12}$.

11. Change the following mixed numbers to equivalent improper fractions: $3\frac{2}{3}$, $6\frac{4}{7}$, $8\frac{1}{8}$, $4\frac{3}{5}$, $8\frac{16}{25}$, $10\frac{17}{35}$, $14\frac{8}{13}$, $12\frac{7}{12}$, $18\frac{3}{20}$, $9\frac{5}{24}$.

12. When we reduce $\frac{8}{12}$ to $\frac{2}{3}$, do we increase or decrease the size of the fractional unit? Do we increase or decrease the number of fractional units?

13. Which fraction is the greatest, $\frac{7}{8}$, $\frac{5}{6}$, $\frac{3}{4}$, $\frac{13}{15}$?

ADDITION AND SUBTRACTION OF FRACTIONS

46. Finding the Least Common Denominator. Only like quantities can be added or subtracted. It is not possible to add or subtract 4 lb. and 2 oz. or $\frac{1}{2}$ and $\frac{1}{4}$ until they are reduced to the same denomination.

The common denominator should be the smallest number which will exactly contain all of the denominators. It may usually be found by inspection.

Any number which contains all of the factors of another number will also contain that number. Thus, 24 contains all of the factors of 12, and it therefore contains 12.

The common denominator must, therefore, contain all of the factors of the denominators.

Hence, to find the least common denominator of two or more fractions, *find the prime factors of the denominators; multiply the different prime factors, using each factor the greatest number of times it is contained in any one denominator.*

Example. What is the least common denominator of fractions with the denominators 8, 9, and 12?

Solution.

$$8 = 2 \times 2 \times 2.$$
$$9 = 3 \times 3.$$
$$12 = 2 \times 2 \times 3.$$
$$\text{l. c. m. of denominators} = 2 \times 2 \times 2 \times 3 \times 3.$$

47. Reducing Fractions to a Common Denominator. After the least common denominator has been found, the fractions may be reduced to this denominator.

Example. Reduce $\frac{3}{5}$, $\frac{2}{3}$, and $\frac{5}{6}$ to a common denominator.

Solution. The common denominator is 30.

$$\tfrac{3}{5} = \tfrac{18}{30}.$$
$$\tfrac{2}{3} = \tfrac{20}{30}.$$
$$\tfrac{5}{6} = \tfrac{25}{30}.$$

State the process of reducing fractions to a common denominator.

Written Work

Reduce the fractions in each example to fractions having the least common denominator:

1. $\frac{1}{2}$ and $\frac{1}{3}$. **2.** $\frac{1}{3}$ and $\frac{1}{4}$. **3.** $\frac{1}{4}$ and $\frac{1}{8}$.
4. $\frac{1}{6}$, $\frac{1}{3}$, and $\frac{1}{12}$. **5.** $\frac{1}{8}$, $\frac{1}{12}$, $\frac{1}{20}$. **6.** $\frac{2}{3}$, $\frac{4}{9}$, $\frac{5}{12}$.
7. $\frac{3}{7}$, $\frac{2}{11}$, $\frac{5}{42}$. **8.** $\frac{7}{12}$, $\frac{9}{32}$, $\frac{13}{56}$, $\frac{4}{9}$, $\frac{5}{14}$. **9.** $\frac{3}{20}$, $\frac{17}{36}$, $\frac{8}{49}$, $\frac{12}{35}$.
10. $\frac{11}{18}$, $\frac{9}{28}$, $\frac{1}{42}$, $\frac{11}{15}$. **11.** $\frac{17}{96}$, $\frac{23}{42}$, $\frac{25}{63}$, $\frac{19}{27}$.

48. Rule for Addition of Fractions.

Reduce the fractions to a common denominator. Add the numerators to form the numerator of the sum. The common denominator is the denominator of the sum. Reduce the result to the simplest form.

Proper fractions should be reduced to their lowest terms. Improper fractions should be changed to mixed numbers.

Examples. 1. Add $\frac{2}{3}$ and $\frac{3}{4}$.

Solution. The least common multiple of the denominators is 12.

$$\frac{2}{3} = \frac{8}{12}.$$
$$\frac{3}{4} = \frac{9}{12}.$$
$$\frac{8}{12} + \frac{9}{12} = \frac{17}{12} = 1\tfrac{5}{12}.$$

2. Add $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{10}$.

Solution. The least common multiple of the denominators is 60.

$$\frac{3}{4} = \frac{45}{60}.$$
$$\frac{5}{6} = \frac{50}{60}.$$
$$\frac{7}{10} = \frac{42}{60}.$$
$$\frac{45}{60} + \frac{50}{60} + \frac{42}{60} = \frac{137}{60} = 2\tfrac{17}{60}.$$

49. Rule for Subtraction of Fractions.

The process is the same as for Addition of Fractions except that the numerators are subtracted instead of added after the fractions have been reduced to a common denominator.

Oral Work

State the sum, then the difference of the following:

1. $\frac{1}{2}, \frac{1}{3}$. **2.** $\frac{1}{3}, \frac{1}{4}$. **3.** $\frac{1}{5}, \frac{1}{7}$. **4.** $\frac{1}{4}, \frac{1}{6}$. **5.** $\frac{1}{3}, \frac{1}{5}$.
6. $\frac{1}{2}, \frac{1}{8}$. **7.** $\frac{1}{7}, \frac{1}{9}$. **8.** $\frac{1}{9}, \frac{1}{10}$. **9.** $\frac{2}{3}, \frac{2}{7}$. **10.** $\frac{5}{6}, \frac{3}{4}$.
11. $\frac{7}{9}, \frac{2}{3}$. **12.** $\frac{5}{6}, \frac{4}{5}$. **13.** $\frac{4}{7}, \frac{1}{3}$. **14.** $\frac{3}{4}, \frac{3}{5}$. **15.** $\frac{3}{5}, \frac{2}{7}$.

Note. The Short Methods for the Addition and Subtraction of Fractions, explained on pages 69–71, may be studied at this time.

50. Rule for Addition of Mixed Numbers.

Add the fractions.

If the sum is an improper fraction, reduce it to a mixed number.

Add the sum of the fractions to the sum of the integers.

Example. Add $3\frac{3}{4}$ and $4\frac{7}{8}$.

Solution.
$$\frac{3}{4} + \frac{7}{8} = \frac{13}{8} = 1\tfrac{5}{8}.$$
$$3 + 4 = 7.$$
$$7 + 1\tfrac{5}{8} = 8\tfrac{5}{8}.$$

Written Work

Find the sums:

1. $4\frac{1}{2}+3\frac{1}{3}$. **2.** $2\frac{1}{5}+5\frac{1}{7}$.
3. $7\frac{1}{8}+5\frac{2}{3}$. **4.** $17\frac{1}{5}+18\frac{2}{7}$.
5. $275\frac{2}{3}+78\frac{1}{5}$. **6.** $3\frac{7}{8}+5\frac{1}{4}$.
7. $9\frac{1}{10}+17\frac{2}{5}$. **8.** $3\frac{1}{3}+4\frac{7}{8}+3\frac{5}{12}$.
9. $23\frac{1}{3}+16\frac{5}{12}+7\frac{3}{4}$. **10.** $18\frac{5}{6}+42\frac{1}{10}+37\frac{3}{5}$.

51. Rule for Subtraction of Mixed Numbers.

If the fraction of the minuend is larger than that of the subtrahend, the above rule for addition can be easily changed to apply to subtraction.

Example. Subtract $4\frac{2}{5}$ from $7\frac{5}{6}$.

SOLUTION.
$\frac{5}{6}-\frac{2}{5}=\frac{13}{30}$.
$7-4=3$.
Result $3\frac{13}{30}$.

If the fraction of the minuend is smaller than that of the subtrahend, we may proceed as follows:

Example. Subtract $4\frac{2}{3}$ from $9\frac{1}{5}$.

SOLUTION. $\frac{2}{3}$ cannot be subtracted from $\frac{1}{5}$, but $9\frac{1}{5}$ may be regarded as $8\frac{6}{5}$. $\frac{6}{5}-\frac{2}{3}=\frac{8}{15}$. Therefore, $9\frac{1}{5}-4\frac{2}{3}=4\frac{8}{15}$.

Written Work

Add as indicated:

1. $\frac{3}{7}+\frac{2}{5}$. **2.** $\frac{6}{13}+\frac{8}{9}$. **3.** $\frac{7}{9}+\frac{4}{15}$.
4. $\frac{2}{15}+\frac{7}{24}$. **5.** $\frac{3}{5}+\frac{9}{25}+\frac{9}{16}$. **6.** $\frac{15}{32}+\frac{7}{24}+\frac{11}{36}$.
7. $3\frac{5}{8}+\frac{1}{7}$. **8.** $12\frac{9}{16}+\frac{7}{18}$. **9.** $13\frac{5}{7}+22\frac{9}{15}$.

Subtract as indicated:

10. $\frac{4}{7}-\frac{3}{11}$. **11.** $\frac{15}{32}-\frac{7}{20}$. **12.** $2\frac{9}{14}-\frac{3}{16}$. **13.** $4\frac{7}{8}-\frac{6}{7}$.
14. $3\frac{2}{3}-2\frac{1}{2}$. **15.** $4\frac{7}{9}-2\frac{1}{6}$. **16.** $5\frac{9}{25}-2\frac{8}{9}$.

Add upward and across:

17.

$$\begin{array}{ccccc}
\frac{1}{4} & + & \frac{1}{3} & + & \frac{1}{2} \\
\frac{2}{3} & + & \frac{3}{4} & + & \frac{5}{12} \\
\frac{5}{6} & + & \frac{7}{8} & + & \frac{7}{6} \\
\hline
? & + & ? & + & ?
\end{array}$$

18.

$$\begin{array}{ccccc}
\frac{5}{8} & + & \frac{3}{16} & + & \frac{2}{3} \\
\frac{7}{24} & + & \frac{5}{6} & + & \frac{1}{24} \\
\frac{1}{8} & + & \frac{5}{6} & + & \frac{7}{4} \\
\hline
? & + & ? & + & ?
\end{array}$$

19. Add upward; subtract across:

$$\begin{array}{r}
14\frac{2}{3}-4\frac{1}{5}\\
18\frac{5}{6}-7\frac{2}{3}\\
5\frac{1}{12}-3\frac{1}{4}\\
15\frac{3}{4}-7\frac{5}{6}\\
\hline
?\quad-\;?
\end{array}$$

MULTIPLICATION AND DIVISION OF FRACTIONS

52. Rule for Multiplication of Fractions.

Multiply the numerators to form the numerator of the product. Multiply the denominators to form the denominator of the product. Reduce the product to its simplest form.

Thus, $\frac{3}{5}\times\frac{4}{7}=\frac{12}{35}$.

NOTE. The word "of" placed between fractions indicates multiplication.

Mixed numbers should be changed to improper fractions before multiplying.

Example. $8\times 6\frac{3}{4}\times\frac{2}{3}\times 3\frac{4}{5}=?$

SOLUTION. Reducing mixed numbers to improper fractions: $\frac{8}{1}\times\frac{27}{4}\times\frac{2}{3}\times\frac{19}{5}=?$

$$\frac{\overset{2}{\cancel{8}}}{1}\times\frac{\overset{9}{\cancel{27}}}{\underset{1}{\cancel{4}}}\times\frac{2}{\underset{1}{\cancel{3}}}\times\frac{19}{5}.$$

Multiplying the resulting numerators: $2\times 9\times 2\times 19=684$, the numerator of the product.

Multiplying the resulting denominators: $1\times 1\times 1\times 5=5$, the denominator of the product.

Changing to a mixed number: $\frac{684}{5}=136\frac{4}{5}$, the product.

Written Work

Perform the following multiplications:

1. $\frac{2}{3}\times\frac{4}{5}$.
2. $\frac{3}{8}\times\frac{5}{7}$.
3. $\frac{6}{5}\times\frac{8}{7}$.
4. $\frac{1}{2}\times\frac{5}{12}$.
5. $\frac{3}{4}\times\frac{7}{8}$.
6. $3\frac{1}{5}\times 4\frac{2}{3}$.
7. $\frac{8}{9}\times\frac{3}{4}$.
8. $\frac{2}{3}$ of $\frac{3}{4}\times\frac{1}{2}$.
9. $\frac{3}{7}\times\frac{3}{5}$ of $\frac{4}{3}$.
10. $\frac{8}{12}\times\frac{5}{7}\times\frac{4}{9}\times\frac{3}{10}$.
11. $5\frac{2}{3}\times 3\frac{1}{4}\times 4\frac{2}{5}$.
12. $\frac{3}{8}\times 4\times\frac{7}{2}\times\frac{3}{15}$.

53. Rule for Division of Fractions.

Invert the divisor, and multiply the fractions, using cancellation when possible.

Examples. 1. $\frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \times \frac{7}{5} = \frac{14}{15}$.

2. $\frac{8}{9} \div \frac{4}{5} = \frac{8}{9} \times \frac{5}{4} = \frac{10}{9} = 1\frac{1}{9}$.

3. $8\frac{2}{3} \div \frac{4}{9} = ?$

SOLUTION.
$$8\tfrac{2}{3} \div \tfrac{4}{9} = \frac{\overset{13}{\cancel{26}}}{\cancel{3}} \times \frac{\overset{3}{\cancel{9}}}{\underset{2}{\cancel{4}}} = \frac{39}{2} = 19\tfrac{1}{2}.$$

Written Work

1. $\frac{3}{5} \div \frac{2}{3}$.
2. $\frac{4}{7} \times \frac{9}{8}$.
3. $\frac{2}{3}$ of $\frac{3}{4} \times \frac{1}{6}$.
4. $5\frac{1}{2} \div 4\frac{2}{3}$.
5. $7\frac{1}{8} \div 3\frac{1}{4}$.
6. $\frac{3}{8} + \frac{2}{3}$.
7. $\frac{4}{7} - \frac{2}{5}$.
8. $3\frac{1}{5} + 5\frac{2}{3}$.
9. $\frac{3}{4}$ of $\frac{3}{7}$.
10. $9\frac{1}{2} \times 12\frac{2}{3} \div 3\frac{1}{4}$.
11. $\frac{7}{8}$ of $\frac{2}{11} \div \frac{14}{22}$.
12. $\frac{3}{5} + \frac{2}{9}$.
13. $4\frac{2}{5} - 3\frac{1}{3} + 5\frac{3}{4}$.
14. $17\frac{2}{3} \times 5\frac{1}{2} \div 4\frac{5}{12}$.
15. $\frac{3}{8}$ of $\frac{2}{3} \div \frac{2}{5}$ of $\frac{4}{9}$.

Review Exercises

1. Multiply each of the fractions in the column at the left by each of the fractions at the top. Enter the products in the spaces formed by the horizontal and vertical lines. (For example, the space marked "X" is to contain the product of $\frac{5}{8}$ and $\frac{7}{8}$.)

MULTIPLY	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{5}{6}$	$\frac{7}{8}$
$\frac{5}{6}$				
$3\frac{1}{3}$				
$\frac{5}{8}$				X
$\frac{7}{8}$				

2. In the following form divide each of the fractions in the column at the left by the fractions in the upper row. Rule a blank similar to the form given and record the quotients in the proper spaces.

	$\frac{3}{5}$	$\frac{7}{8}$
$\frac{5}{6}$		
$\frac{8}{3}$		
$\frac{3}{4}$		
$\frac{2}{5}$		
$\frac{4}{9}$		
$\frac{5}{4}$		
$\frac{3}{8}$		
$\frac{5}{12}$		
$\frac{8}{9}$		
$\frac{5}{6}$		

Miscellaneous Problems

1. If $\frac{1}{3}$ of a number is 5, what is the number?

2. If $\frac{2}{3}$ of a number is 12, what is the number?

3. If $\frac{4}{7}$ of a number is 20, what is the number?

4. If $\frac{3}{5}$ of $\frac{2}{3}$ of a number is 15, what is the number?

5. After spending $\frac{1}{4}$ of his monthly salary for board and room, and $\frac{1}{10}$ of it for clothes, a man has $65 left. What is his salary?

6. If 5 yards of cloth cost 47½ ¢, what will 1 yard cost at the same rate?

7. A train runs $\frac{2}{3}$ of a mile in $\frac{5}{6}$ of a minute. What is the rate per hour?

8. Which will give the larger result, multiplying or dividing an integer by a proper fraction? Multiplying or dividing by an improper fraction?

9. If you are told the amount of a man's wages and the fractional part of his wages which he spent, how can you find the fractional part which he saved?

10. State two ways of finding the amount saved, and state which you think is the easier.

11. If you were told what $\frac{3}{4}$ of $\frac{2}{3}$ of a number is, how could you find the number?

12. A man owned $\frac{3}{4}$ of a farm and sold $\frac{1}{5}$ of what he owned. If he received $3000 for the land sold, what was the value of the entire farm at the same rate?

13. A mechanic works 8 hours a day. Two and one half hours are spent at a bench and the remainder at a machine. What fractional part of the day is spent at a machine?

14. In a certain family $\frac{1}{4}$ of the yearly income is spent for rent, $\frac{1}{5}$ for clothes, $\frac{1}{4}$ for food, and $\frac{1}{2}$ of the remainder for travel. If the yearly income is $3000, how much is spent for travel?

15. A man sold $\frac{2}{3}$ of an acre of land for $76. At that rate what is his farm of 120 acres worth?

16. $\frac{4}{7}$ of the number of students in a high school are girls. The number of boys is 120. How many students are there in the school?

17. Mr. Jones bought some land and sold it so as to realize $\frac{1}{5}$ more than the cost. If the selling price was $360, what did he pay for the land?

18. Mr. Williams sold some goods for $\frac{1}{6}$ less than the cost. If he received $70 for the goods, what was the cost?

19. In making up a certain cake recipe for 6 people, $1\frac{1}{2}$ cups of sugar and $\frac{2}{3}$ of a cup of butter are used. How much sugar and butter should be used in making up the recipe for 4 people?

20. A boy works $3\frac{1}{2}$ days at the rate of $6.50 a week of 6 days. How much does he earn?

21. Find the total cost of $8\frac{2}{3}$ yards of ribbon at 28 cents per yard, $3\frac{1}{2}$ yards of insertion at $12\frac{1}{2}$ cents per yard, and $5\frac{1}{3}$ yards of silk at \$1.35 per yard.

22. A dealer bought oranges at the rate of 4 for 10 cents and sold them at the rate of 3 for 10 cents. How much did he gain on each orange?

23. Mr. Rankin owned $\frac{2}{3}$ of a store and sold $\frac{2}{3}$ of his share to Mr. Johnson for \$3600. At this rate what was the store worth?

24. If a man drives his automobile 18 miles in $\frac{2}{3}$ of an hour, how long would he require, at the same rate, to travel 60 miles?

25. When $\frac{3}{4}$ of a yard of cloth costs \$1.80, what is the price of $\frac{4}{5}$ of a yard?

26. A girl grew $1\frac{1}{4}$ inches during one year and $\frac{5}{8}$ of an inch during the next year. How much more did she grow during the first year than during the second?

27. The entire length of a skirt is to be $26\frac{3}{4}$ inches, and the ruffle at the bottom is to be $3\frac{7}{8}$ inches wide. What will be the length of the skirt above the ruffle?

28. The record for the hundred yard dash is $9\frac{3}{5}$ seconds. A boy can run the distance in $11\frac{4}{5}$ seconds. What is the difference between his time and the record time?

29. A farmer raised $296\frac{1}{3}$ bushels of wheat on $15\frac{1}{4}$ acres. What was the average yield per acre?

30. Three and one half bushels of seed were sown and the yield was $21\frac{1}{2}$ bushels. What was the average yield per bushel of seed?

SHORT METHODS INVOLVING FRACTIONS

54. To add fractions when the numerators are the same.

Add the denominators and multiply this sum by the common numerator to obtain the numerator of the result. Multiply the denominators to obtain the denominator of the result. Reduce to simplest form.

Examples. 1. Add $\frac{1}{3}$ and $\frac{1}{7}$.

SOLUTION. $3 + 7 = 10$, the numerator of the result.
$3 \times 7 = 21$, the denominator of the result.
Hence, $\frac{1}{3} + \frac{1}{7} = \frac{10}{21}$.

2. Add $\frac{2}{5}$ and $\frac{2}{9}$.

SOLUTION. $5 + 9 = 14$. $14 \times 2 = 28$, the numerator of the result.
$5 \times 9 = 45$, the denominator of the result.
Hence, $\frac{2}{5} + \frac{2}{9} = \frac{28}{45}$.

Oral Work

Add as indicated:

1. $\frac{1}{5} + \frac{1}{3}$. **2.** $\frac{1}{7} + \frac{1}{9}$. **3.** $\frac{1}{5} + \frac{1}{8}$. **4.** $\frac{1}{9} + \frac{1}{16}$.
5. $\frac{1}{7} + \frac{1}{8}$. **6.** $\frac{1}{12} + \frac{1}{7}$. **7.** $\frac{1}{9} + \frac{1}{14}$. **8.** $\frac{2}{7} + \frac{2}{13}$.
9. $\frac{1}{3} + \frac{1}{6}$. **10.** $\frac{2}{3} + \frac{2}{8}$. **11.** $\frac{2}{11} + \frac{2}{7}$. **12.** $\frac{3}{5} + \frac{3}{7}$.
13. $\frac{3}{8} + \frac{3}{13}$. **14.** $\frac{3}{5} + \frac{3}{11}$.

55. To subtract fractions when the numerators are the same.

Subtract the denominators and multiply this difference by the common numerator to form the numerator of the result. Multiply the denominators to form the denominator of the result. Reduce to simplest form.

Example. Subtract $\frac{2}{5}$ from $\frac{2}{3}$.

SOLUTION. $5 - 3 = 2$. $2 \times 2 = 4$, the numerator of the result.
$5 \times 3 = 15$, the denominator of the result.
Hence, $\frac{2}{3} - \frac{2}{5} = \frac{4}{15}$.

Oral Work

Perform the following subtractions:

1. $\frac{1}{6} - \frac{1}{8}$. **2.** $\frac{1}{7} - \frac{1}{13}$. **3.** $\frac{1}{8} - \frac{1}{12}$.
4. $\frac{2}{7} - \frac{2}{15}$. **5.** $\frac{3}{7} - \frac{3}{16}$. **6.** $\frac{5}{9} - \frac{5}{14}$.

56. To add two fractions by "cross multiplication."

Multiply the numerator of each fraction by the denominator of the other fraction. Add these products to form the numerator. Multiply the denominators to form the denominator. Reduce to lowest terms.

Example. Add $\frac{2}{3}$ and $\frac{3}{4}$.

SOLUTION.
$2 \times 4 = 8$.
$3 \times 3 = 9$.
$9 + 8 = 17$, the numerator.
$3 \times 4 = 12$, the denominator.

Hence, $\frac{2}{3} + \frac{3}{4} = \frac{17}{12}$, $\frac{17}{12} = 1\frac{5}{12}$.

Oral Work

Add as indicated:

1. $\frac{3}{5}+\frac{2}{7}$. 2. $\frac{2}{3}+\frac{3}{7}$. 3. $\frac{1}{2}+\frac{3}{8}$.
4. $\frac{3}{10}+\frac{7}{16}$. 5. $\frac{8}{9}+\frac{7}{8}$. 6. $\frac{3}{5}+\frac{7}{8}$.
7. $\frac{5}{12}+\frac{3}{8}$. 8. $\frac{6}{7}+\frac{6}{11}$. 9. $\frac{5}{7}+\frac{9}{16}$.

57. To subtract fractions by "cross multiplication."

Multiply the numerator of each fraction by the denominator of the other fraction. Subtract these products to form the numerator. Multiply the denominators to form the denominator. Reduce to lowest terms.

Example. Subtract $\frac{2}{3}$ from $\frac{5}{6}$.

SOLUTION.

$$\begin{aligned} 5\times 3 &= 15.\\ 6\times 2 &= 12.\\ 15-12 &= 3, \text{ the numerator.}\\ 6\times 3 &= 18, \text{ the denominator.}\end{aligned}$$

Hence, $\frac{5}{6}-\frac{2}{3}=\frac{3}{18}$, $\frac{3}{18}=\frac{1}{6}$.

Written Work

Subtract as indicated:

1. $\frac{4}{7}-\frac{1}{3}$. 2. $\frac{8}{11}-\frac{3}{5}$. 3. $\frac{5}{8}-\frac{3}{10}$. 4. $\frac{7}{8}-\frac{7}{11}$.
5. $\frac{8}{15}-\frac{4}{11}$. 6. $\frac{13}{25}-\frac{4}{11}$. 7. $\frac{5}{3}-\frac{6}{17}$.

58. To find the approximate product of mixed numbers.

Multiply the integers. Multiply each integer by the other fraction to the nearest unit. Add these three products.

Example. Multiply $13\frac{1}{2}$ by $6\frac{2}{3}$.

SOLUTION.

$$\begin{array}{r} 13\frac{1}{2}\\ \underline{6\frac{2}{3}}\\ 13\times 6 = 78\\ \frac{1}{2}\text{ of } 6 = 3\\ \frac{2}{3}\text{ of } 13 = \underline{9}\\ 90 \end{array} \text{ (to the nearest unit).}$$

Written Work

Find approximate products:

1. $43\frac{1}{3}$, $62\frac{1}{5}$ 2. $89\frac{1}{4}$, $7\frac{1}{2}$ 3. $76\frac{1}{4}$, $82\frac{1}{7}$ 4. $15\frac{2}{5}$, $40\frac{2}{7}$ 5. $98\frac{1}{6}$, $47\frac{1}{3}$

6. $17\frac{3}{8}$	**7.** $13\frac{2}{7}$	**8.** $79\frac{2}{9}$	**9.** $38\frac{2}{5}$	**10.** $73\frac{5}{8}$
$46\frac{4}{5}$	$16\frac{1}{4}$	$63\frac{1}{5}$	$71\frac{1}{6}$	$96\frac{7}{8}$

59. To find the product of any two numbers ending in $\frac{1}{2}$.

(*a*) When the sum of the integers is an even number.

To the product of the integers, add one half of their sum and annex $\frac{1}{4}$ to the result.

Example. Multiply $39\frac{1}{2}$ by $3\frac{1}{2}$.

Solution.

$$\begin{array}{rr} & 39\frac{1}{2} \\ & 3\frac{1}{2} \\ \hline 39 \times 3 = & 117 \\ \frac{1}{2}\text{ the sum of 39 and 3} = & 21 \\ \hline & 138\frac{1}{4} \end{array}$$ *Result.*

Written Work

Multiply:

1. $44\frac{1}{2}$	**2.** $18\frac{1}{2}$	**3.** $38\frac{1}{2}$	**4.** $75\frac{1}{2}$	**5.** $29\frac{1}{2}$	**6.** $81\frac{1}{2}$
$64\frac{1}{2}$	$12\frac{1}{2}$	$14\frac{1}{2}$	$35\frac{1}{2}$	$33\frac{1}{2}$	$75\frac{1}{2}$

(*b*) When the sum of the integers is an odd number.

To the product of the integers, add the result obtained by taking half of one less than their sum. To this result annex $\frac{3}{4}$.

Example. Multiply $39\frac{1}{2}$ by $6\frac{1}{2}$.

Solution.

$$\begin{array}{r} 39\frac{1}{2} \\ 6\frac{1}{2} \\ \hline 234 \\ 22 \\ \hline 256\frac{3}{4} \end{array}$$

Written Work

Multiply:

1. $73\frac{1}{2}$	**2.** $57\frac{1}{2}$	**3.** $92\frac{1}{2}$	**4.** $35\frac{1}{2}$	**5.** $98\frac{1}{2}$
$42\frac{1}{2}$	$24\frac{1}{2}$	$47\frac{1}{2}$	$58\frac{1}{2}$	$17\frac{1}{2}$
6. $29\frac{1}{2}$	**7.** $83\frac{1}{2}$	**8.** $76\frac{1}{2}$	**9.** $43\frac{1}{2}$	**10.** $98\frac{1}{2}$
$36\frac{1}{2}$	$96\frac{1}{2}$	$19\frac{1}{2}$	$46\frac{1}{2}$	$23\frac{1}{2}$
11. $48\frac{1}{2}$	**12.** $95\frac{1}{2}$	**13.** $73\frac{1}{2}$	**14.** $47\frac{1}{2}$	**15.** $27\frac{1}{2}$
$72\frac{1}{2}$	$81\frac{1}{2}$	$71\frac{1}{2}$	$33\frac{1}{2}$	$71\frac{1}{2}$

CHAPTER VIII

DECIMAL FRACTIONS

60. Comparison of Common and Decimal Fractions.

The denominator of a common fraction may be any number, and it is always expressed.

The denominator of a decimal fraction is always some power of 10, and the power is indicated by the number of figures to the right of the decimal point.

Thus, $.7 = \frac{7}{10}$. $\qquad .0007 = \frac{7}{10000}$.

$.07 = \frac{7}{100}$. $\qquad .423 = \frac{423}{1000}$.

$.007 = \frac{7}{1000}$. $\qquad .027 = \frac{27}{1000}$.

61. Reading and Writing Decimals.

The names of the various decimal orders are stated in the following table :

	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths	Ten-millionths
.	1	2	3	4	5	6	7

The names of the various orders should be learned. You should know that the *name* of the third decimal order is thousandths; that of the fifth is hundred-thousandths ; that of the second is hundredths, etc.

To read a decimal fraction, read the number as if it were an integer and then state the name of the last decimal order which a digit of the number occupies.

Pronounce the word "and" at the decimal point and omit it elsewhere, except in reading complex decimals as noted below:

.045 is read forty-five thousandths.
.505 is read five hundred five thousandths.
500.005 is read five hundred *and* five thousandths.
.4206 is read four thousand two hundred six ten-thousandths.
.0$\frac{2}{3}$ is read two thirds of a tenth.
2.0$\frac{2}{3}$ is read two *and* two thirds of a tenth.
.2$\frac{2}{3}$ is read two and two thirds tenths (a complex decimal).

Read:

1. .043.
2. .1764.
3. 2.04.
4. 500.17.
5. 400.004.
6. .00$\frac{2}{3}$.
7. 214.0014.
8. .000562.
9. 64.3.
10. 14.003.
11. .003726.
12. 542.54.
13. 3005.016.
14. .00024$\frac{1}{3}$.
15. 7.00707.

Write decimally:

1. Seven tenths; forty-two thousandths; five hundred four thousandths.

2. Four hundred and three thousandths; eight hundred two ten-thousandths.

3. Three fourths of a tenth; two and one seventh tenths.

4. Twenty-four ten-thousandths; five millionths; seventeen hundred-thousandths; two hundred and three hundredths.

5. Five hundred seventeen millionths; sixteen ten-thousandths; eighteen hundred and fifteen ten-thousandths.

6. Five thousand four hundred-millionths; eighteen and four thousandths.

62. Addition of Decimal Fractions.

Place the decimals so that the decimal points fall vertically. Add as with integers. Place the decimal point of the sum directly below those in the numbers added.

Example. Find the sum of 4.023, .507, 2.003, and 1.125.

SOLUTION.

$$\begin{array}{r} 4.023 \\ .507 \\ 2.003 \\ \underline{1.125} \\ 7.658 \end{array}$$

Written Work

1. Add 2569.327, 1462.978, 47.9634, 2693.072, and .019.
2. Add 27.9548, 91.0005, 37.427, and 27563.974.
3. Add 2752.9374, .0003, 23.247, and 259.6347.

Change the following common fractions to decimal fractions, and add:

4. $\frac{24}{100}, \frac{372}{1000}, \frac{19}{10}, \frac{254}{100}$.
5. $\frac{3}{1000}, \frac{24}{10000}, \frac{675}{10}, \frac{170}{10000}$.
6. $\frac{36.5479}{100}, \frac{762}{1000}, \frac{7642}{10000}, \frac{271}{1000}$.
7. Copy, find the totals required, and check:

$ 324.80	$ 3764.20	$7436.08	?
17.04	762.83	427.36	?
9320.87	9402.03	92.18	?
473.26	1.18	4724.63	?
791.14	13420.03	924.56	?
?	?	?	?

63. Subtraction of Decimal Fractions.

Place the decimals so that the points fall vertically. Subtract as with integers, then place the decimal point in the remainder immediately below the decimal points above.

Example. Subtract 23.213 from 34.047.

SOLUTION.

$$\begin{array}{r} 34.047 \\ \underline{23.213} \\ 10.834 \end{array}$$

Subtract as indicated. Check the results.

1. 394.237 – 1.027. **2.** .47 – .0003. **3.** .394237 – .15.

4. 7763.421 – 28.796. **5.** 87.5932 2.3579.

Change to decimal fractions and subtract:

6. $\frac{465}{1000} - \frac{23}{100}$. **7.** $\frac{76943}{100} - \frac{235}{1000}$. **8.** $\frac{8732}{10} - \frac{236547}{100000}$.

9. The sum of two numbers is 342.086. One of the numbers is 206.78. What is the other number?

10. A man deposited $5764.80 in a bank. He later drew out $3780.92 and then he deposited $4814.60. How much had he then?

64. Multiplication of Decimal Fractions.

Multiply as with integers. From the right of the product, point off as many places as the sum of the number of decimal places in the multiplicand and the multiplier.

Examples. 1. Multiply 4.625 by .05.

Solution.

$$\begin{array}{r} 4.625 \\ .05 \\ \hline .23125 \end{array}$$

2. Multiply .00362 by .06.

Solution.

$$\begin{array}{r} .00362 \\ .06 \\ \hline .0002172 \end{array}$$

Written Work

Multiply as indicated. Check the results.

1. 25.763 × .1463. **2.** 75.46 × .03.

3. .2462 × .347. **4.** .083 × .0462.

5. 78.23 × .000007. **6.** 17.13 × .042.

7. 570.04 × .00326. **8.** $374.26 × .055.

9. $9037.28 × .035. **10.** $473.54 × .335.

11. Find the cost of 46 lots at an average cost of $847.60.

12. Find the cost of 143 barrels of apples at an average price of $3.64 per barrel.

65. Division of Decimal Fractions.

a. When the divisor is an integer.

Examples. 1. .12 ÷ 4 = .03; just as \$12 ÷ 4 = \$3.

2. .426 ÷ 3 = .142; just as \$426 ÷ 3 = \$142.

In making the division the decimal point should be written in the quotient when it is reached in the dividend.

3. Divide 157.25 by 37.

SOLUTION.

$$\begin{array}{r}
4.25\\
37\overline{)157.25}\\
\underline{148}\\
9.2\\
\underline{7.4}\\
1.85\\
\underline{1.85}
\end{array}$$

b. When the divisor is not an integer.

Multiply both dividend and divisor by the least power of ten that will make the divisor an integer. Divide as in case *a.*

Example. Divide .028 by .4.

SOLUTION. First multiply both dividend and divisor by 10, to make the divisor an integer. Then divide the resulting dividend, .28, by 4. The result is .07.

Similarly, .0456 ÷ .03 = 4.56 ÷ 3 = 1.52.
Similarly, 5.64 ÷ .0004 = 56400 ÷ 4 = 14100.

If there is a remainder after all of the decimal places in the dividend have been used, zeros may be annexed to the dividend and the division may be carried as far as is desired.

Oral Work

1. .08 by .4.
2. .014 by .2.
3. .364 by .04.
4. 3300 by .11.
5. 8.48 by .008.
6. .220 by .0011.
7. 1 by .01.
8. .10 by .1.
9. .018 by 18.
10. .0001 by .00001.
11. .042 by 2.1.
12. 1.1 by .011.

Written Work

1. $268632 \div 36.9$.

2. $26863.2 \div 36.9$.

3. $26863.2 \div 369$.

4. $2686.32 \div 36.9$.

5. $26.8632 \div 36.9$.

6. $2686.32 \div .369$.

7. $26.8632 \div .369$.

8. $26.8632 \div .0369$.

9. $2.68632 \div .369$.

10. $2.68632 \div 3.69$.

11. $2.68632 \div 36.9$.

12. $2.68632 \div 369$.

13. $.268632 \div 369$.

14. $2.68632 \div 3690$.

15. $293.45 \div 14.24$.

16. $2.734 \div 1.32$.

17. $.73469 \div 127.9638$.

18. $762.397 \div 36947.28$.

19. $.1479376 \div 293.4798$.

20. $.0097 \div 12.34692$.

21. The multiplier is .045, the product is .01665. What is the multiplicand?

22. At $.24 per dozen, how many eggs can be bought for $4.08?

23. If a man's annual income is $5420 and his annual expenses are $4262, what are his average weekly savings?

24. What number is $\frac{3}{7}$ as large as .0427?

25. What number divided by 4.28 gives a quotient of .07 and a remainder of .04?

26. In a certain factory 30 men are employed at $2.15 per day, 10 men at $3.60 per day, 28 men at $2.90 a day, and 12 men at $1.80 a day. Find the average daily wage.

27. Using the current market price find the cost of the following: 1 pk. of apples, $\frac{1}{2}$ bu. of sweet potatoes, $\frac{1}{2}$ lb. of tea, 6 bars of laundry soap, $1\frac{1}{2}$ lb. porterhouse steak.

28. A train ran at the rate of .87 mile a minute. At this rate how many miles would it travel in $3\frac{1}{2}$ minutes? In 10 minutes? In 1 hour?

29. An automobile traveled 72.25 miles in $2\frac{1}{3}$ hours. What was the average rate of speed per hour?

66. Changing Decimal Fractions to Equivalent Common Fractions.

Omit the decimal point; write for the denominator 1 *with as many zeros annexed as there are places in the decimal. Reduce to lowest terms.*

Examples. 1. Change .75 to a common fraction.

SOLUTION. $.75 = \frac{75}{100} = \frac{3}{4}$.

2. Change .00864 to a common fraction.

SOLUTION. $.00864 = \frac{864}{100000} = \frac{27}{3125}$.

Written Work

Change the following decimals to equivalent common fractions and reduce to lowest terms:

1. .48. **2.** .095. **3.** .3705. **4.** .0012. **5** 3.6425.
6. .125. **7.** .9825. **8.** .625. **9.** .1875. **10** .0015.
11. $.06\frac{1}{4}$. **12.** $.002\frac{1}{2}$. **13.** $.37\frac{1}{2}$. **14.** .333. **15.** $.012\frac{1}{2}$.
16. $.006\frac{2}{3}$. **17.** .00625. **18.** .0036. **19.** $.008\frac{1}{3}$. **20.** $.009\frac{1}{11}$.
21. $.001\frac{1}{4}$.

67. Changing Common Fractions to Equivalent Decimal Fractions.

Since a common fraction may be regarded as an indicated division, the reduction may be made by the methods of division previously explained.

Example. Change $\frac{4}{5}$ to a decimal fraction.

SOLUTION. $\frac{4}{5} = \frac{1}{5}$ of 4. Place a decimal point to the right of the 4, annex zeros, and divide.

$$\begin{array}{r} .8 \\ 5)\overline{4.0} \end{array}$$

Similarly, $\frac{7}{8} = \frac{1}{8}$ of 7.

$$\begin{array}{r} .875 \\ 8)\overline{7.000} \end{array}$$

Similarly, $\frac{2}{375} = \frac{1}{375}$ of 2.

$$\begin{array}{r} .0053+ \\ 375)\overline{2.000} \\ \underline{1.875} \\ .1250 \\ \underline{.1125} \\ 125 \end{array}$$

It is usually not necessary to carry the division more than three or four places.

For example, $\frac{1}{7}$ may be expressed decimally as $.142\frac{6}{7}$ or as $.1429^-$.

Written Work

Considering the figures in the top row as numerators, and those at the left as denominators, change the common fractions thus formed to decimals, entering the results in the proper places. Carry the results to three decimal places.

Prepare the work on a carefully ruled form.

68. Approximate Results. For most practical purposes it is unnecessary to carry a result to more than three decimal places. Thus, $25.86347 is stated with sufficient accuracy as $25.86.

In finding approximate decimal fractions, follow these directions.

Carry the division at least one more place than the number of digits desired in the result. If the digit at the right of the last place desired in the result is less than 5, *drop it; if it is* 5 *or larger than* 5, *increase the preceding digit by* 1.

DENOMINATORS	NUMERATORS								
	3	7	6	26	47	126	79	814	1216
18									
27									
94									
38									
172									
9									
66									
213									
793									
2361									
43									
109									

Written Work

The following table gives the land area, in acres, of the different geographical sections of the United States, and the number of acres in each section devoted to farming.

Prepare a ruled form similar to the model.

Find what decimal part of the land area of each section, and of the United States, is improved farm land. (Approximate results to the nearest thousandth.)

Also find the total land area of the United States, the total area of the improved land, and the fraction of the total area which is improved.

Arrange the different sections on the blank in the order of their rank, placing the section with the largest fraction of improved farm land at the top of the blank.

SECTION	TOTAL LAND AREA	IMPROVED FARM LAND	
	Acres	Acres	Decimal Fraction of Total
New England	39,664,640	7,254,904	
Middle Atlantic	64,000,000	29,320,894	
East North Central	157,160,960	88,947,228	
West North Central	326,914,560	164,284,862	
South Atlantic	172,205,440	48,479,733	
East South Central	114,885,760	43,946,846	
West South Central	275,037,440	58,264,273	
Mountain	549,840,000	15,915,002	
Pacific	203,580,800	22,038,008	
United States			

Oral Work

The following table shows:

The value of the butter *sold* in each section of the United States in a recent year.

The fractions which these values were of the *total production* of butter in each section.

VALUE OF BUTTER PRODUCED IN UNITED STATES

SECTION	VALUE OF BUTTER SOLD	FRACTION OF TOTAL PRODUCT SOLD	VALUE OF TOTAL PRODUCTION
New England	$ 8,533,864	.725	$
Middle Atlantic . . .	15,229,862	.655	
East North Central . .	31,855,809	.585	
West North Central . .	20,333,127	.438	
South Atlantic	7,622,916	.275	
East South Central . .	4,842,959	.167	
West South Central . .	5,381,690	.19	
Mountain	2,166,918	.422	
Pacific	4,410,978	.572	

1. By referring to the table we find that .725 of the butter produced in New England was sold. How many tenths of the butter made in New England was sold?

2. If $4,410,978 is about fifty-seven hundredths of the value of all the butter made in the Pacific States in a year, how would you find the total value of the butter production in this section?

NOTE. When in doubt whether the solution of a given problem requires multiplication or division, replace the given numbers by smaller numbers. Reread the problem with these small numbers, and decide upon the process.

3. Will the numbers to be recorded in the last column of the preceding blank be larger or smaller than those in the first column?

Written Work

Rule a blank similar to the one above.

Find the total value of the butter produced in each section. (Approximate results to the nearest dollar.)

Enter the sections on the blank in the order of their rank as butter producers.

REVIEW OF DECIMAL FRACTIONS

Written Work

1. In a recent year 1,591,311,371 dozen eggs were produced in the United States. The following decimals show the fractional part of this number produced in the different geographical sections.

New England	.035
Middle Atlantic	.102
East North Central	.247
West North Central	.28
South Atlantic	.086
East South Central	.081
West South Central	.104
Mountain	.022
Pacific	.043

Find the number of dozen eggs produced in each section, and tabulate this on a ruled form. How can you check the accuracy of your work?

2. The value of the wool produced by the mountain states in 1909 was .608 more than the value of the wool produced by these states in 1899. The increase in dollars was $11,039,843. What was the value in 1899? In 1909?

3. The mountain states include a large portion of the grazing land of the country and for this reason produce a large part of the wool of the country. In 1909 they produced .4332 of the total value of the wool grown in the United States. By referring to your results in the preceding problem, find the total value of the wool grown in the United States in 1909.

4. In 1909 Wyoming produced the largest value of wool grown in any state of the Union. The value of its product was $8,912,608. This was what decimal part of the value of the wool product of the entire country for the year?

5. Montana, the second largest producer of wool, raised wool valued at $8,223,754. Ohio, the third largest producer, raised wool valued at $6,749,005. Fill in the blanks in the following sentences with the proper decimal fractions:

Wyoming produced —— times as much wool as Montana.
Ohio produced —— as much wool as Wyoming.

6. If a boy's average gain in weight is 8.25 lb. per year, how much will he gain in weight in 2½ yr.?

7. If one turn of a screw advances the point .14 in., how far will 7 turns advance it?

8. A gallon of milk weighs 8.622 lb. and a gallon of water weighs 8.355 lb. How much more does ½ gal. of milk weigh than ½ gal. of water?

9. A boy can run 100 yd. in 11.2 sec. At this rate, how long would it take him to run 80 yd.?

10. In a certain city the rainfall during March was 2.46 in., during April 3.15 in., and during May 1.09 in. Find the average for these months.

11. A man sold 24 dozen eggs, which was .4 of all that he had. How many did he have?

12. If .375 of an acre of land sells for $48, what should 2½ acres sell for, at the same rate?

13. An English pound contains 113.00001 grains of pure gold, and an American gold dollar contains 23.22 grains of pure gold. Find the value of an English pound in American money.

CHAPTER IX

SHORT METHODS INVOLVING ALIQUOT PARTS

69. Equivalent Common and Decimal Fractions in Frequent Use.

$.50 = \frac{1}{2}$.
$.33\frac{1}{3} = \frac{1}{3}$.
$.25 = \frac{1}{4}$.
$.20 = \frac{1}{5}$.
$.16\frac{2}{3} = \frac{1}{6}$.
$.14\frac{2}{7} = \frac{1}{7}$.
$.12\frac{1}{2} = \frac{1}{8}$.
$.11\frac{1}{9} = \frac{1}{9}$.
$.10 = \frac{1}{10}$.
$.09\frac{1}{11} = \frac{1}{11}$.
$.08\frac{1}{3} = \frac{1}{12}$.
$.06\frac{2}{3} = \frac{1}{15}$.
$.06\frac{1}{4} = \frac{1}{16}$.
$.05 = \frac{1}{20}$.
$.40 = \frac{2}{5}$.
$.37\frac{1}{2} = \frac{3}{8}$.
$.60 = \frac{3}{5}$.
$.62\frac{1}{2} = \frac{5}{8}$.
$.75 = \frac{3}{4}$.
$.80 = \frac{4}{5}$.
$.83\frac{1}{3} = \frac{5}{6}$.
$.87\frac{1}{2} = \frac{7}{8}$.

Commit these common fractions and their equivalent decimals to memory, as they can be used to shorten many computations.

70. A Short Method of Multiplying by These Fractions.

Which is easier, $1248 \times .16\frac{2}{3} = 208$; or $\frac{1}{6}$ of $1248 = 208$?

State a short method of multiplying by each of the decimal fractions in the table above.

Oral Work

Multiply:

1. $642 \times .50$.
2. $936 \times .33\frac{1}{3}$.
3. $488 \times .25$.
4. $650 \times .20$.
5. $366 \times .16\frac{2}{3}$.
6. $840 \times .14\frac{2}{7}$.
7. $720 \times .12\frac{1}{2}$.
8. $456 \times .08\frac{1}{3}$.
9. $930 \times .06\frac{2}{3}$.
10. $320 \times .06\frac{1}{4}$.

The results in the preceding exercise are integers. The same method may be used when the division results in a fraction.

Example. Multiply 4634 by .25.

SOLUTION. $.25 = \frac{1}{4}$.

$4634 \div 4 = 1158.5$.

Oral Work

Without copying, find the results of the following:

1. $8735 \times .50$.
2. $3940 \times .33\frac{1}{3}$.
3. $267 \times .25$.
4. $379.3 \times .16\frac{2}{3}$.
5. $842 \times .20$.
6. $3824 \times .14\frac{2}{7}$.
7. $5316 \times .12\frac{1}{2}$.
8. $4026 \times .08\frac{1}{3}$.
9. $1848 \times .06\frac{2}{3}$.
10. $1940 \times .06\frac{1}{4}$.
11. $35.69 \times .33\frac{1}{3}$.
12. $649.3 \times .25$.
13. $.8347 \times .50$.
14. $723.68 \times .20$.
15. $36.92 \times .14\frac{2}{7}$.
16. $56.32 \times .12\frac{1}{2}$.
17. $7836 \times .06\frac{1}{4}$.
18. $1223 \times .16\frac{2}{3}$.
19. $3380 \times .06\frac{2}{3}$.

20. Find the value of each of the following:

48 lb. at $37\frac{1}{2}$¢ per pound.
32 lb. at $12\frac{1}{2}$¢ per pound.
21 lb. at $87\frac{1}{2}$¢ per pound.
24 yd. at $12\frac{1}{2}$¢ per yard.
30 yd. at $62\frac{1}{2}$¢ per yard.
21 lb. at $14\frac{2}{7}$¢ per pound.
48 yd. at $6\frac{2}{3}$¢ per yard.
96 lb. at $83\frac{1}{3}$¢ per pound.

71. A Short Method of Dividing by These Fractions.

Which is easier, $6 \div .25 = 24$; or $6 \times 4 = 24$?

State a short method of dividing by each of the decimal fractions in the table on page 85.

Oral Work

Without copying, state the results of the following:

1. $7 \div .12\frac{1}{2}$.
2. $49 \div .50$.
3. $126 \div .33\frac{1}{3}$.
4. $63 \div .14\frac{2}{7}$.
5. $8 \div .06\frac{1}{4}$.
6. $10 \div .06\frac{2}{3}$.
7. $15 \div .25$.
8. $1627 \div .20$.
9. $13.63 \div .16\frac{2}{3}$.
10. $1.43 \div .25$.
11. $3.249 \div .50$.
12. $.023 \div .08\frac{1}{3}$.
13. $63.98 \div .2$.
14. $.043 \div .16\frac{2}{3}$.

72. Aliquot Parts which are not Unit Fractions.

State a short method of multiplying by $.37\frac{1}{2}$; by $.66\frac{2}{3}$; by $.75$; by $.87\frac{1}{2}$.

State a short method of dividing by $.37\frac{1}{2}$; by $.66\frac{2}{3}$; by $.75$; by $.87\frac{1}{2}$.

When dividing by one number and multiplying by another, observe the following directions:

Divide first if the division results in an integer. If the division would result in a mixed number, multiply first.

Examples. 1. Multiply 32 by $.37\frac{1}{2}$.

Solution. $.37\frac{1}{2} = \frac{3}{8}$.
$32 \div 8 = 4$.
$4 \times 3 = 12$. Since 8 is a factor of 32, the division is performed first.

2. Divide 16 by $.37\frac{1}{2}$.

Solution. Substituting the equivalent common fraction, we have
$16 \div \frac{3}{8} = ?$
or $16 \times \frac{8}{3} = ?$ Inverting the divisor.
$16 \times 8 = 128$.
$128 \div 3 = 42\frac{2}{3}$. Since 3 is not a factor of 16, the multiplication is performed first.

What advantage is there in observing these directions?

Written Work

Without recopying, find the results of the following:

1. $64 \times .375$.
2. $96 \times .66\frac{2}{3}$.
3. $480 \times .75$.
4. $168 \times .875$.
5. $248 \times .87\frac{1}{2}$.
6. $3260 \times .75$.
7. $4864 \times .37\frac{1}{2}$.
8. $639 \times .66\frac{2}{3}$.
9. $42 \div .37\frac{1}{2}$.
10. $56 \div .87\frac{1}{2}$.
11. $40 \div .66\frac{2}{3}$.
12. $36 \div .75$.
13. $1462 \div .66\frac{2}{3}$.
14. $465 \div .375$.
15. $11322 \div .75$.
16. $994 \div .875$.
17. $426 \times .875$.
18. $143 \times .37\frac{1}{2}$.
19. $1347 \times .75$.
20. $539 \times .66\frac{2}{3}$.
21. $3.29 \times .75$.
22. $1.267 \times .875$.
23. $139 \div .87\frac{1}{2}$.
24. $1426 \div .37\frac{1}{2}$.
25. $139 \div .66\frac{2}{3}$.
26. $46 \div .75$.
27. $46.9 \div .37\frac{1}{2}$.
28. $.429 \div .66\frac{2}{3}$.

73. General Table of Aliquot Parts. The following table shows the most frequently used decimal parts of 1, 10, 100, and 1000.

FRACTIONS	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{8}$	$\frac{3}{8}$
of 1	.50	$.33\frac{1}{3}$	$.66\frac{2}{3}$	.25	.75	.125	.375
of 10	5.	$3.33\frac{1}{3}$	$6.66\frac{2}{3}$	2.5	7.5	1.25	3.75
of 100	50.	$33.33\frac{1}{3}$	$66.66\frac{2}{3}$	25.	75.	12.5	37.5
of 1000	500.	$333\frac{1}{3}$	$666\frac{2}{3}$	250.	750.	125.	375.

FRACTIONS	$\frac{5}{8}$	$\frac{5}{6}$	$\frac{7}{8}$	$\frac{1}{12}$	$\frac{1}{15}$	$\frac{1}{16}$	$\frac{1}{6}$	$\frac{1}{7}$
of 1	.625	$.83\frac{1}{3}$	.875	$.08\frac{1}{3}$	$.06\frac{2}{3}$	$.06\frac{1}{4}$	$.16\frac{2}{3}$	$.14\frac{2}{7}$
of 10	6.25	$8.33\frac{1}{3}$	8.75	$.82\frac{1}{3}$		.625	$1.6\frac{2}{3}$	1.42
of 100	62.5	$83.3\frac{1}{3}$	87.5	$8.3\frac{1}{3}$		6.25	$16\frac{2}{3}$	14.28
of 1000	625.	$833.3\frac{1}{3}$	875.	$83\frac{1}{3}$		62.5	$166\frac{2}{3}$	142.8

Study this table until you can recognize quickly these fractional parts of 1, 10, 100, and 1000.

74. Rules for Multiplication by Aliquot Parts of 1, 10, 100, and 1000.

a. To multiply by fractional parts of 10.

Multiply by the equivalent fraction; multiply this product by 10, *by moving the decimal point one place to the right, annexing a zero, if necessary.*

Examples. 1. Multiply 16 by $2\frac{1}{2}$.

SOLUTION. $2\frac{1}{2}$ is $\frac{1}{4}$ of 10. $\frac{1}{4}$ of $16 = 4$. Multiply by 10, by annexing a zero; the result is 40.

2. Multiply 26 by 1.25.

SOLUTION. 1.25 is $\frac{1}{8}$ of 10. $\frac{1}{8}$ of $26 = 3.25$. Move the decimal point one place to the right; the result is 32.5.

b. To multiply by fractional parts of 100.

Multiply by the equivalent common fraction; multiply this product by 100.

Examples. 1. Multiply 24 by $87\frac{1}{2}$.

Solution.
$87\frac{1}{2}$ is $\frac{7}{8}$ of 100.
$\frac{7}{8}$ of $24 = 21$.
$21 \times 100 = 2100$.

2. Multiply 4252 by 12.5.

Solution.
12.5 is $\frac{1}{8}$ of 100.
$\frac{1}{8}$ of $4252 = 531.5$.
$531.5 \times 100 = 53{,}150$.

Oral Work

State a short method of multiplying by each of the following:

$.06\frac{1}{4}$; 2.50; $6\frac{2}{3}$; $33\frac{1}{3}$; 250; $87\frac{1}{2}$; 6.25;
$37\frac{1}{2}$; 7.50; 5; $333\frac{1}{3}$; 75; 125; $8\frac{1}{3}$;
$2\frac{1}{2}$; 12.50; $6\frac{1}{4}$; $62\frac{1}{2}$; $1\frac{1}{4}$; $8\frac{3}{4}$; 25;
$66\frac{2}{3}$; 375; 750; 875; 37.50; 7500; 12,500.

Find the products of the following. Do the work orally when possible. Be prepared to explain how the short method was applied in each example.

1. 32×25. **2.** $111 \times 3\frac{1}{3}$. **3.** $390 \times 66\frac{2}{3}$.
4. 648×250. **5.** 724×75. **6.** $832 \times 1\frac{1}{8}$.
7. $96 \times 87\frac{1}{2}$. **8.** $108 \times 83\frac{1}{3}$. **9.** $3264 \times .625$.
10. $144 \times 8\frac{1}{3}$. **11.** 80×875. **12.** 48×375.
13. 464×12.50. **14.** $24 \times 7\frac{1}{2}$. **15.** 3.20×25.
16. $.27 \times 6\frac{2}{3}$. **17.** 8.4×3.75. **18.** 640×6.25.
19. $3.612 \times .83\frac{1}{3}$. **20.** $208 \times 3\frac{3}{4}$. **21.** 143×250.
22. 1936×62.50. **23.** 1721×7.50. **24.** 1239×125.
25. 426×750.

75. Interchanging Multiplicand and Multiplier. When the multiplicand and multiplier are abstract numbers, they may be interchanged.

Example. Multiply 25 by 428.

Solution. Interchanging, 428×25
$\frac{1}{4}$ of $428 = 107$.
$107 \times 100 = 10{,}700$.

Written Work

Perform the following multiplications as indicated. Explain how the short method was applied in each example.

1. 924×12.50.
2. 864×750.
3. 125×488.
4. $264 \times .12\frac{1}{2}$.
5. $45 \times .06\frac{2}{3}$.
6. 250×3288.
7. 62.50×648.
8. $8\frac{1}{3} \times 3.60$.
9. 342×87.50.
10. 375×112.
11. $6\frac{2}{3} \times 456$.
12. 1468×250.
13. 875×88.
14. 1250×96.
15. $1776 \times .625$.
16. $7.2 \times 8\frac{3}{4}$.
17. $1\frac{1}{4} \times 104$.
18. 125×1.248.
19. 66×875.
20. $83\frac{1}{3} \times 9.6$.
21. $62\frac{1}{2} \times 20$.

Find the cost of the following:

22. 12 articles at \$.50 each
16 articles at .25 each
18 articles at $.33\frac{1}{3}$ each
30 articles at .40 each
16 articles at .75 each
Total

23. 64 doz. articles at \$ $.62\frac{1}{2}$ per dozen
76 doz. articles at 8.75 per dozen
58 doz. articles at .75 per dozen
125 doz. articles at .16 per dozen
$6\frac{1}{4}$ doz. articles at .32 per dozen
Total

24. 72 lb. at \$ $.83\frac{1}{3}$ per pound
52 lb. at 1.25 per pound
44 lb. at $.37\frac{1}{2}$ per pound
90 lb. at $.12\frac{1}{2}$ per pound
112 lb. at $.06\frac{1}{4}$ per pound
Total

25. 80 yd. at \$ $.62\frac{1}{2}$ per yard
$66\frac{2}{3}$ yd. at 1.20 per yard
86 yd. at 1.25 per yard
25 yd. at .78 per yard
75 yd. at $.66\frac{2}{3}$ per yard
Total

26. $83\frac{1}{3}$ yd. at $1.80 per yard
$12\frac{1}{2}$ doz. at 1.60 per dozen
$8\frac{3}{4}$ lb. at 1.60 per pound
$6\frac{2}{3}$ yd. at 2.10 per yard
$3\frac{1}{3}$ yd. at 1.80 per yard
Total

27. 135 yd. at $$.06\frac{1}{4}$ per yard
56 yd. at $.08\frac{3}{4}$ per yard
96 yd. at $.16\frac{2}{3}$ per yard
152 yd. at $.12\frac{1}{2}$ per yard
24 yd. at $.07\frac{1}{2}$ per yard
Total

76. Rules for Division by Aliquot Parts of 1, 10, 100, 1000.

a. To divide by fractional parts of 1.

Divide by the equivalent common fraction.

Example. Divide 13 by $.12\frac{1}{2}$. SOLUTION. $.12\frac{1}{2} = \frac{1}{8}$.
$13 \div \frac{1}{8} = 13 \times \frac{8}{1} = 104$.

b. To divide by fractional parts of 10.

Divide by 10; *divide this quotient by the equivalent common fraction.*

Example. Divide 80 by 2.50. SOLUTION. 2.50 is $\frac{1}{4}$ of 10.
$80 \div 10 = 8$.
$8 \div \frac{1}{4} = 8 \times \frac{4}{1} = 32$.

c. To divide by fractional parts of 100.

Divide by 100; *divide this quotient by the equivalent common fraction.*

Oral Work

1. State a rule for dividing by fractional parts of 1000.

2. $100 will pay for how many articles at $$.33\frac{1}{3}$ each?

3. $12 will pay for how many yards of cloth at $.25 per yard?

4. Which is easier —

$14 \div .75 = ?$ or $14 \div \frac{3}{4} = ?$

5. State how to find the total cost of some articles, when the number of articles bought is given and the cost of each is $8\frac{1}{3}$¢; $14\frac{2}{7}$¢; $87\frac{1}{2}$¢; $16\frac{2}{3}$¢; $33\frac{1}{3}$¢; $62\frac{1}{2}$¢; $37\frac{1}{2}$¢.

Without copying, state results for the following examples. State the method of operation in each.

6. $15 \div .12\frac{1}{2}$. **7.** $14 \div .06\frac{1}{4}$.

8. $9 \div .25$. **9.** $\$6 \div \$.75$.

10. $\$14 \div \$.87\frac{1}{2}$. **11.** $\$8 \div \$1.33\frac{1}{3}$.

State the quotients of each of the following:

	DIVIDEND	DIVISOR	QUOTIENT		DIVIDEND	DIVISOR	QUOTIENT
12.	42	$.33\frac{1}{3}$		**13.**	13	.25	
14.	22	$.66\frac{2}{3}$		**15.**	36	.75	
16.	5	$.06\frac{1}{4}$		**17.**	45	$.37\frac{1}{2}$	
18.	17	.125		**19.**	28	87.5	
20.	15	1.25		**21.**	900	6.25	
22.	400	$8.33\frac{1}{3}$		**23.**	150	$.06\frac{2}{3}$	

Written Work

Complete the following:

	TOTAL COST	PRICE EACH	NUMBER PURCHASED
1.	$ 5.25	$ $.37\frac{1}{2}$	
2.	20.25	.75	
3.	56.00	$.66\frac{2}{3}$	
4.	40.00	1.25	
5.	77.00	$.87\frac{1}{2}$	
6.	212.50	2.50	
7.	97.50	3.75	

8. Divide each of the following by $.12\frac{1}{2}$:

$.37\frac{1}{2}$; $.33\frac{1}{3}$; $.625$; $.87\frac{1}{2}$; $.25$; $1.14\frac{2}{7}$.

9. Find the sum of $\frac{1}{4}$, $\frac{1}{5}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, and $\frac{1}{2}$ and express the sum decimally.

10. Divide 36 by each of the following:

$.37\frac{1}{2}$; $.12\frac{1}{8}$; $.14\frac{2}{7}$; $.50$; $.06\frac{1}{4}$.

11. How many yards can be bought in each case:

Total Cost	Cost per Yard	Number of Yards
$16	$ .25	
24	$.33\frac{1}{3}$	
36	$.37\frac{1}{2}$	
20	$.62\frac{1}{2}$	
32	.50	
26	$.66\frac{2}{3}$	
56	1.25	
44	$1.33\frac{1}{3}$	
24	$2.66\frac{2}{3}$	

Review of Short Methods

Perform the following operations, telling what short method was used in each case:

1. $360 \div .37\frac{1}{2}$.
2. $\frac{2}{7} + \frac{3}{8}$.
3. 436×11.
4. 65×25.
5. 463×287.
6. 1016×1005.
7. 3654×2700.
8. 105×126.
9. 95×87.
10. $\frac{1}{3} - \frac{1}{5}$.
11. $48\frac{1}{4} \times 63\frac{7}{8}$.
12. $61\frac{1}{2} \times 12\frac{1}{2}$.
13. 109×113.
14. $\frac{9}{16} + \frac{8}{11}$.
15. 463×416.
16. 96×94.
17. 1015×1014.
18. 13×18.
19. $6\frac{1}{2} \times 4\frac{1}{2}$.
20. 825×927.
21. $.000825 \times 100$.
22. 17×12.
23. 724×999.
24. 116×104.
25. $\frac{3}{5} \div \frac{7}{8}$.
26. 113×102.
27. 1026×1003.
28 $\frac{1}{5} + \frac{1}{3}$.
29. 5362×111.
30. 433×99.
31. 109×112.
32. 976×990.
33. $33\frac{1}{2} \times 12\frac{1}{2}$.
34. 17×19.
35. $13\frac{1}{2} \times 14\frac{2}{5}$.
36. $\frac{3}{7} + \frac{3}{8}$.
37. $\frac{3}{7} - \frac{3}{8}$.
38. $15\frac{1}{2} \times 18\frac{1}{2}$.
39. 96×88.
40. 905.387×3000.
41. 986×993.
42. 16×18.
43. 874×1236.

UNITS OF MEASURE AND THEIR APPLICATIONS

CHAPTER X

DENOMINATE NUMBERS

TABLES OF WEIGHTS AND MEASURES

77. Long Measure.

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet = 1 rod (rd.)
320 rods, or 5280 feet = 1 mile (mi.)
1760 yards = 1 mile

Architects, carpenters, and mechanics frequently write ′ for foot, and ″ for inch. Thus 8′ 7″ means 8 ft. 7 in.

In engineering it is customary to divide the foot and the inch into tenths and hundredths, instead of into halves and fourths. There is a growing tendency to use the decimal division of the units.

Other measures of length are:

1 hand = 4 in. Used in measuring the height of horses.
1 fathom = 6 ft. Used in measuring depths at sea.
1 knot, nautical or geographical mile = $1.1526\frac{2}{3}$ mi. or 6086 ft.

The knot is used in measuring distances at sea. It is equivalent to 1 min. of longitude at the equator.

78. Square Measure.

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards = 1 square rod (sq. rd.)
160 square rods = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)

Sq. ′ and sq. ″ are frequently used for square foot and square inch. Thus, 15 sq. ′ 6 sq. ″ means 15 sq. ft. 6 sq. in.

A *square* is 100 sq. ft. It is used in measuring roofing.

79. Cubic Measure.

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)
128 cubic feet = 1 cord

A cubic yard (of earth) is considered a load.

$24\frac{3}{4}$ cubic feet = 1 perch (P.)

A *perch* of stone or masonry is $16\frac{1}{2}$ ft. (1 rd.) long, $1\frac{1}{2}$ ft. wide, and 1 ft. high.

80. Avoirdupois Weight.

16 ounces (oz.) = 1 pound (lb.)
100 pounds = 1 hundredweight (cwt.)
2000 pounds = 1 short ton (T.)
2240 pounds = 1 long ton (T.)

Avoirdupois weight is used in weighing all ordinary substances except precious metals, jewels, and drugs at retail.

The long ton is used in mining and in the United States custom-house.

81. Liquid Measure.

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)

A gallon contains 231 cu. in.

31.5 gal. are considered 1 barrel (bbl.).

63 gal. = 1 hogshead (hhd.).

82. Dry Measure.

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

A bushel contains 2150.42 cu. in.

A dry quart contains 67.2 cu. in.

A liquid quart contains 57.75 cu. in.

83. Measures of Time.

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
30 days = 1 commercial month (mo.)
52 weeks = 1 year (yr.)
12 months = 1 year
360 days = 1 commercial year
365 days = 1 common year
366 days = 1 leap year
100 years = 1 century

84. Counting.

12 units = 1 dozen (doz.)
12 dozen = 1 gross (gro.)
12 gross = 1 great gross (gr. gro.)
20 units = 1 score

85. Wood Measure.

A straight pile of wood, 4 ft. × 4 ft. × 1 ft., contains 1 cord foot. 8 cord feet, or 128 cubic feet = 1 cord.

86. United States Money.

10 mills = 1 cent
10 cents = 1 dime
10 dimes = 1 dollar
10 dollars = 1 eagle

The mill is not a coin, but the term is frequently used in computations. The term "eagle" is not common in business.

87. English Money.

4 farthings (far.) = 1 penny (*d.*), (plural, "pence")
12 pence = 1 shilling (*s.*) = $\$.243^{+}$
20 shillings = 1 pound sterling (£), $4.8665

88. French Money.

100 centimes (c.) = 1 franc (fr.)

A franc is worth about $.193.

89. German Money.

100 pfennigs (pf.) = 1 mark (M.)

A mark is worth about $0.238.

Written Work

1. Draw a line 1 foot long at the blackboard. Try to form a very definite mental picture of this length so that you may be able to estimate lengths with a fair degree of accuracy.

2. Draw lines of the following lengths without any aid, then check each estimate.

$\frac{1}{2}$ ft., 2 ft., 3 ft., $\frac{2}{3}$ ft., $1\frac{1}{2}$ ft., 7 ft., 3 in., 9 in., 15 in.

3. Estimate the length and width of your desk; the length and width and height of the classroom; the dimensions of the school building and grounds. Check your estimates.

4. Practice estimating and stepping off various distances until you can do so with a fair degree of accuracy.

5. Lay off on the school grounds a square rod and, if possible, an acre. Estimate the areas of various lots, parks, or fields, and check your results, when possible.

6. By lifting various units of weights and then checking your own estimates of the weights of numerous objects, you can acquire the ability to estimate certain weights with but a small error.

90. Changing to Lower Denominations.

Example. How many quarts in 3 bu. 3 pk. 5 qt.?

SOLUTION.
3 bu. = 12 pk.
12 pk. + 3 pk. = 15 pk.
15 pk. = 120 qt.
120 qt. + 5 qt. = 125 qt.
Therefore, 3 bu. 3 pk. 5 qt. = 125 qt.

After studying this solution, state a method for reducing measured quantities to lower denominations.

Oral Work

Reduce as indicated; do as much of the work orally as possible.

1. $15\frac{1}{3}$ yd. to feet.

2. $4\frac{2}{3}$ ft. to inches.

3. 5 gal. 3 qt. 1 pt. to pints.

4. 4 bu. 3 pk. 5 qt. to quarts.

5. 5 lb. 9 oz. to ounces (avoir.).

Example. Reduce .345 mi. to lower denominations.

SOLUTION.
1 mi. = 320 rd.
.345 mi. = 110.4 rd.
1 rd. = 5.5 yd.
.4 rd. = 2.2 yd.
1 yd. = 3 ft.
.2 yd. = .6 ft.
1 ft. = 12 in.
.6 ft. = 7.2 in.

Therefore, .345 mi. = 110 rd. 2 yd. 0 ft. 7.2 in.

After studying this solution, state a method for reducing fractions of a large denomination to units of lower denominations.

Written Work

Reduce:

1. $\frac{8}{9}$ mi. to rods.

2. .236 mi. to feet.

3. .89 bu. to quarts.

4. $\frac{13}{24}$ sq. mi. to square rods.

5. 62 rd. to feet.

6. .85 gal. to pints.

7. 4.3 mi. to feet.

8. A man bought an acre of land and sold it in building lots of 10 sq. rd. each. How many lots did he sell?

91. Changing to Higher Denominations. When a quantity is expressed in a low denomination, it may be desired to express it in higher denominations.

Examples. 1. Express 1316 pints in higher denominations.

Solution. Since 2 pints = 1 quart,
1316 pints = 658 quarts.
Since 4 quarts = 1 gallon,
658 quarts = 164 gals., and 2 qt. remaining.
Therefore, 1316 pints = 164 gal., 2 qt.

After studying this solution, state a method for reducing quantities to units of higher denominations.

2. Reduce 3 qt. 1 pt. to a decimal part of a peck.

Solution. 3 qt. 1 pt. = 7 pt.
1 pk. = 16 pt.
Therefore, 3 qt. 1 pt. = $\frac{7}{16}$ pk.
$\frac{7}{16}$ pk. = .4375 pk.

Written Work

Reduce to units of higher denominations:

1. 367 pints (dry measure). **2.** 133 in. **3.** 47 ft.
4. 12 yd. **5.** 216 oz. (avoir.).
6. 31 pt. (liquid measure). **7.** 2369 sq. in.
8. Reduce 4 sq. ft. 68 sq. in. to a decimal part of a square rod.
9. Reduce 1 qt. 2 pt. to a decimal part of a gallon.
10. Reduce 7 oz. to a decimal part of a pound (avoir.).
11. Reduce $\frac{2}{3}$ yd. to a decimal part of a rod.
12. Reduce $2\frac{1}{2}$ ft. to a decimal part of a yard.
13. Reduce 47 sq. rd. to a decimal part of an acre.
14. Reduce $8\frac{1}{2}$ in. to a decimal part of a foot.
15. Reduce $2\frac{1}{2}$ qt. to a decimal part of a gallon.
16. Reduce 4 ft. 7 in. to a decimal part of a rod.

92. Addition of Denominate Numbers.

Example. Find the sum of 2 yd. 2 ft. 9 in. and 4 yd. 1 ft. 7 in.

Solution.

	2 yd.	2 ft.	9 in.
	4 "	1 "	7 "
	6 yd.	3 ft.	16 in.
or	7 "	1 "	4 "

State a method for adding denominate numbers.

Written Work

Add:

1.

2 yr.	5 mo.	18 da.
5 “	8 “	19 “
6 “	11 “	13 “

2.

5 gal.	3 qt.	1 pt.
4 “	2 “	1 “
5 “	1 “	1 “

3.

2 lb.	5 oz.	17 pwt.
3 “	8 “	19 “
4 “	9 “	12 “

4.

5 A.	126 sq. rd.	
16 “	249 “	“
13 “	168 “	“

5.

7 mo.	12 da.
5 “	5 “

6.

12 mi.	4 rd.
8 “	9 “

7. A rectangular field is 18 rd. 2 yd. 2 ft. long, and 13 rd. 1 yd. 2½ ft. wide. What length of wire will be required for a fence 6 wires high?

8. A dealer bought 17 gal. 3 qt. of milk from each of two men, and 29 gal. 3 qt. 1 pt. from a third man. What was the entire cost at 19 cents a gallon?

93. Subtraction of Denominate Numbers.

If it is required to subtract 1 mi. 42 rd. 13 ft. from 3 mi. 25 rd. 14 ft., we write the numbers as follows:

3 mi.	25 rd.	14 ft.
1 “	42 “	13 “

When, as in this problem, the number of units of some denomination of the minuend is smaller than the number of corresponding units of the subtrahend, combine one unit of the next larger denomination with the number of units of the minuend. The problem thus becomes:

2 mi.	345 rd.	14 ft.
1 “	42 “	13 “
1 “	303 “	1 “

Written Work

Subtract as indicated:

1. 13 bu. 3 pk. 2 qt.
6 " 2 " 7 "

2. 7 A. 5 sq. rd. 6 sq. ft.
3 " 7 " " 2 " "

3. 27 rd. 4 yd. 1 ft.
13 " 5 " 2 "

4. 14 gal. 2 qt. 1 pt.
5 " 3 "

5. 19 lb. 12 oz.
7 " 14 "

6. A man owned a field containing $1\frac{2}{5}$ A. From it he sold two lots each having an area of $11\frac{1}{2}$ sq. rd. What was the area of the part remaining?

7. From a cask which contained 31 gal. 3 qt. 1 pt. of vinegar, 19 gal. 3 pt. were drawn out. How much remained?

94. Multiplication Involving Denominate Numbers.

Example. Multiply 3 yd. 2 ft. 7 in. by 4.

SOLUTION.
3 yd. 2 ft. 7 in.
4
12 yd. 8 ft. 28 in.
or 15 yd. 1 ft. 4 in.

Written Work

Multiply as indicated:

1. 5 gal. 3 qt. 1 pt. by 7.

2. 7 lb. 5 oz. by 12.

3. 5 cu. yd. 19 cu. ft. 364 cu. in. by 13.

4. 4 mi. 19 rd. 3 yd. by 18.

95. Division Involving Denominate Numbers.

Examples. 1. Divide 4568 inches by 8.

SOLUTION.
4568 in. ÷ 8 = 571 in.
571 in. = 2 rd. 8 yd. 2 ft. 7 in.

2. Divide 17 gal. 2 qt. 1 pt. by 3.

SOLUTION.
17 gal. 2 qt. 1 pt. = 141 pt.
141 pt. ÷ 3 = 47 pt.
47 pt. = 5 gal. 3 qt. 1 pt.

3. How many times is 4 gal. 2 qt. contained in 1 barrel ?

SOLUTION.
4 gal. 2 qt. = 18 qt.
1 bbl. = 126 qt.
126 qt. contains 18 qt. 7 times.

Explain the method used in this illustration.

Written Work

Divide as indicated:

1. 27 cu. yd. 15 cu. ft. ÷ 6.

2. 584 A. 8 sq. rd. ÷ 8.

3. 7 sq. ft. 48 sq. in ÷ 16.

4. How many times are 4 yd. 2 ft. 7 in. contained in 29 yd. 6 in. ?

5. Reduce to the decimal part of a mile: 5 rd.; $7\frac{1}{2}$ rd.; 5 ft.; 13 yd.; 5 ft. 9 in.

6. An automobile traveled 1 mi. in $3\frac{1}{2}$ min. What was the rate per hour ?

7. If a man sells 17 loads of wheat, each containing 53 bu. 3 pk., at $97\frac{1}{2}$ cents a bushel, how much should he receive ?

8. Fifteen cans hold an average of 10 gal. 2 qt. 1 pt. How much do they all hold ?

9. Change .427 sq. mi. to units of lower denominations.

10. A cubic foot of water weighs 62.5 lb., cast iron weighs about 7.2 as much as water; what is the weight of $2\frac{3}{8}$ cu. ft. of cast iron ?

11. Determine whether the window space in your school room is as much as $\frac{1}{6}$ of the floor space. It should be at least $\frac{1}{6}$ in a well-lighted room.

12. A train leaves a city at 4 : 30 P.M., and reaches a certain station $87\frac{1}{2}$ mi. distant at 6 : 20 P.M. Allowing 12 min. for station stops, what is the rate of travel per hour ?

13. There are twenty-four students in a class and each needs $2\frac{1}{2}$ cups of hot water for a cooking lesson. How much water must there be in a kettle to supply all the class at one time, allowing 4 cups to a quart ?

CHAPTER XI

THE METRIC SYSTEM

THE **metric system** of weights and measures originated in France about 1800. An international convention met at Paris in 1799 and adopted the system, but it was not until forty years later that it came into general use in France. Since that time its use has become universal in scientific measurements, and it is established as the commercial system in a large part of the civilized world. The United States made the metric system "lawful" in 1866, and obligatory in Porto Rico and the Philippine Islands about 1900. The advantage of the metric system lies in the fact that it is based on a decimal scale. The only multiplier or divisor used is 10 or a power of 10. All reductions are made by merely changing the position of the decimal point.

96. Definitions of Terms. The unit of **length** is the **meter.** It is approximately 39.37 inches.

The unit of **capacity** is the **liter** (lēter), which is equivalent to a cube having an edge one-tenth of a meter long.

The unit of **weight** is the **gram,** which is the weight of a cube of distilled water having an edge of $\frac{1}{100}$ of a meter.

97. Fractions and Multiples of Units.

Fractional parts of these units are expressed by Latin prefixes, as follows:

milli- = one thousandth
centi- = one hundredth.
deci- = one tenth.

Multiples of the units are expressed by Greek prefixes, as follows:

deka- = ten.
hekto- = one hundred.
kilo- = one thousand.
myria- = ten thousand.

Thus,
a millimeter is .001 of a meter.
a milligram is .001 of a gram.
a centigram is .01 of a gram.
a dekaliter is 10 liters.

The prefixes and their meanings should be learned.

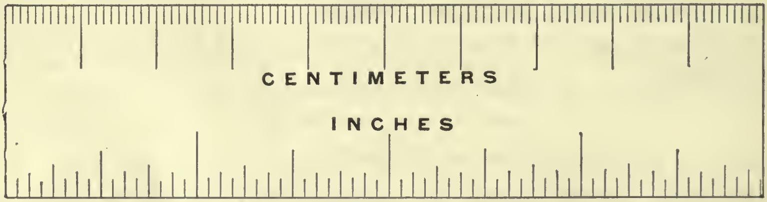
98. Measures of Length.

1 millimeter (mm.) = .001 of a meter.
1 centimeter (cm.) = .01 of a meter.
1 decimeter (dm.) = .1 of a meter.
1 METER (m.) = 1 meter.
1 dekameter (Dm.) = 10 meters.
1 hektometer (Hm.) = 100 meters.
1 kilometer (Km.) = 1000 meters.
1 myriameter = 10,000 meters.

The table may be expressed as follows:

10 millimeters = 1 centimeter.
10 centimeters = 1 decimeter.
10 decimeters = 1 meter.
10 meters = 1 dekameter.
10 dekameters = 1 hektometer.
10 hektometers = 1 kilometer.

The following rule is 10 centimeters, or 1 decimeter, long. The smallest subdivisions are millimeters.



The meter is used for measuring certain merchandise and for other purposes where the foot and yard would be used in the English system. It is also used in engineering and constructions. The kilometer is used for measuring distances which would be expressed in the English system in miles.

The centimeter is used instead of the inch in certain measurements such as in expressing the size of paper and books. The millimeter is used for very fine measurements in machine construction and similar work.

Examples. 1. Reduce 473 m. to Km.

SOLUTION. 473 m. = .473 Km.

2. Reduce .54 Km. to m.

SOLUTION. .54 Km. = 540 m.

Oral Work

1. Reduce 734 m. to Hm.; Km.; dm.
2. Reduce .042 Km. to cm.; Hm.; mm.
3. Reduce 5427 cm. to m.; Km.
4. Reduce .0037 Km. to cm.
5. Reduce 4.72 Km. to dm.
6. Reduce .034 m. to Km.
7. Reduce $1\frac{1}{2}$ cm. to mm.
8. Reduce .0842 Km. to m.

The advantage of the metric system is well illustrated in the measurements of distances. A distance of 1 kilometer, 6 dekameters, 4 decimeters, 8 centimeters, is expressed as 1060.48 meters, and is read "ten hundred sixty meters and forty-eight centimeters, just as we would say "ten hundred sixty dollars and forty-eight cents."

99. Measures of Area.

The table of area is formed by squaring the units of length, as in the English system.

100 sq. millimeters (sq. mm.) = 1 sq. centimeter (sq. cm.).
100 sq. centimeters = 1 sq. decimeter (sq. dm.).
100 sq. decimeters = 1 sq. meter (sq. m.).
100 sq. meters = 1 sq. dekameter (sq. Dm.).
100 sq. dekameters = 1 sq. hektometer (sq. Hm.).
100 sq. hektometers = 1 sq. kilometer (sq. Km.)

The abbreviations of the system are not uniform, mm.^{2} is frequently used instead of sq. mm.; etc.

The more commonly used denominations of square measure are square kilometer (sq. Km.), square meter (sq. m.), square centimeter (sq. cm.), and square millimeter (sq. mm.)

The square meter is used to measure such surfaces as those to which the square yard would apply; the square centimeter is used to measure smaller surfaces.

100. Land Measure.

The **are** is ten meters square. It has a side about 33 feet long.

The **hektare**, containing 100 ares, is about $2\frac{1}{2}$ acres. The hektare is the unit of land measure. A quarter section (160 acres) is almost exactly 64 hektares.

Examples. 1. Reduce .4367 sq. m. to sq. Hm.

SOLUTION. 4367 sq. m. = .4367 sq. Hm.

2. Reduce .0547 sq. Dm. to sq. dm.

SOLUTION. .0547 sq. Dm. = 547 sq. dm.

Oral Work

1. Reduce 427 sq. m. to sq. Dm; sq. Hm.
2. Reduce 37 sq. cm. to sq. m.; sq. mm.
3. Reduce .0354 sq. Hm. to sq. m.; sq. Dm.
4. Reduce .1 sq. m. to sq. cm.; sq. mm.

Written Work

Complete the following, carrying results to three decimal places:

1 square meter = ? square inches.
1 square meter = ? square yards.
1 square meter = ? square feet.
1 square centimeter = ? square feet.
1 square foot = ? square meters.
1 square yard = ? square meters.

101. Measures of Volume.

The cubic meter is the unit of volume. It is sometimes called the **stere**.

1000 cu. millimeters (cu. mm.) = 1 cu. centimeter (cu. cm.)
1000 cu. centimeters = 1 cu. decimeter (cu. dm.)
1000 cu. decimeters = 1 cu. meter (cu. m.)
1000 cu. meters = 1 cu. dekameter (cu. Dm.)
1000 cu. dekameters = 1 cu. hektometer (cu. Hm.)
1000 cu. hektometers = 1 cu. kilometer (cu. Km.)

The abbreviations mm.^{3}, cm.^{3}, etc. are sometimes used.

Examples. 1. Reduce 4386 cu. m. to cu. Dm.

Solution. 4386 cu. m. = 4.386 cu. Dm.

2. Reduce .0427 cu. cm. to cu. mm.

Solution. .0427 cu. cm. = 42.7 cu. mm.

Oral Work

1. Reduce 37864 cu. m. to cu. Dm.
2. Reduce .04276 cu. cm. to cu. mm.
3. Reduce 742863 cu. dm. to cu. Dm.
4. Reduce .0476 cu. m. to cu. cm.

Written Work

A meter is approximately 39.37 inches. Complete the following table of comparisons:

1 cubic meter = ? cubic inches.
1 cubic meter = ? cubic feet.
1 cubic meter = ? cubic yards.
1 cubic foot = ? cubic meters.
1 cubic yard = ? cubic meters.

102. Measures of Capacity.

The unit of capacity is the liter, which is a cubic decimeter. It equals 1.05668 liquid quarts or 0.9081 dry quart.

10 milliliters (ml.) = 1 centiliter (cl.)
10 centiliters = 1 deciliter (dl.)
10 deciliters = 1 liter (l.)
10 LITERS = 1 dekaliter (Dl.)
10 dekaliters = 1 hektoliter (Hl.)
10 hektoliters = 1 kiloliter (Kl.)

The liter serves the same purpose in measuring capacities as the quart and gallon. The hektoliter is used for measuring quantities commonly measured by the bushel in countries where the English system is used.

Oral Work

1. Reduce 427 l. to Dl.; Hl.; cl.
2. Reduce .043 Dl. to dl.; cl.
3. Reduce .04278 Kl. to. l.; dl.; Hl.

Written Work

Complete the following table of comparisons :

1 peck = ? liters.
1 bushel = ? liters.
1 bushel = ? hektoliters.
1 hektoliter = ? bushels ?

103. Measures of Weight.

The gram is the unit of weight. A liter of water weighs a kilogram, or 1000 grams.

10 milligrams (mg.) = 1 centigram (cg).
10 centigrams = 1 decigram (dg.)
10 decigrams = 1 gram (g.)
10 grams = 1 dekagram (Dg.)
10 dekagrams = 1 hektogram (Hg.)
10 hektograms = 1 kilogram (Kg.)

Oral Work

1. Reduce 4736 g. to Kg.; cg.; mg.
2. Reduce .03 cg. to mg.; g.; Dg.
3. Reduce .07428 Kg. to g.; dg.; cg.

Written Work

A kilogram = 2.204622 pounds avoirdupois.

Complete the following table of comparisons:

1 gram = ? pound avoirdupois.
1 gram = ? ounce avoirdupois.
1 kilogram = ? ounces avoirdupois.
1 pound = ? kilograms.

The kilogram and the "half kilo" are the common measures used in trade. A half kilo = ? pounds.

The metric ton or tonneau (1000 kilograms) is used for larger weights.

1 metric ton = ? pounds
1 metric ton = ? short tons.
1 metric ton = ? long tons.

104. Comparative Table of Weights and Measures.

The following table of comparisons will be useful in reducing measurements stated in the English system to equivalent metric measures, and vice versa. It is not to be memorized.

1 inch	= 25.4001 millimeters, or 2.54001 centimeters.
1 foot	= .304801 meter.
1 yard	= .914402 meter.
1 mile	= 1.60935 kilometers.
1 square inch	= 945.16 square millimeters, or 6.452 square centimeters.
1 square foot	= .09290 square meter.
1 square yard	= .8361 square meter.
1 square mile	= 2.59 square kilometers.
1 cubic inch	= 16,387.2 cubic millimeters or 16.3872 cubic centimeters.
1 cubic foot	= .02832 cubic meter.
1 cubic yard	= .7646 cubic meter.
1 acre	= .4047 hectare.
1 liquid quart	= .94636 liter.
1 liquid gallon	= 3.78543 liters.
1 dry quart	= 1.1012 liters.
1 peck	= 8.80982 liters.
1 bushel	= .35239 hectoliter.
1 grain	= .06480 gram.
1 ounce (avoir.)	= 28.3495 grams.
1 ounce (Troy)	= 31.10348 grains.
1 pound (avoir.)	= .45359 kilogram.
1 pound (Troy)	= .37324 kilogram.
1 millimeter	= .03937 inch.

1 centimeter = .3937 inch.
1 meter = 39.37 inches.
1 meter = 3.28083 feet.
1 meter = 1.093611 yards.
1 kilometer = .62137 mile.
1 square millimeter = .00155 square inch.
1 square centimeter = .155 square inch.
1 square meter = 10.764 square feet or 1.196 square yards.
1 square kilometer = .3861 square mile.
1 cubic centimeter = .061 cubic inch.
1 cubic meter = 35.314 cubic feet, or 1.3079 cubic yards.
1 hectare = 2.471 acres.
1 milliliter = .03381 liquid ounce or .2705 apothecaries' dram.
1 liter = 1.05668 liquid quarts, or .26417 liquid gallon.
1 liter = .9081 dry quart.
1 hectoliter = 2.83774 bushels.
1 gram = 15.4324 grains, or .03527 ounce (avoir.), or .03215 ounce (Troy).
1 kilogram = 2.20462 pounds (avoir.), or 2.67923 pounds (Troy).

Exercise

State the result in each of the following in metric units. (Use a meter stick in making the measurements.)

1. Measure the edges of the cover of this book.

2. How many square centimeters in the surface of the cover?

3. How tall are you?

4. Find the dimensions of the top of your desk.

5. What is the length of your classroom? What is its width? Estimate its height.

6. What is the area of the floor?

(Other measurements may be suggested by the teacher).

7. Make a cube from cardboard which will contain 1 liter.

Using the table of comparative weights and measures, make the following reductions:

8. 1 ft. 8 in., to meters.

9. $3\frac{1}{2}$ meters to feet and inches.

10. 6 sq. ft. to square meters.

11. 14 square meters to square yards.

12. 36 cubic inches to cubic centimeters.

13. 130 cubic centimeters to cubic inches.

14. A bin is 8 ft. × 16 ft. × 7 ft. How many cubic meters will it contain?

15. A horse weighs 715 kilograms. How many pounds does he weigh?

16. The distance from Boston to Chicago by rail is 999 miles. How many kilometers?

17. What is the cost of 16 yards of cloth at $1.35 per meter?

18. A barrel (31.5 gal.) will contain how many liters?

19. If a stere is $\frac{3}{11}$ of a cord, how many steres are there in a wood pile 4 ft. × 4 ft. × 17 ft.?

20. How long would a tank 4 feet wide and 3 feet high have to be to contain 6 cubic meters? (State your result to the nearest foot).

21. The Eiffel tower in Paris is about 300 m. high. State this height in feet?

22. Olive oil weighs .92 as much as an equal volume of water. What is the weight of 1 liter of olive oil?

23. If a stream flows at the rate of $1\frac{1}{2}$ Km. per hour, what is its rate of flow per second? Express the result in centimeters.

24. A liter of mercury weighs 13.596 Kg. How many mm^3. of mercury weigh 3 g.?

25. Fifty-four miles are how many kilometers, to the nearest unit?

26. 428 ft. are how many meters, to the nearest unit?

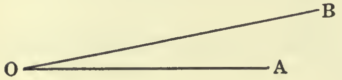
27. Sound travels 332 m. per second, how long will it take it to travel 4.5 Km.?

CHAPTER XII

PRACTICAL BUSINESS MEASUREMENTS

PLANE FIGURES

105. Rectilinear Figures. An **angle** is the opening between two lines which meet.

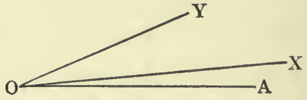


Thus the angle AOB is formed by the lines AO and OB, which meet (intersect) at O.

The lines which intersect are called the **arms** of the angle and their point of intersection is called the **vertex** of the angle.

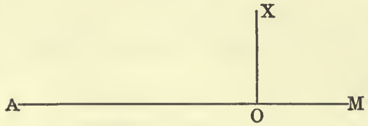
Thus AO and OB are the arms of the angle and O is the vertex.

Two angles which have the same vertex and a common arm between them are called **adjacent** angles.



Thus the angles AOX and XOY are adjacent angles.

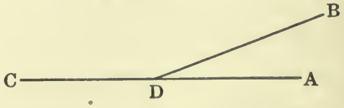
A **right angle** is an angle formed when one straight line meets another straight line so as to make the adjacent angles equal. The lines forming the angles are said to be **perpendicular** to each other.



Thus the angles MOX and XOA are right angles and the lines AM and OX are perpendicular to each other.

An **acute** angle is an angle less than a right angle.

An **obtuse** angle is an angle greater than a right angle.



Thus the angle ADB is acute and the angle BDC is obtuse.

A **surface** has length and breadth but no thickness.

A **plane**, or **plane surface**, is a level surface such as that of still water.

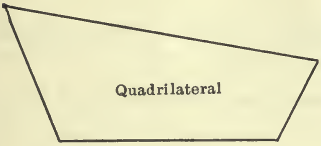
A **plane figure** is a figure all of whose points lie in the same plane.

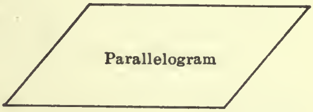
A **quadrilateral** is a plane figure bounded by four straight lines.

A **parallelogram** is a quadrilateral whose opposite sides are parallel.

A **rectangle** is a parallelogram whose angles are right angles.

A **square** is a rectangle whose sides are all equal.



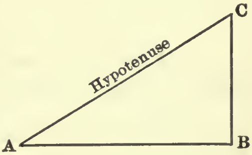


A **diagonal** of a quadrilateral is the straight line connecting two of its opposite vertices.

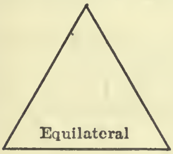
A **triangle** is a plane figure bounded by three straight lines.

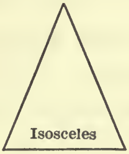
A **right triangle** is one that has a right angle.

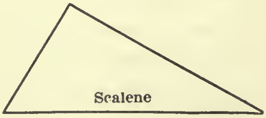
The **hypotenuse** of a right triangle is the side opposite the right angle.



A triangle which has three sides equal is called **equilateral.** If two sides are equal it is called **isosceles.** If it has no equal sides it is called **scalene.**



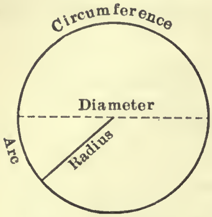




The **perimeter** of a plane figure is the sum of its sides.

A **circle** is a plane figure bounded by a curved line called the circumference, every point of which is equally distant from a point within called the center.

The **diameter** of a circle is a straight line passing through the center and terminated by the circumference.



The **radius** of a circle is the straight line from the center of the circle to the circumference. It is equal to one half of the diameter.

An **arc** of a circle is any part of the circumference.

The **base** of a plane figure is the side on which it is supposed to stand.

The **altitude** of a plane figure is the perpendicular distance from the highest point above the base to the base or to the base produced.

An area is always expressed in terms of some square unit, such as square inch, square foot, or square yard.

To find the area of any figure is to find the number of square units it contains.

A square whose side is one unit is said to have an area of one square unit. Thus, a square whose side is one inch has an area of one square inch.

Oral Work

1. Into how many squares 1 in. on a side can you divide a rectangle 5 in. long and 1 in. wide?

2. Into how many squares 1 in. on a side can you divide a rectangle 5 in. long and 3 in. wide?

3. How many square feet in the area of a rectangle 7 ft. long and 3 ft. wide?

AREAS OF PLANE FIGURES

The number of square units in the area of a rectangle is equal to the number of units in the length times the number of units in the width.

Find the areas of the rectangles having the following dimensions:

4. 12 ft. by 8 ft.

5. $3\frac{1}{2}$ rd. by 6 rd.

6. 25 rd. by 25 rd.

7. 11 in. by 15 in.

8. 15 in. by 15 in.

9. 35 rd. by 35 rd.

10. 16 ft. by 12 ft.

11. 3.5 ft. by 3.5 ft.

Written Work

1. How many acres are there in a rectangular field 130 rd. long and 80 rd. wide?

2. Find the cost of painting a rectangular floor 30 ft. long and 18 ft. wide at 80 cents a square yard.

3. A rectangular floor 24 ft. long and 22 ft. wide is to be covered with tiles 8 in. square. How many tiles will be required?

4. A football field is 120 yards long and $53\frac{1}{3}$ yards wide. How many acres does it contain?

5. A tennis court is 78 ft. long and 36 ft. wide. What part of an acre does it contain?

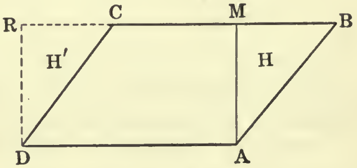
6. A field 96 rd. by 130 rd. produces 2340 bu. of wheat. What is the average yield per acre?

7. Find the area of a square field whose perimeter is 128 rd.

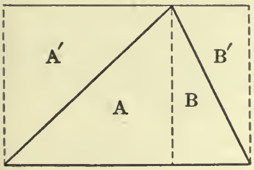
8. A rectangular field is 42 rd. long and 26 rd. wide. Find the cost of fencing it at $1.20 a rod.

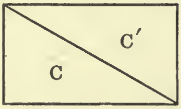
9. Find the cost of painting the four walls of a room 14 ft. long, 10 ft. 6 in. wide, and 9 ft. high at 8 cents per square yard, no allowance being made for openings.

The parallelogram *ABCD* may be shown to be equivalent to the rectangle *AMRD* by cutting off the area *H* and placing it in the position *H′*



It is evident that the rule for finding the area of a parallelogram is the same as for finding the area of a rectangle. State the rule.





A triangle may be shown to be equivalent to one half the area of a rectangle with the same base and altitude.

The number of square units in the area of a triangle is equal to one half the number of units in the base times the number of units in the altitude.

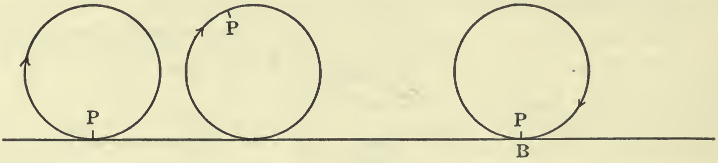
Oral Work

State the area of the triangles whose bases and the altitudes are as follows :

	Base	Altitude		Base	Altitude
1.	10 ft.	6 ft.	**2.**	12 rd.	7 rd.
3.	4.2 rd.	20 rd.	**4.**	$10\frac{1}{2}$ ft.	12 ft.
5.	18 yd.	11 yd.	**6.**	1 ft. 4 in.	9 in.
7.	3 yd. 1 ft.	$2\frac{1}{2}$ ft.	**8.**	4 ft. 6 in.	20 in.

106. Circumference and Area of a Circle. Find the length of the circumferences of several circles of different sizes. This may be done as follows:

Mark a point on the circumference and roll the circle along a level surface. Determine the distance between the places where the point comes in contact with the level surface.



In the figure above, the length of the line PB is equal to the length of the circumference of the circle.

After the circumferences of several circles of different sizes have been measured, find the ratio of the length of the circumference of each circle to the length of its diameter. Find the average of these ratios. If the measurements were made with care you will find that, whatever the radius of the circle, the length of the circumference divided by the length of the diameter gives a quotient slightly larger than three. It is shown in geometry that the ratio is always the same and that it is approximately 3.1416 or $\frac{22}{7}$. This constant ratio is called π (pronounced pī). Length of circumference $\div$ length of diameter $= \pi$.

This is usually expressed as follows: $\frac{C}{D} = \pi$, or $C = \pi D$.

If the diameter of a circle is known, the circumferences may be found by multiplying the diameter by the value of π.

If the circumference is known, the diameter may be found by dividing the circumference by the value of π.

Note. For most practical purposes $\frac{22}{7}$ may be used as the value of π.

Written Work

Find the circumferences of the circles with the following diameters:

1. 16 ft. **2.** 32 in. **3.** 18 yd. **4.** 3 ft. 4 in.
5. 9 ft. 2 in. **6.** 110 yd. **7.** 2 rd. **8.** 6 ft. 8 in.
9. 2 rd. 1 yd. **10.** 7 ft. 2.5 in.

Find the diameters of circles with the following circumferences:

11. 110 yd. **12.** 44 ft. **13.** 5000 ft.
14. 5 ft. 6 in. **15.** 1 mi.

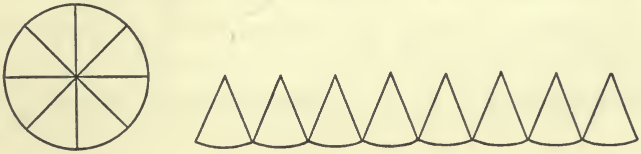
16. A bicycle wheel has a diameter of 28 in. How many times will the wheel turn in going 1 mile?

17. The inner circumference of a circular race track is one mile. The track is 90 ft. wide. How long is the outer circumference?

If a circle be divided as shown in the figure below, it is evident that the circle is composed of figures which are approximately triangles. The area of the circle is equal to the sum of the areas of the approximate triangles.

The altitude of each of the triangles is equal to the radius of the circle, and the sum of the bases of the triangles is equal to the circumference of the circle.

We may conclude, therefore, *that the number of units in the area of the circle is equal to one half the number of units in the circumference, times the number of units in the radius.*



This may be expressed as follows: area of circle $= \frac{C \times R}{2}$, where C represents the number of units in the circumference and R the number of units in the radius.

Since $C = \pi D$, or $2\pi R$, the area is equal to $\frac{2\pi R \times R}{2}$, or πR^2.

Hence, *to find the area of a circle, square the radius and multiply the result by* π.

Note. The parts into which the circle is divided are not exact triangles but it is proved in geometry that the area of the circle is the same as that of a triangle having a base equal to the circumference and an altitude equal to the radius.

Written Work

1. Find the area of a circle the radius of which is 6 ft.

2. Find the area of a circle the radius of which is 12 ft. Compare with the preceding area.

3. Find the cost of painting a circular area having a diameter of $7\frac{1}{2}$ ft. at 75 cents per square yard.

4. Find the area inclosed by a circular running track with an inner circumference of $\frac{3}{8}$ mi.

5. Find the area of a circular window 6 ft. in diameter.

6. Find the cost of refinishing the top of a circular table which has a diameter of 4 ft. 4 in., at 25 cents per square foot.

Compute the areas of the following:

7. A triangle having a base of 11 in. and an altitude of 9 in.

8. A parallelogram having a base of 12 in. and an altitude of 3.5 in.

9. A circle having a radius of 3 ft. 7 in.

10. A circle having a circumference of 14 ft. 3 in.

11. A rectangle having an altitude of 7.5 ft., and a base of 3.45 ft.

12. A triangle having an altitude of 4 rd. 3 yd., and a base of 12.3 rd.

13. A square which has a perimeter of 14.72 in.

14. What must you know and what must you do to find the area of each of the following: circle; square; triangle; parallelogram; rectangle?

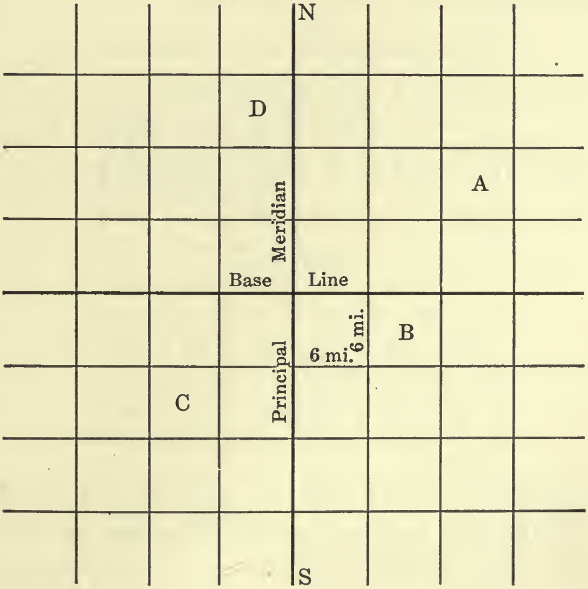
LAND MEASURE

107. Description of Farm Lands.

When the land of the Central and Western states was surveyed the following methods were employed:

a. Imaginary lines running north and south were established and called *principal meridians.* These principal meridians were numbered from 1 to 24. The first runs through Ohio and the last through Oregon.

b. Imaginary lines running east and west were also established, and called *base lines.*



c. Lines were then run at intervals of six miles parallel to the principal meridians, and others at intervals of six miles parallel to the base lines.

d. The land was thus divided into tracts 6 miles square. Each such area is called a township. The townships are described by their relation to the principal meridians and to the base line.

The township marked *A* is described in real estate records thus:

Twp. 2 N, R 3 E of the ——* principal meridian.

Twp. 2 N means that it is in the second row of townships north of the base line; while R 3 E means that it is in the third row east of the specified principal meridian.

Description of *C*: Twp. 2 S, R 2 W.

Describe townships marked *D* and *B*.

What is meant by Twp. 3 N, R 1 W? Twp. 3 S, R 2 E?

e. Each township is divided into sections, one mile square. Each section, therefore, contains 640 acres. The sections are numbered as shown in the following plot.

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

Assuming that this is a plot of township *A* in the figure on page 119, the 25th section would be described as follows:

Section 25, Twp. 2 N, R 3 E of the —— principal meridian.

Fractions of a section are often sold, and the following method of description is used.

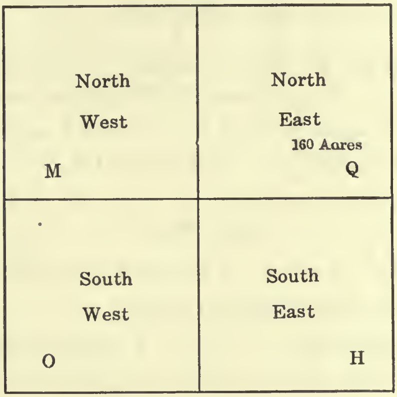
* Here is stated the number of the meridian.

The section is divided into quarters.

M is described as the northwest $\frac{1}{4}$ of Section 25, Twp. 2 N, R 3 E.

H is described as the southeast $\frac{1}{4}$ of Section 25, Twp. 2 N, R 3 E.

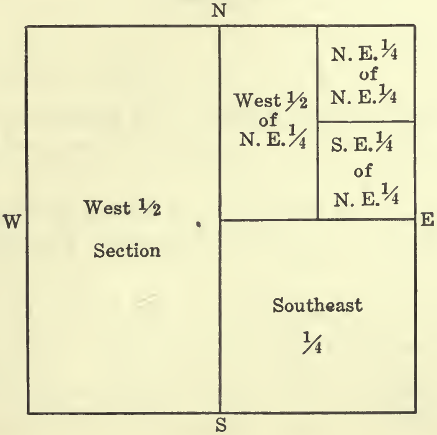
Section 25


North
West
M
North
East
160 Acres
Q
South
West
O
South
East
H

If *M* and *Q* were sold together, they would be described as the north $\frac{1}{2}$ of Section 25, Twp. 2 N, R 3 E.

The following diagram shows a section divided into various farms, with their descriptions.

Section 25


N
W
E
S
West ½
Section
West ½
of
N. E. ¼
N. E. ¼
of
N. E. ¼
S. E. ¼
of
N. E. ¼
Southeast
¼

Written Work

Since a section is a mile square, each side of the section is 320 rd. in length. State the dimensions of each of the farms shown in the diagram on page 121; thus,

The west $\frac{1}{2}$ of Section 25 is 160 rods east and west by 320 rods north and south and contains 320 acres.

108. Measurements of Farm Lands. The unit of farm land measure is the acre, which contain 160 square rods.

Thus, a piece of land 10 rods wide and 16 rods long contains an acre, or a strip of land 1 rod wide and 160 rods long contains an acre.

Oral Work

1. Locate farms on plot of Section 25 having the following dimensions. How many acres in each?

a. 160 rods × 80 rods.
b. 160 rods × 160 rods.
c. 320 rods × 160 rods.
d. 80 rods × 80 rods.

2. The owner of the southwest $\frac{1}{4}$ sells a strip 20 rods wide along the south side of his farm. How much does he receive at $150 per acre? Describe the property sold.

3. The owner of the S. E. $\frac{1}{4}$ of the N. E. $\frac{1}{4}$ sells the north half of his farm for $125 per acre.

How much land did he sell? Describe it. What was the total selling price?

SQUARE ROOT AND ITS APPLICATIONS

109. Extracting the Square Root. The **square** of a number is the product obtained by using the number twice as a factor. Thus, since $3 \times 3 = 9$; 9 is the square of 3.

The **square root** of a number is one of the two equal factors of the number. Thus, the square of 4 is 16. The square root of 16 is 4.

The square root of a number may be indicated by writing the number under the **radical sign** or by placing "$\frac{1}{2}$" above and to the right of the number. Thus, both $\sqrt{144}$ and $144^{\frac{1}{2}}$ indicate the square root of 144.

Oral Work

What is the square root of each of the following: 36, 64, 81, 169, 225, 25, 49, 1, 256, 400, 100, 10,000?

How many digits in the square of each of the following: 1, 9, 10, 99, 100, 999, 1000, 9999?

How does the number of digits in the square of a number compare with the number of digits in the number?

The square of 53 may be found as follows:

$$\begin{array}{r} 50+3 \\ \underline{50+3} \\ (50\times 3)+3^2 \\ \underline{50^2+(50\times 3)} \\ 50^2+2(50\times 3)+3^2 \end{array} \qquad\qquad \begin{array}{rcr} 50^2 & = & 2500 \\ 2(50\times 3) & = & 300 \\ 3^2 & = & \underline{9} \\ & & 2809 \end{array}$$

By squaring several numbers, as illustrated above, you will understand that *the square of a number is equal to the square of the tens, plus twice the product of the tens by the units, plus the square of the units.*

This principle may be applied in finding the square root of numbers whose square root cannot be found by inspection.

The square of a number contains twice as many digits, or one less than twice as many digits, as the original number. Hence, if an integer be separated into groups or periods of two digits each, from right to left, there will be as many digits in the square root as there are groups of digits in the original number.

Example. Find the square root of 576.

SOLUTION. Beginning at the right, separate the digits into groups of two each. The greatest square in 5 is 4, and the square root of 4 is 2. Two is therefore the tens digit of the root.

Find the remainder and annex the next group (76).

The result is 176. We have taken the square of the tens from the number, hence the remainder (176) must contain twice the product of the tens by the units plus the square of the units. Twice two tens is four tens or 40. 40 is contained 4 times in 176. Hence, 4 is the units' digit of the root. Twice the tens, times the units, plus the square of the units, is the same as the sum of twice the tens, and the units, times the units. Therefore, add 4 units to the 40 and multiply the result by 4. The result is then 176. Therefore, the square root of 576 is 24.

$$\begin{array}{r|l} \multicolumn{2}{r}{2\;4} \\ \multicolumn{2}{r}{\overline{5}\;\overline{76}} \\ \multicolumn{2}{r}{\underline{4}} \\ 40 & 176 \\ \underline{4} & \underline{176} \\ 44 & \end{array}$$

To find the square root of a number.

Begin at the units and separate the number into groups of two digits each. Find the greatest square in the left-hand group, and write its square root for the first digit of the required root. Subtract the square of the root digit from the left-hand group and then annex the second group for a dividend.

Annex a zero to the part of the root already found and multiply the result by two. Divide the dividend by this product. Place the quotient as the next figure of the root. Add the quotient to the divisor and multiply the result by the digit of the root just found.

Continue in like manner until all of the groups have been used. The result will be the square root. (It may not be the exact square root, as indicated in the note on page 125.)

If the number contains a decimal fraction, begin at the decimal point and separate the number into groups of two digits each way from the decimal point. If the last group on the right of the decimal point has but one figure, annex a zero. Each decimal group must contain two figures.

To find the square root of a common fraction whose numerator and denominator are not perfect squares, first reduce the common fraction to a decimal, then extract the root.

Examples. 1. Extract the square root of 16,641.

SOLUTION.

```
      1  2  9
      1 66 41
      1
   20 |66
    2 |44
   22 |2241

  240 |
    9 |
  249 |2241
```

2. Extract the square root of 18.0625.

SOLUTION.

```
      4. 2  5
     18. 06 25
     16
   80 |206
    2 |164
   82 |  4225
  840 |
    5 |
  845 |  4225
```

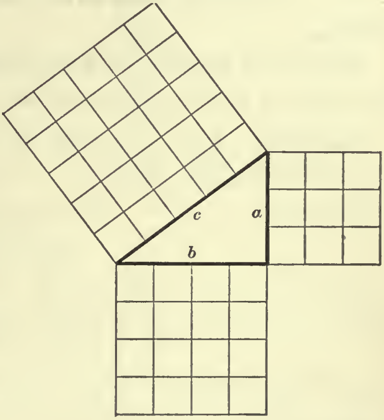
Written Work

Find the square root of:

1. 3025.
2. 9025.
3. 89.25.
4. .1764.
5. 52.9984.
6. .005476.
7. 5.
8. $\frac{3}{4}$.
9. .0256.
10. 3.
11. $\frac{2}{5}$.
12. 7.

Note. If a number is not a perfect square, its approximate square root may be found by placing the decimal point in its proper place and annexing zeros to the right of the last digit. It is usually not necessary to find a root to more than two or three decimal places.

110. The Square and the Right Triangle. Since the number of units in the area of a square is found by squaring the number of units in one side, it follows that the square root of the number of units in the area is the number of units in one side.



It is proved in geometry that *the square on the hypotenuse of a right triangle is equal to the sum of the squares on the other two sides.* This is illustrated in the figure.

The length of the hypotenuse is equal to the square root of the sum of the squares on the other two sides. To find the length of either side, extract the square root of the difference between the square on the hypotenuse and on the other side.

Written Exercise

1. A square field contains 9025 sq. rd. Find the length of one side.

2. The base lines on a ball field are each 90 ft. in length, and are at right angles to each other. What is the distance from home plate to second base?

3. A tennis court is 78 ft. long, and 36 ft. wide. What is the length of the diagonal?

4. A ladder 19 ft. long placed on level ground reaches 13 ft. up a wall. How far is the foot of the ladder from the wall?

5. The rafters of a building are 25 ft. long. The ridge of the roof is 18 feet above a line joining the foot of the rafters. How wide is the building?

6. A rectangular corner lot is 90 ft. long and 60 ft. wide. A path runs diagonally across it. What distance is saved by using the path?

7. A guy rope 80 ft. long is attached to a flagpole at a point 54 ft. above the ground. How far from the foot of the pole may the rope be made to touch the ground when stretched taut?

111. To find the area of a triangle when the length of each side is known.

When the altitude of a triangle is not known but the length of the sides is known, the area may be found as in the following:

Example. Find the area of a triangle the sides of which are 12 ft., 14 ft., and 22 ft.

SOLUTION.

$$12 + 14 + 22 = 48.$$
$$\tfrac{1}{2} \text{ of } 48 = 24.$$
$$24 - 12 = 12.$$
$$24 - 14 = 10.$$
$$24 - 22 = 2.$$
$$24 \times 12 \times 10 \times 2 = 5760.$$

$\sqrt{5760} = 75.88$, the number of square feet in the area.

Rule.

From one half the sum of the three sides subtract each side separately. Find the product of the half sum and of the three remainders. Extract the square root of this product. The result is the number of units in the area.

Written Work

1. Find the area of an equilateral triangle the sides of which are 8 inches.

2. The perimeter of an isosceles triangle is 28 inches. One of the equal sides is 9 inches. Find the area.

3. Compare the area of an equilateral triangle with a side of 4 inches with that of one with a side of 8 inches.

4. Find the value at $160 an acre of a triangular field having sides of 60 rd., 78 rd., and 84 rd. respectively.

ROOFING

112. Roofing is generally measured by the *square* of 100 square feet.

Shingles average 4 in. in width, and are usually laid 4 in., 4½ in., 5 in., or 5½ in. to the weather. A bunch of shingles contains 250. Part of a bunch is not sold.

The following table shows the usual estimate per square.

EXPOSED TO WEATHER	NUMBER PER SQUARE
4 in.	1000
4½ in.	900
5 in.	800
5½ in.	700

Slate varies in size. It is usually 16″ by 24″, or 6″ by 12″.

Written Work

1. When shingles are laid 4″ to the weather, how many 4-in. shingles cover a square foot? How many to the square foot when they are 5″ to the weather?

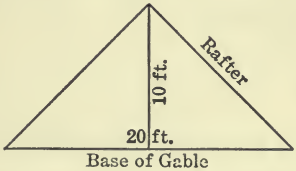
2. A roof is 42 ft. long and 28 ft. wide on each side. How many bunches of shingles will be required to cover it, if the shingles are laid 5 in. to the weather?

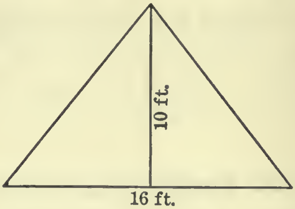
3. At $4.50 a square what will be the cost of tinning a roof 22 ft. by 18 ft.?

4. A roof is 40 ft. long and 30 ft. wide on each side. How many shingles, laid 5 in. to the weather, will be required to cover it?

5. How many bundles of shingles, laid 5 in. to the weather, will be required for a roof 28 ft. by 21 ft.?

The pitch of a roof is found by dividing the rise in the rafters by the width of the base of the gable. If the rafters rise 10 ft. and the base of the gable is 40 ft., the pitch of the roof is $\frac{1}{4}$. A pitch of $\frac{5}{8}$ is called the Gothic pitch.





Written Work

Find the pitch of the roof in each of the following:

	Width of Base	Height
1.	45 ft.	20 ft.
	36 ft.	18 ft.
	64 ft.	16 ft.
	20 ft.	5 ft.
	24 ft.	15 ft.

2. Find the height of the gable.

Width of Base	Pitch of Roof
50 ft.	$\frac{1}{2}$
20 ft.	$\frac{1}{4}$
32 ft.	Gothic

FLOORING

113. Flooring is usually measured by the **square** or by the **thousand square feet.** There is some waste when lumber is "tongued" and "grooved." Dealers measure the width before the lumber is matched. The amount of waste depends upon the width of the lumber. It is customary to allow $\frac{1}{4}$ for flooring 3 in. or more in width and $\frac{1}{3}$ for flooring less than 3 in. in width.

ILLUSTRATION. How many square feet of 4-in. flooring will be required for a room 28 ft. by 24 ft.?

$28 \times 24 = 672$, the number of square feet to be covered.

$1\frac{1}{4} \times 672$ sq. ft. $= 840$ sq. ft., the quantity required.

Written Work

1. A room is 24 ft. by 36 ft. Find the cost of the $2\frac{1}{2}$-in. flooring at $50 per thousand square feet.

2. Compute the cost of flooring a hall 72 ft. by 50 ft., with 4-in. lumber at $30 a thousand, incidentals $28.40 extra.

3. How many feet of 4-in. flooring will be required for a hall 60 ft. by 42 ft.?

4. What will the 4-in. flooring for a hall 60 ft. by 42 ft. cost at $36 per thousand?

PLASTERING, PAPERING, PAINTING, CARPETING

114. Plastering is usually measured by the square yard. There is no uniform rule regarding allowances to be made for openings.

To find the area of the walls, multiply the perimeter by the height.

The dimensions of a room are usually stated in the following order: length, breadth, height.

Thus the dimensions of a room 18 ft. long, 14 ft. wide, and 9 ft. 6 in. high would be indicated as follows: 18 ft. × 14 ft. × 9 ft. 6 in. or $18' \times 14' \times 9'\ 6''$.

Wall paper is generally 18 inches wide. It is sold by the single roll of 8 yards or by the double roll of 16 yards. Complete rolls must be bought. There is no uniform procedure in making allowance for openings. Some paper hangers deduct the total width of all openings from the perimeter of the room, and cover the spaces about the openings with the parts of strips left from the rolls.

Painting is usually measured by the *square yard*. It is not customary to make any allowance for openings.

Carpet is sold by the linear yard regardless of its width. Ingrain carpets are usually 1 yard wide, other carpets are usually $\frac{3}{4}$ yard wide.

Carpets are generally laid lengthwise of the room. It is sometimes more economical to lay a carpet crosswise. Fractional lengths, but not fractional widths, may be bought. Allowance must sometimes be made for matching designs.

Written Work

The six rooms of a house have the following dimensions and openings: Living room, 20′ × 16′, 3 windows 3′ × 6′ 6″, and two doors 3′ 6″ × 7′ 6″. Dining room, 15′ × 13′ 6″, 2 windows 4′ × 6′, and two doors 3′ 6″ × 7′. Two bedrooms, each 14′ 6″ × 12′ 6″, 2 windows 4′ × 6′ 6″, and 2 doors 3′ 6″ × 7′. Kitchen, 15′ × 15′ 6″, 3 windows 4′ × 7′, and 2 doors 3′ 6″ × 7′. Bathroom, 10′ 6″ × 8′ 6″, 1 window 3′ × 5′, 1 door 3′ 6″ × 7′. All rooms are 9 ft. high and the baseboards are 1 ft. wide.

1. Find the cost of plastering the walls and ceiling of each room at 35 cents a square yard, allowing for one half the openings.

2. Find the cost of papering the walls of the living room and the dining room at $1.10 a double roll, making one half allowance for openings.

3. Find the cost of calcimining the ceilings of these two rooms at 60 cents a square yard.

4. Find the cost of papering the walls of the two bedrooms at 90 cents a double roll, making full allowance for openings.

5. How many square yards of linoleum will be required to cover the floors of the bathroom and the kitchen?

6. Find the cost of painting the baseboards in the six rooms, at 40 cents a square foot, allowance being made for all openings.

7. How many gallons of paint will be required for two coats on a building 32 ft. by 24 ft. by 18 ft., if the first coat takes one gallon for 52 sq. yd. and the second one gallon for 74 sq. yd.?

SOLIDS

115. Rectangular Solids. A **solid** has three dimensions: length, breadth, and thickness.

A **rectangular solid** is bounded by six rectangular surfaces. Such a solid is called a **prism**.

The bases of a prism are equal and parallel.

A **cube** is a rectangular solid having six square faces.

A cube 1 unit long, 1 unit wide, and 1 unit thick contains

1 cubic unit. Volumes are always measured in terms of some cubic unit.

How many cubic inches in a block 1 in. long, 1 in. wide, and 1 in. thick? In a prism 5 in. long, 1 in. wide, and 1 in. thick? in a prism 5 in. long, 3 in. wide, and 1 in. thick? in a prism 5 in. long, 3 in. wide, and 2 in. thick?

It is evident that *the number of cubic units in the volume of a rectangular solid is equal to the product of the number of units in its three dimensions.*

Written Work

Compute the volumes of the following:

1. A cube with an edge of 35 in.

2. A rectangular solid 7′ × 8′ × 5′ 6″.

3. A piece of ice $2\frac{1}{2}'$ × $1\frac{2}{3}'$ × 9″.

4. A piece of timber 22 ft. long, $1\frac{1}{2}$ ft. wide, and $1\frac{1}{2}$ in. thick.

5. How many cubic feet of earth must be removed in digging a cellar 20′ × 18′ × 7′?

6. Compute the cost of a stone wall 24 ft. long, 3.5 ft. wide, and 4 ft. high at $3.65 a cubic yard.

7. A swimming tank is 44′ by 28′ by 9′. How many cubic feet of water does it contain when filled to a depth of 4′? 8′?

8. A ditch for some sewer pipe is to be made 1200 yd. long, 3 ft. wide, and 7 ft. deep. Compute the cost of making the excavation at 28 cents per cubic yard.

9. Compute the number of cubic feet of air space allowed each student in your schoolroom.

10. Compare the volumes of two rectangular pieces of ice having the same length and breadth, the thickness of one being twice that of the other.

11. Compute the volume of a rectangular bin 18′ × 16′ × 7′.

12. Find the capacity in bushels of a grain elevator 60′ × 42′ × 24′. (A bushel is equivalent to $1\frac{1}{4}$ cu. ft.)

116. A **cord** of wood or stone is a pile 8 ft. long, 4 ft. wide, and 4 ft. high. It contains 128 cu. ft. In certain localities a cord of wood means a pile 8 ft. long, and 4 ft. high, the price varying with the length of the stick.

Written Work

1. A pile of four-foot wood ten feet high and forty-two feet long was cut from a ten-acre tract. What was the average number of cords per acre?

2. How many cords of wood in a pile 30 ft. by 4 ft. by 9 ft.?

3. How many cords of 4 ft. wood in a pile 16 ft. by 8 ft. by 12 ft.?

117. A **cylinder** is a solid bounded by two bases which are equal parallel circles and by a uniformly curved surface.

The curved surface of a cylinder is called its **lateral surface.**

If the lateral surface of a cylinder be exactly covered with paper and the paper then removed, the paper will be seen to have the form of a rectangle the base of which is the length of the circumference of the cylinder and the altitude is the height of the cylinder.

Hence, *the number of units in the lateral area of a cylinder equals the product of the number of units in the circumference by the number of units in the altitude. To find the entire surface of the cylinder the areas of the two circular bases must be added to the lateral area.*

It is shown in geometry that the number of units in the volume of a cylinder equals the product of the number of units in the area of the base and the number of units in the altitude.

Written Work

1. Compute the entire surface of a cylindrical cup having a diameter of 3 in. and an altitude of 7 in.

Determine the cubic contents of this cup.

2. A cylindrical cistern having a radius of $4\frac{1}{2}'$ is filled with water to a depth of $9'$. How many cubic feet of water are there in the cistern?

3. A cylindrical tank 22 ft. in diameter is filled with water to the depth of 18 ft. What is the weight of the water (1 cubic foot of water weighs 62.5 lb.)?

4. A man contracted to dig a cistern 7 ft. in diameter, 24 ft. deep at the rate of $1.20 per cu. yd. Find the entire cost.

5. Find the cost of cementing this cistern at $1.80 per square yard.

6. How many yards of sheet iron will be required for a smokestack 3 ft. in diameter and 24 ft. high, allowing 1 in. for the lap? What will be the cost at 14 cents per square foot?

7. If a steam roller is 5 ft. high and 7 ft. long, how many square feet of ground will it roll at each revolution?

8. At $7\frac{1}{2}$ gal. to the cubic foot what will be the capacity of a cylindrical tank having a base of 6 ft. and an altitude of 14 ft.?

9. A farmer has a roller $2\frac{1}{2}$ ft. in diameter and 6 ft. wide. Allowing 4 in. for lapping, how far must the team travel in rolling a rectangular field 80 rd. long and 56 rd. wide?

10. At 60 cents a square yard find the cost of polishing a cylindrical monument 14 ft. high and 4 ft. in diameter.

118. The sphere. It is shown in geometry that *the number of units in the surface of a sphere is equal to* 4π *times the square of the number of units in the radius; and the number of units in the volume of a sphere is equal to* $\frac{4}{3}\pi$ *times the cube of the number of units in the radius.*

Written Work

Find the surfaces and volumes of spheres of radii:

1. 3″. **2.** 7″. **3.** 2.5″. **4.** 13′. **5.** 24.2″.
6. 8.24′. **7.** 6.3″. **8.** 14.1′. **9.** 8.7′. **10.** 12.3″.

11. A ball with a radius of 18 in. is to be gilded. How many square inches of gilding will be required?

119. Capacity. Grain is usually estimated by the *stricken bushel* of 2150.42 cu. in.; fruits, vegetables, coal, and corn on the cob by the *heaped bushel* of 2747.71 cu. in.

Since a stricken bushel is 2150.42 cu. in., and there are 1728 cu. in. in 1 cu. ft., therefore, 1 cu. ft. contains $\frac{1728}{2150.42}$ bushels or approximately .8 of a bushel.

To find the number of bushels of grain a bin will hold, multiply its capacity in cubic feet by .8.

A cubic foot of space will contain $\frac{1728}{2747.71}$ heaped bushels, or .63 heaped bushels.

To find the number of heaped bushels that a bin will hold, multiply its capacity in cubic feet by .63.

Written Work

1. How many bushels of potatoes will just fill a bin 18′ × 7′ × 5′?

2. How many bushels of wheat will just fill a bin 30′ × 6′ × 6′?

3. A wagon bed is 10′ long, 4′ wide, and $2\frac{1}{2}$′ deep. How many bushels of wheat will it hold?

4. Mr. Williams wishes to build a bin that will store 250 bushels of wheat. He expects to make the bin 24′ long and $6\frac{1}{2}$′ wide. What must be the depth?

5. A car 34′ long and 7′ wide is filled with oats to a depth of $4\frac{1}{2}$′. How many bushels of oats are in the car?

A gallon is equal to 231 cu. in. A cubic foot of space, therefore, contains $\frac{1728}{231}$ gallons, or 7.48 gallons.

To find the number of gallons when the cubic contents may be found, multiply the capacity in cubic feet by 7.48.

(In practice 1 cu. ft. is considered equal in capacity to $7\frac{1}{2}$ gallons.)

Written Work

Find the capacity in gallons, of:

1. A cistern 12′ in diameter and 18′ deep.

2. A tank 12′ long, $4\frac{1}{2}$′ wide, and 5′ deep.

3. A cistern 14′ in diameter and 20.5′ deep.

4. What is the capacity in gallons of a cylindrical tank 18.5′ in diameter and 32′ deep?

5. A cylindrical standpipe 60′ high has a diameter of 14′. How many gallons of water will it hold?

120. Board Measure. The **board foot** is the unit of lumber measure. It is a board 1 ft. long, 1 ft. wide, and 1 in. thick. The volume of a board foot is therefore, 144 cu. in. (except when the lumber is less than 1 in. in thickness). Boards having a thickness of less than 1 in. are considered as having a thickness of an inch.

A board 16 ft. long, 12 in. wide, and 1 in., or less, in thickness contains 16 board feet. If the thickness were $1\frac{1}{2}$ in., the board would contain 24 board feet.

In measuring boards that taper uniformly the average width is used.

The width of boards, except in expensive lumber, is generally taken as the next smaller half inch.

When no thickness is mentioned the thickness is understood to be 1 inch.

It should be evident that *the number of board feet is the product of the width in inches, by the thickness in inches, by the length in feet, by the number of pieces, divided by* 12.

ILLUSTRATION 1. How many board feet in 24 scantlings, 2″ × 4″, 16′?

$$\frac{24 \times 2 \times 4 \times 16}{12} = 256, \text{ the number of board feet.}$$

ILLUSTRATION 2. How many board feet in 18 pieces, each 4″ × 9″, 16′.

$$\frac{18 \times 4 \times 9 \times 16}{12} = 864, \text{ the number of board feet.}$$

Oral Work

Compute the number of board feet in each of the following:

1. A board 12 ft. long, 1 ft. wide, and 1 in. thick.
2. A board 7 ft. long, 2 in. wide, and 1 in. thick.
3. Two boards each 10 ft. long, 4 in. wide, and 1 in. thick.
4. Two boards each 14′ × 4″ × 12″.
5. Twenty boards each 10′ × 3″ × $\frac{3}{4}$″.
6. One hundred boards each 14′ × 4″ × 2″.
7. Twenty-five boards each 9′ × 2″ × 1″.
8. Three hundred boards each 15′ × 4″ × $\frac{3}{4}$″.
9. Fifty boards each 12′ × 6″ × 2″.

Dealers sometimes use a table in determining the number of board feet in a given piece of timber. A portion of such a table is given below.

SIZE IN INCHES	LENGTH IN FEET				
	10	12	14	16	18
2 × 4	6⅔	8	9⅓	10⅔	12
2 × 6	10	12	14	16	18
2 × 8	13⅓	16	18⅔	21⅓	24
2 × 10	16⅔	20	23⅓	26⅔	30
2 × 14	23⅓	24	32⅔	37⅓	42
2½ × 12	25	30	35	40	45
2½ × 14	29⅙	35	40⅚	46⅔	52½
3 × 6	15	18	21	24	27
3 × 8	20	24	28	32	36

Written Work

Determine the number of board feet in:

1. 40 pieces of white pine 2″ × 10″, 18′.
2. 130 joists 2″ × 8″, 16′.
3. 36 beams 4″ × 8″, 20.
4. 20 planks 2½″ × 8″, 18′.
5. 80 joists 4″ × 4″, 20′.
6. At $15 a thousand, find the cost of the following:

14 joists,	2″ × 6″, 18′
24 joists,	2″ × 10″, 32′
80 boards,	¾″ × 4″, 16′
30 scantlings,	2″ × 4″, 16′
20 beams,	8″ × 9″, 16′.

CHAPTER XIII

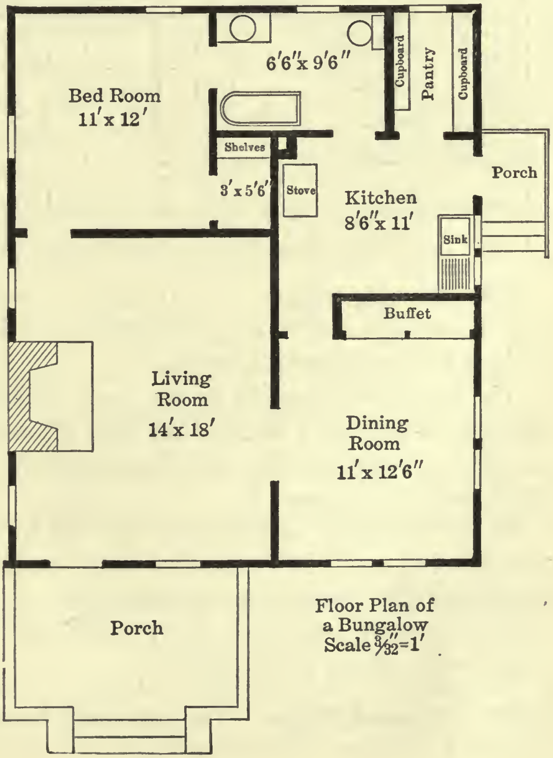
DRAWINGS AND GRAPHS

NEWSPAPERS, magazines, and trade journals make frequent use of drawings or graphs in order to make clear the relations between magnitudes. A graph will often reveal a relationship much more quickly and more clearly than a table of statistics.

121. Purposes of Drawings and Graphs. Drawings and graphic representations are extensively employed for two purposes:

a. For reference, to show quickly and conveniently the shape and dimensions of fields, rooms, buildings, furniture, machinery, etc.

Dimensional drawings and diagrams, similar to this one, are of great value. An architect designs a building, and the contractor builds it in accordance with the plan shown by the drawing. Tools and machinery are usually constructed in accordance with drawings prepared by draftsmen and designers.


Bed Room
11'x 12'
6'6"x 9'6"
Cupboard
Pantry
Cupboard
Shelves
3'x 5'6"
Stove
Kitchen
8'6"x 11'
Porch
Sink
Buffet
Living
Room
14'x 18'
Dining
Room
11'x 12'6"
Porch

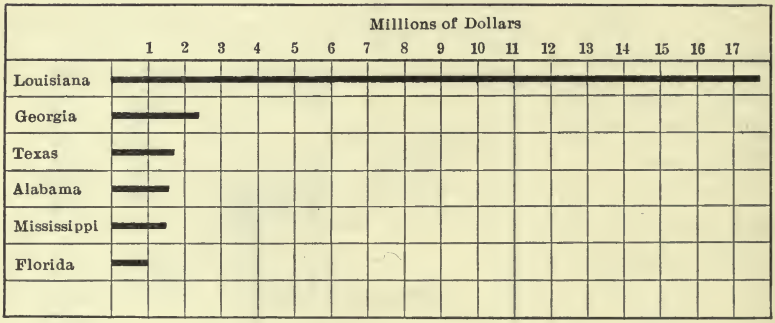
Floor Plan of
a Bungalow
Scale $\frac{3}{32}'' = 1'$

b. To present statistics in a manner

which will show their meaning and their relationship more clearly than can be done by columns of figures. For example, the relative importance of Louisiana as a producer of cane sugar is made clearer by the following graph than by the table of statistics.

VALUE OF CANE SUGAR PRODUCED IN CERTAIN STATES IN 1909

STATE	DOLLARS
Louisiana	17,752,537
Georgia	2,268,110
Texas	1,669,683
Alabama	1,527,166
Mississippi	1,506,887
Florida	1,089,698



122. Drawing to Scale. The ability to read drawings and graphic representations intelligently depends somewhat upon a knowledge of how to draw figures to scale.

The values represented by the lines in drawings and graphs can be easily determined because the lines are drawn to scale; that is, a certain distance on the drawing represents a specified amount of the quantity represented.

In the floor plan, on page 137, one foot of true distance is represented by $\frac{3}{32}$ of an inch plotted distance on the drawing. All lines in the drawing correspond to this scale.

In the graph showing the production of cane sugar, $\frac{3}{8}$ inch represents a value of $1,000,000.

123. Types of Scales. The scale may be stated in various ways, the most common of which are:

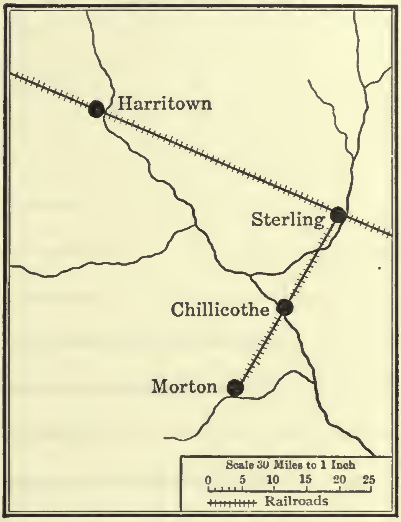
a. A fraction.

For example, a scale of $\frac{1}{4}$ means that the true distances or values are four times as great as the plotted distances or values.

b. Statement of equivalent dimensions or values.

For example, $1'' = 1'$. (Read 1 in. = 1 ft.) One inch of the drawing represents one foot of the real object. In the graph showing cane sugar production the scale is $\frac{3}{8}'' =$ \$1,000,000, that is, $\frac{3}{8}$ inch represents \$1,000,000.

c. A diagram or scaled rule accompanying the drawing. This is the method commonly used in map drawing. Thus, in the map shown here, the scale is indicated.



d. The graph may be drawn on ruled paper, the distance between the ruled lines representing a specified value.

Exercise

1. A scale of $1'' = 1'$, is equivalent to what fractional scale?

2. A scale of $1'' = 5''$, is equivalent to what fractional scale?

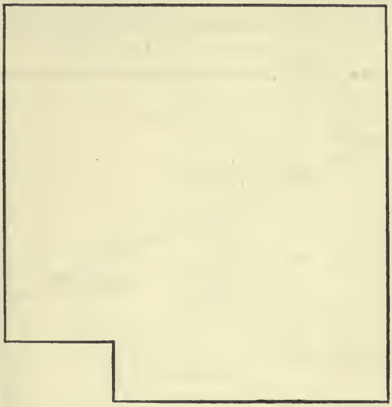
3. A scale of $1'' = 5'$, is equivalent to what fractional scale?

4. If the scale is $\frac{1}{12}$, what distance on the drawing would represent $6'\ 3''$? $4'\ 9''$?

5. If the scale is $\frac{1}{300}$, what plotted distance would represent 750 feet? 750 inches?

6. If the scale is $1'' = 1000$ bushels, what distance on a graph would represent 5125 bushels? 8500 bushels? 1750 bushels?

The following floor plan is drawn to a scale of $\frac{1}{96}$.



7. One inch of plotted distance on the floor plan is equivalent to how many feet of true distance?

8. One foot of true distance on the floor plan is represented by what part of an inch on the drawing?

9. What are the true dimensions of this room?

10. Assume that this drawing is made to the scale, $\frac{1}{8}$ in. = 1 ft. ($\frac{1}{8}'' = 1'$). What are the true dimensions of the room?

11. The map on p. 139 is drawn to the scale $1'' = ?$ miles.

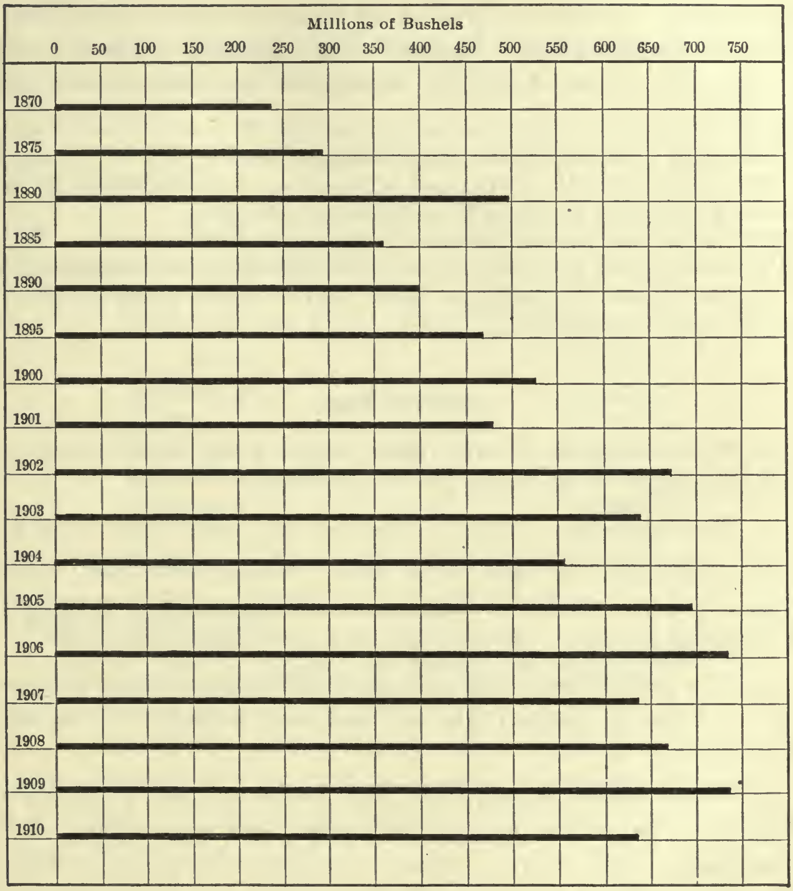
12. $\frac{1}{8}''$ on this map represents what true distance?

13. What is the true distance between each of the towns on the railroad? Measure distances from centers of towns.

14. An electric railroad is in process of construction between Morton and Harritown. It is surveyed in a direct line. How much shorter will this road be than the one via Sterling?

15. It is planned to locate a power house halfway between Sterling and Harritown. Indicate its position on the map and determine its distance from the two towns. How far will it be from the nearest point on the new electric line?

The following is an illustration of a graph drawn on ruled paper. The figures along the top of the graph indicate the number of millions of bushels of wheat grown. Quantities smaller than 10,000,000 bushels can be approximated with sufficient accuracy for general comparative purposes. For example, the wheat crop of 1870 is shown by the graph to have been about 236,000,000 bushels, and that of 1904, about 553,000,000 bushels.



Oral Work

1. What was the approximate crop of each of the years shown?

2. What was the record crop shown by the graph?

124. Determining the Scale. When a drawing or a graph is to be made, it is first necessary to determine the scale. The scale depends, in general, upon three things:

a. The size of the paper to be used.

b. The largest dimension to be shown on the graph.

c. Convenience in showing fractional parts of the scale.

Example. A room is 15′ 6″ long by 10′ 3″ wide. What scale should be used to show a diagram of this room on a sheet of paper 8″ by 10″, leaving a margin of at least one inch around the drawing?

SOLUTION. The largest dimension of the room is 15′ 6″, or 186″.

The largest dimension of the paper is 10 inches. After deducting 2 inches for margins, there is a space of 8″ available for the drawing.

The largest scale possible, therefore, is 8″ represents 186″.

This is equivalent to a scale of 1″ = $23\frac{1}{4}$″. (How was this determined?)

For convenience in showing fractional parts of a foot, it will be better to use the scale of 1″ = 24″.

Written Work

1. With the scale 1″ = 2′, what length lines would represent the dimensions of the room 15′ 6″ × 10′ 3″?

2. Use this scale to make a drawing of a room 12′ 6″ by 9′ 9″.

3. A field is 120 rods by 80 rods. Scale, 1″ = 1 rod. How large a paper is needed to diagram the field, leaving a margin of 2 inches all around? Would this be a reasonable scale to use?

4. If a scale 1 inch = 20 rods is used, what size paper is necessary in order to diagram this field and leave a margin of one inch on all sides?

5. What is the largest scale that could be used to plot this field on a paper the size of this page, leaving margins of at least $\frac{3}{4}$″?

6. It is desired to show by a graph the imports of coffee into various countries of the world. The year chosen is 1910. The United States was the largest importer, with an importation in round numbers of 804,000,000 pounds. The graph is to be placed on paper the same size as this page, with the same margins. How long a line would represent 100,000,000 bushels?

7. If the names of the countries require one inch, the scale might be —— inch = 100,000,000 bushels.

8. Draw a diagram of your schoolroom, entering the dimensions as illustrated in the drawing on page 137.

9. Draw a diagram of a room in your home. Do not enter the dimensions but state the fractional scale used, and draw all lines very carefully in accordance with this scale.

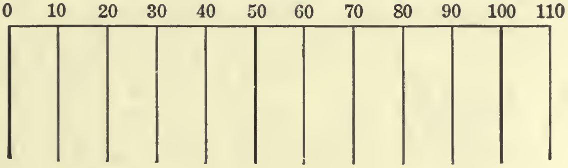
NOTE TO TEACHER. When these drawings are brought to class, they may be exchanged, and the students required to compute the dimensions in accordance with the scale stated.

10. Draw a line graph (similar to the one on page 138) to express the following statistics.

EXPORTS OF TEA FROM VARIOUS COUNTRIES IN 1910

COUNTRY	POUNDS
British India	258,000,000
Ceylon	182,000,000
China	207,000,000
Dutch East Indies	33,000,000
Formosa	24,000,000
Japan	40,000,000
Singapore	2,000,000

Rule the paper thus:



Use the scale $\frac{1}{4}'' = 10$ million pounds. (How large a paper will you need?)

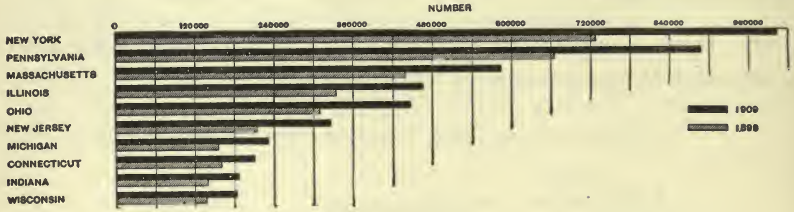
Arrange the countries so that the longest line will be at the top and the shortest at the bottom, the others being placed in the order of their lengths.

VARIOUS TYPES OF GRAPHS

125. Colored and Shaded Graphs. By the use of different colors of ink, or different shadings, comparisons may be made to show increase, decrease, or various changes in statistics from year to year.

The following graph shows the average number of wage earners employed in manufacturing industries in 1899 and 1909. The graph is limited to the ten states employing the largest number of men.

Average Number of Wage Earners, by States: 1909 and 1899[1]

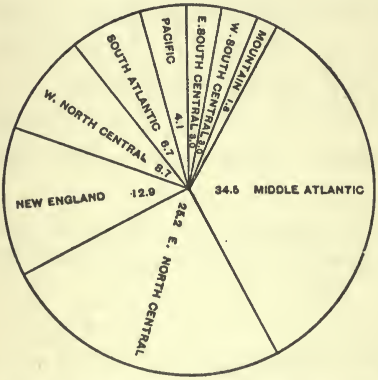


Oral Work

1. By referring to the graph, determine the approximate number of wage earners in each state in 1899 and in 1909.

2. Which state shows the greatest increase in the number of wage earners between the years 1899 and 1909?

126. The Circle is frequently used for graphic purposes. It is particularly valuable because it shows two things very clearly:



Value of Manufactured Products[1]

a. The relation of each magnitude to each of the others.

b. The relation of each magnitude to the sum of all.

This graph shows clearly that the states of the Middle Atlantic Division produce a greater value of manufactured articles than the states of any other division, and that they produce about one third of the entire value of the manufactures of the country.

The scale used in making circular graphs is based on the degree. If the circumference of a circle were divided into 360 equal parts, each part would be one degree of arc. Degrees may be measured by an instrument called a protractor.

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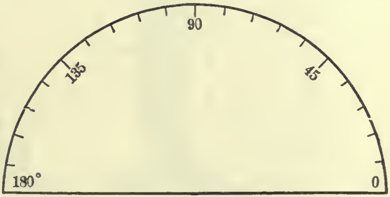
[1] Reprinted by permission of the United States Bureau of the Census, Department of Commerce.

The method of locating the lines in a circle graph may be explained by showing how the space occupied by the Middle Atlantic section was determined.



This section produces .345 of the total value of manufactured products. It should, therefore, occupy .345 of the circle.

.345 of 360° is 124.2°.

Points as nearly as possible 124.2° apart are marked off on the circumference with the aid of the protractor, and lines are drawn from the center of the circle to these points on the circumference.

Written Work

1. The total exports of coffee from all the countries of the world in 1910 were 2,163,764,874 pounds. Brazil exported 1,286,217,168 pounds or —— of the world's trade. It should therefore occupy what part of a circle designed to show the coffee exports of the world? How many degrees of arc?

2. The next largest exporter was the Netherlands, with 173,823,451 pounds. What part of the world's supply was provided by this country? What part of the circle should represent the amount of coffee exports of the Netherlands?

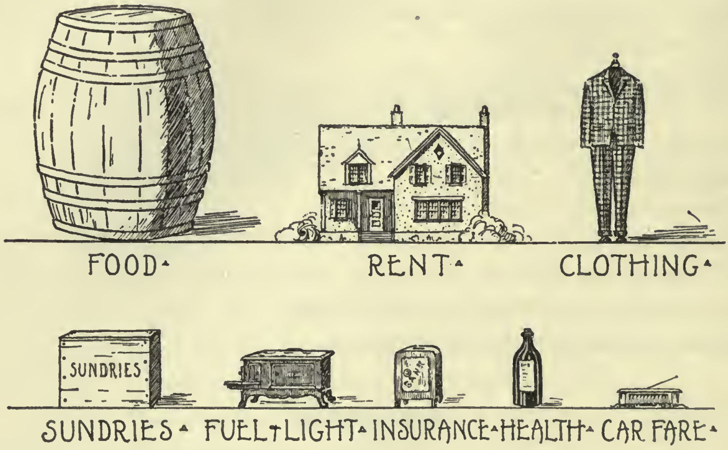
3. Five departments of a wholesale store produced profits as follows:

Dept.	Profits
I	$2165.00
II	3297.00
III	794.00
IV	1719.00
V	4625.00

Prepare a circle graph showing what per cent of the entire profit was made by each department.

127. Graphic Pictures. Another kind of graph frequently used is illustrated below:

WORKINGMEN'S EXPENDITURES



Food	$335.82
Rent	161.36
Clothing	98.79
Sundries	60.28
Fuel and light	36.94
Insurance	18.24
Health	14.02
Car fares	10.53

From *The Independent.*

These figures are based on a study of the wages of a number of workingmen, whose average annual income was $749.83 and whose average annual expenditure was $735.98.

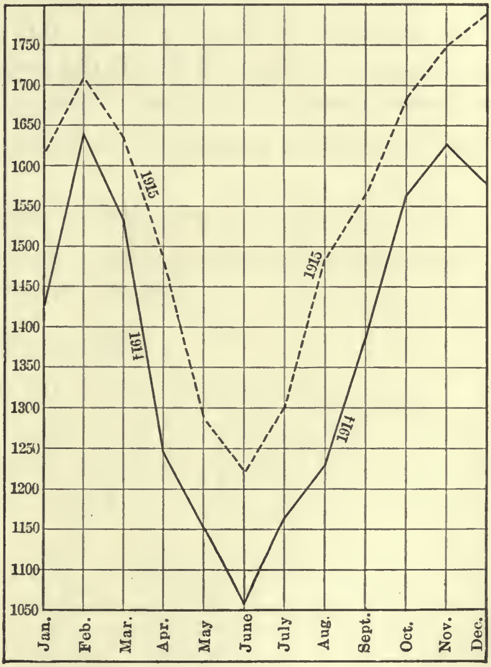
128. The Graphic Curve. Graphs drawn on cross-sectioned paper are especially valuable when it is desired to show the variation of statistics during consecutive intervals, as from month to month, or from year to year. The divisions on the vertical line may represent multiples of some unit of value and those on the horizontal line represent various intervals of time.

The following graph shows the variations in the monthly sales of a store during two years. The unbroken line ————

indicates the sales for the year 1914 and represents the following statistics:

Sales for January	$1426.80	Sales for July	$1167.85
February	1638.80	August	1232.65
March	1526.75	September	1386.64
April	1246.70	October	1566.45
May	1147.10	November	1626.30
June	1060.50	December	1575.25

It is sufficient for comparative purposes to show the approximate sales in dollars.



129. How this Graph is Drawn. An unbroken line represents the data for 1914. A point was placed on the January line at a point representing approximately $1425.00. A second point was placed on the February line at a point representing approximately $1638.00. Other points were properly located on the other lines, and an unbroken line, or curve, was then drawn connecting all the points. In the same way the dotted line was drawn to represent the data for 1915.

Exercise

Study the graph and answer the following questions:

1. Was there any uniformity of trade from year to year? That is, did the busy and the dull months follow in about the same order each year?

2. As a rule, during what seasons did this store have the largest trade?

3. What was its "dullest" month?

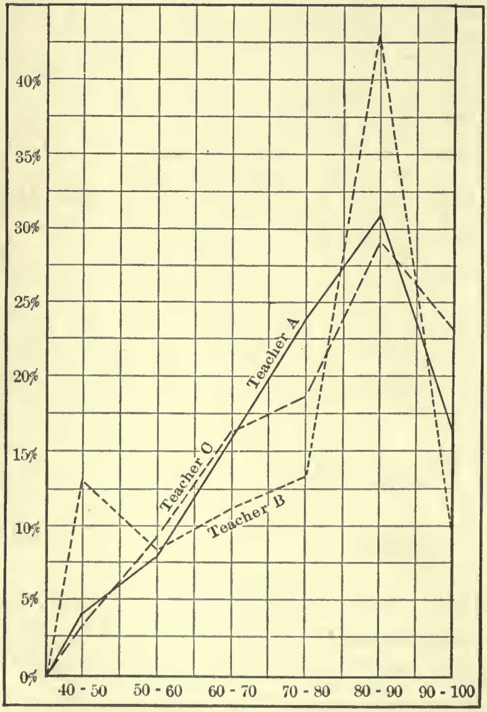
4. How did the business of 1915 compare with that of 1914?

5. About how many more dollars' worth of goods were sold during August, 1915, than during August, 1914?

6. Estimate the sales for each month of the year 1914. If your answers are within $2.00 of the correct amount, they will be accurate enough for purposes of comparison.

7. Discuss the value of such a graph.

130. The following graph shows a slightly different use of the curve. The curves serve to compare the grades given by three teachers in a certain school, showing what per cent of the students in their classes received certain grades.



This graph was made in the following manner:

A table was prepared showing what per cent of each teacher's students received grades between 40 and 50, 50 and 60, etc.

For example: Eight students of teacher A received grades 60–70. This was 16% of the students in A's classes.

Grade	Teacher A		Grade	Teacher A	
	No. of Students	Per Cent of Total Number		No. of Students	Per Cent of Total Number
40–50	2	4	70–80	12	24
50–60	4	8	80–90	16	32
60–70	8	16	90–100	8	16

The curve for Teacher A was plotted thus:

On the horizontal line above 40–50, a point was placed to indicate 4 %.

On the horizontal line above 50–60, a point was placed to indicate 8 %, etc.

All of the points thus located to represent grades given by A were then connected.

Oral Work

Interpret the curves, stating what per cent of the students of each teacher received the grades specified.

Written Review

Use paper $8\frac{1}{2}''$ by $11''$, or larger, for the following graphs.

1. By a graph similar to the first one on page 144, show the following statistics of exports from the United States to the United Kingdom. Use black ink for the data of 1910; red ink for those of 1911.

	1910	1911
Cattle	£ 2,578,000	£ 3,056,000
Bacon	4,453,000	5,067,000
Hams	2,329,000	2,712,000
Fresh Beef	1,070,000	397,000
Lard	4,201,000	4,014,000
Leather	4,057,000	3,828,000
Machinery	2,287,000	2,894,000
Copper	2,568,000	3,027,000
Paraffin Wax	871,000	617,000
Petroleum	3,745,000	3,370,000
Tobacco	2,815,000	3,278,000
Fish	1,021,000	702,000

2. Show graphically what part of the coinage of the years named in the following table was gold, what part silver, and what part other coin.

	Gold	Silver	Other Coin
1909	88,776,908	8,087,852	1,756,389
1910	104,723,735	3,740,468	3,036,929
1911	56,176,822	6,457,301	3,156,726
1912	17,498,522	7,340,995	2,577,386

3. By drawings of corn stalks, ears of corn, corn cribs, or any other suitable design, graph the following statistics of corn crops in the United States.

Year	Bushels	Year	Bushels
1870	1,094,000,000	1900	2,105,000,000
1880	1,717,000,000	1910	2,886,000,000
1890	1,489,000,000		

4. By a graph similar to the illustration of monthly sales (p. 147), show the variation in the wholesale price of corn per bushel by months for the following years.

	1910	1911	1912
	Cents	Cents	Cents
January	62.3	48.2	62.2
February	65.2	49.0	64.6
March	65.9	48.9	66.6
April	65.5	49.7	71.1
May	63.5	51.8	79.4
June	65.2	55.1	82.5
July	66.2	60.0	81.1
August	67.2	65.8	79.3
September	66.3	65.9	77.6
October	61.1	65.7	70.2
November	52.6	64.7	58.4
December	48.8	61.8	48.7

5. Show by a graph the data for the attendance at your school for a school year. (The principal can supply the data.)

6. Graph the temperatures for a certain hour of several successive days.

7. Make a graph showing your progress in speed and accuracy in addition, subtraction, multiplication, and division for a given set of examples.

8. What is the largest scale that could be used to represent a square farm of 160 acres on a paper 18″ by 20″, leaving a margin of at least 1″?

9. Two cities known to be 60 miles apart are $2\frac{1}{2}$ inches apart on a map. What is the scale of the map?

PERCENTAGE

CHAPTER XIV

PERCENTAGE

131. Relation of Percentage to Common and Decimal Fractions. **Percentage** is the process of computing by hundredths. The symbol % stands for per cent.

Thus, 15 % means $\frac{15}{100}$ or .15.

8 % means $\frac{8}{100}$ or .08.

300 % means $\frac{300}{100}$ or 3.00.

$\frac{1}{2}$ % means $\frac{\frac{1}{2}}{100}$ or $.00\frac{1}{2}$ or .005.

$\frac{2}{3}$ % means $\frac{\frac{2}{3}}{100}$ or $.00\frac{2}{3}$.

$7\frac{1}{2}$ % means $\frac{7\frac{1}{2}}{100}$ or $.07\frac{1}{2}$ or .075.

Oral Work

Express each of the following as a common fraction and as a decimal fraction:

1. 7 %
2. 4 %
3. 19 %
4. 24 %
5. $2\frac{1}{3}$ %
6. $\frac{1}{4}$ %
7. $\frac{1}{10}$ %
8. 200 %
9. 184 %
10. $5\frac{1}{6}$ %
11. $\frac{1}{100}$ %
12. $16\frac{2}{3}$ %

Express with the symbol %:

13. .74
14. .03
15. $.07\frac{1}{4}$
16. 2.00
17. $.32\frac{1}{5}$
18. .001
19. 1.00
20. .0045

21. $.5$ **22.** $.005$ **23.** $.16\frac{2}{3}$ **24.** $.14\frac{2}{7}$

25. $\frac{1}{2}$ **26.** $\frac{1}{4}$ **27.** $\frac{1}{5}$ **28.** $\frac{3}{2}$

29. $\frac{3}{4}$ **30.** $\frac{3}{5}$ **31.** $\frac{4}{5}$ **32.** $\frac{1}{3}$

33. 40 % of a quantity is how many hundredths of it? What fractional part of it?

34. 100 % of a number is how many times the number?

35. 300 % of a number is how many times the number?

36. 140 % of a number is how many times the number?

37. 2 times a number is what per cent of the number?

38. 5 times a number is what per cent of the number?

39. What part of a number is:

20 % of it?
80 % of it?
100 % of it?
$37\frac{1}{2}$ % of it?
$6\frac{2}{3}$ % of it?
12 % of it?
75 % of it?
$\frac{1}{2}$ % of it?
$87\frac{1}{2}$ % of it?
$62\frac{1}{2}$ % of it?
13 % of it?
$\frac{1}{5}$ % of it?
$12\frac{1}{2}$ % of it?
50 % of it?
$66\frac{2}{3}$ % of it?

40. Express each of the following as a decimal fraction and with the symbol %.

$\frac{1}{2}, \frac{1}{5}, \frac{1}{3}, \frac{5}{6}, \frac{3}{8}, \frac{2}{5}, \frac{1}{4}, \frac{3}{2}, \frac{2}{3}, \frac{4}{5}, \frac{1}{10}, \frac{5}{12}, \frac{7}{8}, \frac{3}{4}.$

41. The following table states the fractional part of the total number of colonies of bees in the United States owned in each section of the country. State what per cent of the colonies are owned in each section.

New England	.012
Middle Atlantic	.085
East North Central	.158
West North Central	.159
South Atlantic	.197
East South Central	.147
West South Central	.11
Mountain	.05
Pacific	.082

42. The following table shows the per cent of the total value of live stock owned in each section of the United States. State these per cents as decimal fractions.

New England	2 %
Middle Atlantic	7.1 %
East North Central	19.8 %
West North Central	31.5 %
South Atlantic	7.4 %
East South Central	7.5 %
West South Central	12 %
Mountain	7.9 %
Pacific	4.8 %

132. Terms Used in Percentage.

Base. The number of which a given per cent is to be taken is called the **base.**

Rate. The per cent of the base to be taken is called the **rate.**

Percentage. The result obtained by taking a certain per cent of the base is called the **percentage.** Thus,

$$64 \times .25 = 16;$$

or $$\textbf{base} \times \textbf{rate} = \textbf{percentage.}$$

FUNDAMENTAL PROCESSES OF PERCENTAGE

Computations in percentage are based on the following:

$$\text{base} \times \text{rate} = \text{percentage.}$$
$$64 \times .25 = 16.$$

Since the percentage is the product of the base and rate, it is evident that,

a. The percentage divided by the base equals the rate. For example:
$$16 \div 64 = .25 = 25\ \%.$$

b. The percentage divided by the rate equals the base. For example:
$$16 \div 25\ \% = 16 \div .25 = 64.$$

From these statements it is evident that if any two of the terms are known, the other one can be found.

In the chapter on Decimal Fractions you learned how to multiply and divide by any number expressed decimally. Percentage involves the mathematical principles of decimal fractions. The exercises in decimal fractions involved computations with tenths, hundredths, thousandths, etc. In Percentage, computation is by hundredths.

All mathematical computations involved in percentage may be grouped under three headings:

a. To find a given per cent of a number.

b. To find what per cent one number is of another.

c. To find a number when a certain per cent of it is known.

133. To find a given per cent of a number; that is, to find the percentage.

Examples. 1. Find 1 % of 428.

SOLUTION. 1 % = .01. .01 of 428 = 4.28.

2. Find 7 % of 378.

SOLUTION. .07 × 378 = 26.46.

3. Find 14 % of $360.

SOLUTION.

$$\begin{array}{r} \$360 \\ .14 \\ \hline 1440 \\ 360 \\ \hline \$50.40 \end{array}$$

To find the percentage, multiply the base by the rate.

Oral Work

1. Find 1 % of each of the following: 18; 146; 198; 7; 03; 876; .045.

2. Find 5 % of each of the following: 80; 60; 25; 42.

3. Find 10 % of $18.20; of 478 mi.; of 32 yd.; of 648 A.

4. How many places should be pointed off in finding each of the following:

17 % of 4762?	17 % of 476.2?	17 % of 47.62?
1.7 % of 4762?	.17 % of 4762?	17 % of .4762?

5. Find 10 % of 400: 560; 8.46.

6. Find 3 % of 40; 60; 15. **7.** Find 100 % of 15; 40; 3.

8. Find 20 % of 80; 70; 60. **9.** Find 50 % of 40; 2; 10.

10. Find 30 % of 30; 40; 80. **11.** Find $12\frac{1}{2}$ % of 480; 96; 640.

12. Find $37\frac{1}{2}$ % of 24; 80; 72. **13.** Find 200 % of 9; 32; $18\frac{1}{2}$.

14. Find $14\frac{2}{7}$ % of 21; 35; 350. **15.** Find 25 % of 12; 17; 96.

16. Find $\frac{1}{2}$ % of 120; 34; 18.

Written Work

Find:

1. 17 % of 472. **2.** 92 % of $537. **3.** $16\frac{2}{3}$ % of $420.

4. 3 % of 48,729. **5.** 200 % of 37 ft. **6.** 1.8 % of 42,690.

7. 17.5 % of 479,362. **8.** 250 % of $142.

9. 128 % of 346 yd. **10.** 5 % of 83.6.

11. $7\frac{1}{2}$ % of 42.736. **12.** $\frac{1}{2}$ % of 8476.

13. 32 % of $4.76. **14.** 19 % of .084.

15. 25 % of 264 sq. mi. **16.** $37\frac{1}{2}$ % of 24 A.

17. $12\frac{1}{2}$ % of $488. **18.** $87\frac{1}{2}$ % of 648 mi.

19. $33\frac{1}{3}$ % of $\frac{1}{5}$. **20.** 20 % of $\frac{7}{8}$.

21. $6\frac{2}{3}$ % of 32 cu. ft. **22.** 40 % of 85 sq. rd.

23. $62\frac{1}{2}$ % of 40 A. **24.** $33\frac{1}{3}$ % of $36.

25. $66\frac{2}{3}$ % of 120 ft. **26.** $12\frac{1}{2}$ % of 64 yd.

27. A school has an enrollment of 450 students. How many students are absent when 4 % are absent?

28. What is $200 increased by 4 % of itself?

29. In 1900 the population of a certain city was 52,840. In 1910 the population was 15 % larger than in 1900. What was the population in 1910?

30. A boy made a journey of 48 miles. He rode 85 % of the distance in an automobile and walked the remaining distance. How far did he walk?

31. An architect's fees for designing a house are 4 % of its cost. The cost is $7360. What is the fee?

32. A man whose salary is $3200 spends 19 % of it for rent. How much does he spend for rent?

33. A clerk receives 2 % of the amount of his sales. If his sales for Monday were $160.40, how much did he receive?

34. The report of a school with an attendance of 400 showed the per cent of tardiness to be $1\frac{1}{2}$. How many were tardy?

35. A man earns $1400 a year and saves 18 % of it. How long will it take him to save $756?

36. In a recent year, the following live stock was reported on farms in the United States:

	Number
Cattle	61,803,866
Horses	18,833,113

The following table shows the per cents of the total number of cattle and horses, in terms of the above data, in each section of the country. Thus 5.2 % of 61,803,866 was the number of cattle in the Pacific States.

Section	Cattle	Horses
New England	2.2 %	1.8%
Middle Atlantic	6.8	6.2
East North Central	15.9	22.2
West North Central	28.6	34.3
South Atlantic	7.8	5.6
East South Central	6.4	5.8
West South Central	17.3	11.8
Mountain	9.8	7.2
Pacific	5.2	5.1

Rule a form with the headings suggested below. Enter the facts given above; find the number of cattle and horses in each section of the country and enter the statistics on the blanks.

CATTLE AND HORSES IN UNITED STATES

Section	Cattle		Horses	
	United States Total 61,803,866		United States Total 18,833,113	
	Per Cent	Number	Per Cent	Number

134. To find what per cent one number is of another; that is, to find the rate.

Method. Since the percentage is the product of the base and the rate, and since the product may be divided by one factor to determine the other factor, we have the following:

percentage ÷ base = rate.

Examples. 1. 6 is what per cent of 12?

SOLUTION. $\frac{6}{12} = \frac{1}{2} = .50$ or 50 %.

2. A farmer had 72 sheep and sold 18 of them. What per cent of his flock did he sell?

SOLUTION. $18 \div 72 = .25$ or 25 %.

3. George had one brother and three sisters. What per cent of the children were boys?

SOLUTION. There were 5 children in the family.
$\frac{2}{5}$ of the children were boys.
$2 \div 5 = .4$ or 40 %.

Oral Work

1. What per cent of 8 is 4?
2. What per cent of 8 is 2?
3. What per cent of 24 is 3?
4. What per cent of 12 is 18?
5. What per cent of 9 is 3?
6. What per cent of 25 is 5?
7. What per cent of 25 is 20?
8. What per cent of 15 is 30?
9. What per cent of 10 is 8?
10. What per cent of 50 is 20?
11. What per cent of 60 is 40?
12. 15 is what per cent of 20?
13. 20 is what per cent of 40?
14. 18 is what per cent of 24?
15. 1 ft. is what per cent of 1 yd.?
16. 20 bu. are what per cent of 60 bu.?

17. 4 mo. are what per cent of 1 yr.?

18. 3 da. are what per cent of 1 wk.?

19. If you sleep 8 hr. out of 24, what per cent of the time do you sleep?

20. If a man earns $24 a week and saves $4, what per cent of his earnings does he save?

21. An agent sold goods amounting to $1800 and received $200 for his services. What per cent did he receive on the sales?

Written Work

1. A baseball team played 60 games and won 42. What per cent of the games played did the team win?

2. In a school of 250 students 45 were absent. What per cent of the students was absent?

3. In a school auditorium there are 650 seats. Four hundred eighty of the seats are reserved for the students. What per cent of the seats is not reserved?

4. A building worth $14,500 is insured for $10,150. For what per cent of its value is it insured?

5. In a spelling lesson a boy spelled all but 4 of the 30 words correctly. What per cent of the words did he spell correctly?

6. A corporation has a capital stock of $164,000. Dunham owns $41,000 of the stock. What per cent of the stock does he own?

7. A business man invested $3000. His income from the investment the first year was $420. What per cent did he earn?

8. In a certain school there are 224 boys and 312 girls. What per cent of the total enrollment is boys? girls?

9. Eighteen of the boys in this school failed to pass in at least one study. What per cent of the boys failed?

10. Twenty-three of the girls failed. What per cent of the girls failed?

11. What per cent of the entire school failed?

12. In 1910, the total population of the United States, 10 years of age and over, was 71,580,270. Of this number, 5,517,608 were unable to either read or write. What per cent of the population was illiterate?

13. The total population of the United States in 1910 was 91,972,266. The number of foreign-born inhabitants was 13,345,545. What per cent of the total population was foreign born?

14. The largest number of foreign-born people were Germans. They numbered 2,501,181. What per cent of the entire population was of German birth?

15. What per cent of the foreign-born population was of German birth?

16. The forests of the United States originally contained about 5200 billion board feet of timber. Cutting, clearing, and fire have reduced the stand to about 2500 billion board feet. What per cent of the timber remains?

17. The following table is similar to one used by a clothing store to determine the tailoring firm which furnishes suits having the best sale. Record is kept of the number of suits purchased from each firm, and of the number of these suits sold.

Find what per cent of the suits purchased from each of the manufacturers was sold, and enter all statistics on a ruled form similar to the model.

Of what value is this table of statistics to the merchant?

PER CENT OF SALES FROM STOCK PURCHASED FROM CLOTHING MANUFACTURERS

MANUFACTURER	NUMBER BOUGHT	NUMBER SOLD	PER CENT SOLD
TAYLOR, BRIGHT & CO.	368	293	
WELLS & BAILEY	493	316	
F. H BAINUM	639	582	
DUDLEY & COLE	523	416	
WILLIAMS, ELDRIDGE & CO.	863	719	
M. J HARTMAN & SONS	1392	1202	
W.C DENNISTON & CO.	1020	905	

18. The following table shows one of the ways in which merchants determine which clerk is selling the most goods.

Rule a form similar to the model; enter on the blank the facts which are given in the model; find the total sales made by all of the salesmen. Find what per cent of the total sales was made by each salesman. Enter all of these facts on the blank.

TABLE SHOWING PER CENT OF MONTH'S SALES MADE BY VARIOUS SALESMEN

SALESMAN'S NAME	SALES		PER CENT OF TOTAL
J. B. DENNY	$ 1264	90	
O. F. ALBRIGHT	1913	52	
G. F. SLATER	1732	69	
WILLIAM WARNER	2213	72	
C. B. WILEY	1963	45	
H. WALKER	1627	32	
FRANK McGEE	1692	18	
F. L. MONTGOMERY	2138	45	
C. H. HENRY	1728	46	
TOTAL			

19. The following tabulation is a record of a week's sales in four departments of a store.

Rule a blank similar to the model.

Enter the statistics on the blank.

Find the total sales for each day made in all departments; enter these totals in the "total" column at the right.

Find the total sales made during the week in each department; enter these totals on the line at the bottom marked "total."

Find (in two ways) the total sales made in all of the departments for the entire week. Enter in the space marked Grand Total.

Find what per cent of the grand total sales was made each day; enter these per cents in the column at the right.

Find what per cent of the grand total sales was made in each department. Enter these per cents on the line at the bottom marked per cent.

One of the qualities which a business man desires in his employees is the ability to follow directions. This exercise will give you an opportunity to prove your ability along this line. Get a clear understanding of the instructions, and follow them by preparing the exercise without asking for any further directions.

A WEEK'S SALES IN A DEPARTMENT STORE

DAY	DEPT. I		DEPT. II		DEPT. III		DEPT. IV		TOTAL		PER CENT
Monday	$362	50	$562	83	$862	94	$126	39	$		
Tuesday	415	75	475	92	732	83	143	62			
Wednesday . . .	396	21	415	60	769	42	129	38			
Thursday . . .	472	96	516	29	640	20	145	17			
Friday	387	29	429	36	721	32	96	27			
Saturday	493	89	562	64	816	25	163	92			
Total									Grand Total		
Per Cent . . .											

135. To find a number when a certain per cent of it is known; that is, to find the base.

Method. Since the *percentage* is the product of two factors, the base and the rate, and since any product can be divided by one of its factors to find the other factor, it is evident that,

$$\text{percentage} \div \text{rate} = \text{base}.$$

Therefore, *to find the base, divide the percentage by the rate expressed as a decimal.*

Examples. 1. 18 is 25% of what number?

SOLUTION. 18 is the percentage, 25% is the rate; the base is not known.

$18 \div .25 =$ the required number, or base.

$18 \div .25 = 72.$

2. 45 % of the number of students in a school are boys. If there are 135 boys, how many students are in the school?

Solution. $135 \div .45 = 300$, the total number of students.

This may be explained as follows:

Since 45 % of the number of students = 135,
1 % of the number of students = $\frac{1}{45}$ of $135 = 3$
and 100 % of the number of students = $100 \times 3 = 300$.

Oral Work

Find the number of which:

1. 40 is 20 %.
2. 12 is 4 %.
3. 10 is 5 %.
4. 86 is 2 %.
5. $45 is 15 %.
6. 40 is 80 %.
7. 12 is $\frac{1}{2}$ %.
8. 42 is 100 %.
9. 60 is 200 %.
10. 12 is $33\frac{1}{3}$ %.
11. 35 is 7 %.
12. 19 is $\frac{1}{2}$ %.
13. 8 is $\frac{2}{5}$ %.
14. 2700 is $66\frac{2}{3}$ %.

15. A boy lost 40 cents, which was 10 % of what he had. How much did he have?

16. A man sold 32 acres of land, which was 80 % of all that he owned. How many acres did he own?

Written Work

1. In a certain school 5 % of the pupils are absent and 475 pupils are present. What is the enrollment?

2. By saving 28 % of his salary a man saved $2240 in 5 years. What was his salary?

3. 18.75 % of a class failed. If 26 passed, how many were in the class?

4. The distance from New York to San Francisco via the Panama Canal is 5278 miles. This distance is 60.44 % less than the distance via Cape Horn. What is the distance via Cape Horn?

5. A boy paid $3 for a pair of shoes and had 70 % of his money left. How much money had he at first?

6. Four partners engage in business. The first invests $4000; the second invests $3000; each of the others invests 25 % of the total capital. What is the total capital?

7. The Washington Monument is 555 ft. and Eiffel Tower is 984 ft. high. The height of the monument is what per cent of the height of the tower?

8. The average velocity of wind is 18 mi. per hour and that of sound is 1090 ft. per second. The velocity of wind is what per cent of the velocity of sound?

9. The mean annual rainfall in Denver is 14 inches, and that in New York City is 44.8 inches. Each is what per cent of the other?

10. The length of the Hudson River is 280 miles; the length of the Ohio is 950 miles; and the length of the Mississippi is 3160 miles. The length of each is what per cent of the length of the others?

11. The record for the 100-yard dash is now $9\frac{3}{5}$ sec. It was formerly $9\frac{4}{5}$ sec. By what per cent was the record reduced?

12. In 1776 there were 13 states in the Union. There are now 48 states. What has been the per cent of increase?

13. A certain basketball player can shoot on an average 90% of his free chances. If there are 20 fouls called on his opponents in a game, how many points should he make on fouls?

14. The United States Census divides persons engaged in manufacturing into three classes: wage earners, clerks, and proprietors.

The table on page 165 shows the number of persons in the first class; that is, the wage earners. It also shows the per cent of the total number engaged in each line of manufacturing who were wage earners.

For example, there were 50,551 wage earners engaged in the manufacture of agricultural implements. The wage earners were 83.9% of the total number of persons engaged in this work. What was the total number of persons engaged in this work?

Rule a form and find the total number of persons engaged in such industries as the teacher directs.

Since the per cents are only approximate, the results will be only approximate.

Indicate the ten industries having the largest number of employees. Mark the industry with the largest number of employees "1," the second largest "2," etc.

Industry	Wage Earners Av. Number	Per Cent	Total Employees	Rank
All Industries	6,615,046	86.1		
Agricultural Implements	50,551	83.9		
Automobiles	75,721	88.7		
Boots and Shoes	198,297	91.8		
Brass and Bronze	40,618	89.4		
Bread	100,216	69.4		
Butter, Cheese	18,431	58.5		
Canning	59,968	83.3		
Carriages	69,928	84.3		
Cars and Shop Construction	282,174	93.7		
Cars, Steam	43,086	91.5		
Chemicals	23,714	85.3		
Clothing, men's	239,696	88.3		
Clothing, women's	153,743	85.9		
Confectionery	44,638	81.4		
Copper, Tin	73,615	84.7		
Cotton Goods	378,880	97.7		
Electrical Machinery	87,256	82.6		
Flour Mill	39,453	59.7		
Foundry	531,011	86.3		
Furniture	128,452	89.1		
Gas	37,215	73.0		
Hosiery	129,275	94.9		
Iron, Steel, Blast	38,429	89.2		
I. and S. Steel Works	240,076	92.1		
Leather Goods	34,907	80.2		
Leather	62,202	92.7		
Liquors, Distilled	6,430	77.2		
Liquors, Malt	54,579	81.8		
Lumber	695,019	88.5		
Marble	65,603	84.9		
Oil	17,071	80.2		
Paint and Varnish	14,240	65.0		
Paper	75,978	93.3		
Patent Medicines	22,895	55.7		
Petroleum	13,929	8.37		
Printing and Publishing	258,434	66.5		
Silk	99,037	94.1		
Slaughtering	89,728	82.5		
Smelting, Copper	15,628	92.8		
Smelting, Lead	7,424	92.1		
Sugar and Molasses	13,526	86.4		
Tobacco Manufacture	166,810	84.4		
Woolen, Worsted	168,722	96.3		
All Other Industries	1,648,441	86.0		

PERCENTAGE OF INCREASE AND DECREASE

Percentage is frequently employed to find the relation between numbers, or to find how much larger or smaller one number is than another. This does not involve any new mathematical principles.

Example. What number increased by 20 % of itself is 360?

SOLUTION.
100 % of the number = the number,
20 % of the number = the increase,
120 % of the number = 360,
1 % of the number = $360 \div 120 = 3$,
100 % of the number = $100 \times 3 = 300$,

or we may say: 1.20 times the number = 360, therefore the number $= \frac{360}{1.20} = 300$.

Oral Work

(Use equivalent fractions if more convenient.)

What number increased by

1. 20 % of itself is 240?
2. 25 % of itself is 250?
3. $12\frac{1}{2}$ % of itself is 360?
4. 200 % of itself is 1800?
5. $14\frac{2}{7}$ % of itself is 88?
6. 75 % of itself is 14?
7. 50 % of itself is 600?
8. 5 % of itself is 420?
9. 300 % of itself is 2400?
10. $37\frac{1}{2}$ % of itself is 330?
11. 1 % of itself is 202?
12. 7 % of itself is 428?
13. $6\frac{2}{3}$ % of itself is 4800?
14. $66\frac{2}{3}$ % of itself is 40?
15. 60 % of itself is 64?
16. 100 % of itself is 600?

17. How much is $40 increased by 20 % of itself?

18. How much is 60 increased by 10 % of itself?

19. How much is $42 increased by 10 % of itself?

20. How much is 3 increased by 100 % of itself?

21. How much is 24 bu. increased by $37\frac{1}{2}$ % of itself?

22. How much is 12 yd. increased by $33\frac{1}{3}$ % of itself?

23. How much is 10 mi. increased by 7 % of itself?

24. How much is $18 increased by 200 % of itself?

25. How much is 32 acres increased by $12\frac{1}{2}$ % of itself?

136. Per Cent of Increase.

To find the increase, *multiply the base by the per cent of increase.*

Example. Mr. Smith, whose salary was $1200.00, received a 5 % increase. How much was the increase?

SOLUTION.

$1200	Base
.05	
$60.00	Increase

To find the per cent of increase, *divide the increase by the base.*

Example. Mr. Smith, whose salary was $1200.00, received an increase of $60.00. What was the per cent of increase?

SOLUTION. $60.00 ÷ $1200.00 = .05, or 5 %.

To find the base.

a. Given the increase and per cent of increase.

Example. Mr. Smith received a 5 % increase in salary. The increase was $60.00. What was his original salary?

SOLUTION. $60.00 ÷ .05 = $1200.00.
Increase ÷ per cent of increase = base.

b. Given the increase and the amount.

Example. Mr. Smith's salary was increased $60.00, after which he received $1260.00. What was the per cent of increase?

SOLUTION. $1260 − $60 = $1200, the base.
$60 ÷ $1200 = .05.
Hence, the per cent of increase is 5 %.

c. Given the per cent of increase and the amount.

Example. After receiving a 5 % increase, Mr. Smith's salary was $1260.00. What was his original salary and what was the increase?

SOLUTION. His original salary was 100 % of itself. His increase was 5 % of the original salary. Therefore, the new salary was 105 % of the old one.

$1260 ÷ 1.05 = $1200.

Written Work

1. A merchant's sales during 1914 were $6238.92. By advertising, he increased his sales $2485.35 the following year. What was the per cent of increase, and what was the amount of the sales in 1915?

2. The deposits of the Wells Street Bank on June 30, 1914, were $365,894.56. On June 30, 1915, the deposits had increased to $396,268.55. What was the increase and the per cent of increase?

3. A factory produced 240 articles of a certain kind per day. By installing new machinery, the output was increased $12\frac{1}{2}\%$. What was the amount of the increase, and how many articles were produced per day after the new machinery was installed?

4. Williams paid $16.87 more taxes this year than he did last year; an increase of 7.8%. What was the amount of his taxes each year?

5. The payroll of the Maynard Manufacturing Company for the week ending April 4, 1914, was $428.62. The payroll for the week ending April 3, 1915, was 13% larger. What was the pay roll for the week in 1915, and what was the increase?

6. A merchant built an addition to his store which increased the floor space 28%. After the addition was completed, the floor space was 1280 square feet. What was the number of square feet of floor space in the original store, and what was the amount of the addition?

137. Per Cent of Decrease.

To find the per cent of decrease, *divide the decrease by the base.*

Example. Mr. Smith's coal bill in 1914 was $80.00. In 1915 his bill was $4.00 less. What was the per cent of decrease?

SOLUTION. Decrease ÷ base = per cent of decrease.

Thus, $4.00 ÷ $80.00 = 5%.
The decrease was 5%.

Written Work

1. The proprietor of a retail grocery employed five clerks at a total annual expense for wages of $5000.00. In order to reduce the expense, he released one clerk whose wages were $60.00 per month. How much did he decrease the cost of the clerk hire per year? What was the per cent of decrease? What was the annual cost of the clerk hire after one clerk was dismissed?

2. A paper mill which burned 2400 tons of soft coal per year remodeled its furnaces and boilers and now burns 2000 tons of coal per year. How many tons less does this factory consume per year than formerly, and what is the per cent of decrease?

3. Previous to 1914, Mr. Waterbury owned the only drug store in the town of X. When a new drug store was opened, the competition decreased Mr. Waterbury's annual sales $1067.60, or 17%. What was the amount of Mr. Waterbury's sales the year before the new store opened?

4. In 1914 Mr. Snowden, a grocer, lost $327.00 from bad debts. A credit-rating association was formed in the town; the merchants informed each other of persons who did not pay their bills; and the following year Mr. Snowden decreased his loss from bad debts 18%. How much did the credit-rating association save Mr. Snowden, and what was his loss from bad debts in 1915?

5. Rathbun moved his store into a new building where the rent was $5.00 per month less than the rent of the building which he formerly occupied. The rent in the new location was $125.00 per month. What was the per cent saved?

6. Barnes said to his partner, "We cut down our delivery expense 30% when we sold the horses and bought an auto truck. Our annual cost for making deliveries after we made the change was only $1645.00. When we delivered with horses, the cost per year was $———, so we are saving $———."

7. The following table shows one of the uses which business men make of per cent of increase and decrease.

This table compares the sales on the first Tuesday in June, 1914, with the sales on the first Tuesday in June, 1915. The merchant assumes that trade conditions were about the same on the two days. A table of this kind is prepared every day, comparing the business for the day with the business of the corresponding day of the previous year.

Prepare a table similar to the model, and enter the facts given. Show increases and per cents of increase in black ink; decreases and per cents of decrease in red ink.

What was the increase in business in department 1? What was the per cent of increase?

Complete the table.

COMPARATIVE SALES SHEET

DEPARTMENT NUMBER	SALES, TUESDAY, JUNE 2, 1914		SALES, TUESDAY, JUNE 1, 1915		INCREASE OR DECREASE		PER CENT INCREASE OR DECREASE
1	$ 826	95	$ 914	32	$		
2	1034	78	1231	64			
3	1237	62	1196	14			
4	2643	80	2843	27			
5	1413	80	1376	29			
6	962	40	1235	96			
7	2642	16	2927	92			
8	1964	39	2129	80			
9	1636	48	1596	27			
10	1213	42	1723	96			
11	1394	29	1146	92			
12	415	75	496	25			
13	3460	00	3246	29			
14	2690	70	2889	00			
15	878	25	794	60			
Total							

8. The following table shows the value of irrigating arid land. For example, the government records show that, in the regions where rainfall is insufficient, the potato crop on an acre of irrigated land is worth $15.37 more than the crop grown on an acre of unirrigated land. This is an increase of 34.4% due to irrigation. What is the average value per acre of the potato crops grown on unirrigated land and on irrigated land?

Complete the table.

COMPARATIVE VALUE OF CROPS GROWN ON IRRIGATED AND UNIRRIGATED LAND

CROP	EXCESS OF AVERAGE VALUE OF CROP PER ACRE GROWN ON IRRIGATED LAND, OVER THAT GROWN ON UNIRRIGATED LAND			AVERAGE VALUE OF CROP PER ACRE ON UNIRRIGATED LAND		AVERAGE VALUE OF CROP PER ACRE ON IRRIGATED LAND	
Potatoes	$15	37	34.4%				
Sugar Beets	5	39	10.4				
Wheat	8	65	58.6				
Alfalfa	5	97	35.2				
Oats	7	36	63.2				
Barley	6	51	55.1				
Corn	3	51	24.0				
Timothy and Clover, mixed	3	63	27.6				
Timothy alone	3	08	24.1				

138. Per Cent of Maximum. It is sometimes desired to compare a certain number with a larger number. For example, a certain packing plant is equipped to slaughter and pack a maximum of 2000 hogs daily. On October 22, this plant packed 1832 hogs.

The day's pack was $\frac{1832}{2000}$ of the maximum capacity, or 91.6%. State a rule for finding per cent of maximum.

Written Work

The table on page 172 shows the number of wage earners employed in coal mining in the United States in a recent year. The number of employees varies with the season. Some idea of the steadiness of employment in this industry may be formed by comparing the number of employees each month with the maximum.

During what month was the maximum number of persons employed in coal mining?

What per cent of this number was employed in January?

Complete the table.

WAGE EARNERS EMPLOYED IN COAL MINING IN THE UNITED STATES

MONTH	WAGE EARNERS EMPLOYED	PER CENT OF MAXIMUM
January	691,244	
February	686,322	
March	679,791	
April	649,870	
May	646,592	
June	652,894	
July	659,434	
August	667,146	
September	685,234	
October	704,939	
November	720,341	
December	729,273	

139. Per Cent of Average. Percentage is also used to show the relation between different numbers by comparing them with their average.

Written Work

The following table shows the monthly sales of a number of clerks:

CLERK NO.	SALES	PER CENT	CLERK NO.	SALES	PER CENT
201	$1412.32		205	$1836.24	
202	1218.16		206	1216.26	
203	967.32		207	1375.85	
204	1046.89		208	1493.85	

Find the average sales.

The sales of Clerk No. 201 are what per cent of the average?

Complete the table.

What is the value of such a table for the manager of a business?

Review Work

1. If a merchant sells all goods at an advance of 10 % of the cost, what will be the selling price of an article which cost $ 3.40? What will be the profit on this article? If this merchant's annual sales are $ 24,126.35, what is his gross profit?

He marked all goods at 25 % increase on the cost the following year and his sales dropped to $ 15,625.90. What was the cost of the goods sold, and the gross profit?

2. In 1910 the total area of the Indian reservations in the United States, exclusive of Alaska, was 77,446 square miles. In 1890 the area of such reservations was 243,991 square miles. What per cent did the area decrease in twenty years?

3. In 1910 the Indian population was 300,121, in 1890 it was 243,524. What per cent did the population increase?

4. What was the average number of acres of land in reservations per Indian for each of the years named?

5. What was the per cent of increase or decrease in the average acreage?

6. Complete the following table showing the average size of farms in different divisions of the country.

Division	Average Size of Farms (Acres)		Increase (+) Decrease (−)	
	1910	1900	Acres	Per Cent
United States	138.1	146.2		
New England	104.4	107.1		
Middle Atlantic	92.2	92.4		
East North Central	105.0	102.4		
West North Central	209.6	189.5		
South Atlantic	93.3	108.4		
East South Central	78.2	89.9		
West South Central	179.3	233.8		
Mountain	324.5	457.9		
Pacific	270.3	334.8		

7. The following table shows the amount of sugar produced in and imported to the United States. Complete the table.

	Long Tons (2240 lb.)	
Sugar imported from Hawaii and Porto Rico	718,788	
Sugar imported from other countries	1,674,776	
Total imports		x,xxx,xxx
Domestic production of cane sugar	409,960	
Domestic production of beet sugar	434,000	
Total domestic production		x,xxx,xxx
Total consumption of sugar in United States . .		x,xxx,xxx

a. What per cent of the total consumption of sugar was imported?

b. What per cent of the sugar consumed was produced in this country?

c. What per cent of the sugar made in this country was cane sugar? What per cent was beet sugar?

8. Complete the following table.

Class	Persons Engaged in Manufactures				
	1904		1909		Per Cent of Increase, 1904–1909
	Number	Per Cent of Total	Number	Per Cent of Total	
Proprietors and firm members	225,673		273,265		
Salaried employees	519,556		790,267		
Wage earners (average number)	5,468,383		6,615,046		
Total					

140. Percentage Analysis of a Business. Mr. E. C. Barton is the proprietor of a wholesale store. At the end of each month he analyzes the records of his business. The following analysis was made June 30, 1915. Results should be approximate to the nearest hundredth of a per cent.

1. The purchases during June, 1915, were $8146.90. Some of the goods purchased were defective, and were returned. The value of goods returned was $123.60. The returned goods were what per cent of the purchases?

2. Gross sales for June, 1915, were \$11,216.29. Goods returned to the store by dissatisfied customers, \$97.65. The returned sales were what per cent of the gross sales?

3. Gross sales for June, 1914, were \$9862.15. Returned sales, \$101.80. What per cent of the sales were returned in June, 1914?

4. What was the increase in the gross sales, and what was the per cent of increase?

5. Gross Sales – Returned Sales = Net Sales.

What were the net sales for June, 1914? For June, 1915? What was the increase in net sales, and the per cent of increase?

6. The merchandise on hand June 1, 1915, was worth \$14,162.45 at selling prices. The net sales for the month were found in problem 5. What per cent of the stock on hand at the beginning of the month was sold during the month?

7. Net Sales – Cost of Goods Sold = Gross Profit.

The gross profit for June, 1914, was \$1688.54. The gross profit was what per cent of the net sales for the month? (For net sales see results to problem 5.)

8. The gross profit for June, 1915, was \$2001.36. The gross profit was what per cent of the net sales for that month?

9. The expenses of the business in June, 1914, were \$1015.60. The expenses had increased in June, 1915, to \$1140.26. What was the per cent of increase in the expenses?

10. Net Sales – Gross Profit = Cost of Goods Sold.

What was the cost of goods sold in June, 1914?

What was the cost of goods sold in June, 1915?

11. Gross Profit – Expenses = Net Profit.

What was the net profit for June, 1914?

What was the net profit for June, 1915?

12. What was the per cent of increase or decrease in the net profit?

13. Net Profit ÷ Net Sales = Per Cent of Net Profit on Sales.

What was the per cent of net profit for June, 1914?

What was the per cent of net profit for June, 1915?

TRADING ACTIVITIES. PROFIT AND LOSS

CHAPTER XV

BUYING AND SELLING MERCHANDISE

141. The Invoice. When a merchant sells goods, he usually gives the customer a bill or invoice. The invoice states the following facts:

The date of the sale.

The name and address of the seller.

The name and address of the purchaser.

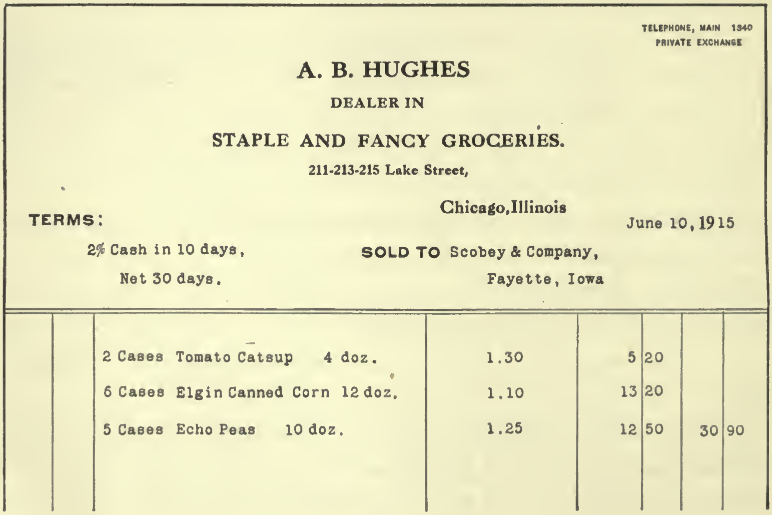
The terms of the sale.

A detailed list of the articles sold, including the price of each item and the total of the invoice.

If prepaid freight is to be charged to the purchaser, the amount of freight is added to the invoice.

If the invoice has been paid, a statement of the receipt of payment is made by the seller.

Study the following invoice and state as many as possible of the facts enumerated above.



TELEPHONE, MAIN 1340
PRIVATE EXCHANGE

A. B. HUGHES

DEALER IN

STAPLE AND FANCY GROCERIES.

211-213-215 Lake Street,

Chicago, Illinois

June 10, 1915

TERMS:

2% Cash in 10 days,
Net 30 days.

SOLD TO Scobey & Company,
Fayette, Iowa

		2 Cases Tomato Catsup 4 doz.	1.30	5	20		
		6 Cases Elgin Canned Corn 12 doz.	1.10	13	20		
		5 Cases Echo Peas 10 doz.	1.25	12	50	30	90

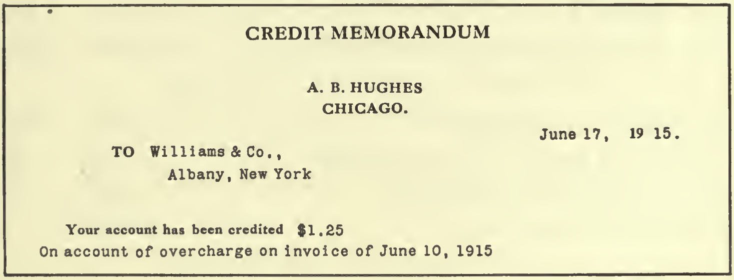
142. What the Purchaser Does. After the purchaser receives both the goods and the invoice, he inspects the merchandise to see if it agrees with the items for which he has been charged on the invoice. If stock of the correct kind and quantity is received, a check mark is entered on the invoice opposite each item in the column at the left.

The invoice is next checked to ascertain:

a. Whether the correct prices have been charged.

b. Whether any error has been made in the extension of the cost of each item, or in the total of the invoice.

143. Credit Memorandum. In case an error is found, notice is sent to the firm from which the goods were purchased. When credit is to be given by the seller on account of an error in the invoice, or for imperfect goods, or for any other cause, the purchaser will usually receive a credit memorandum, notifying him of the amount of credit entered to his account.



CREDIT MEMORANDUM

A. B. HUGHES
CHICAGO.

June 17, 19 15.

TO Williams & Co.,
Albany, New York

Your account has been credited $1.25
On account of overcharge on invoice of June 10, 1915

Written Work

Check the invoice on page 176. If the multiplications are found to be correct, enter a check mark at the right of each item. If the total is correct, check it also.

When an invoice has been checked and found to be correct, the extensions and footings are checked as shown in the illustration.

$16	50 ✓			
23	00 ✓			
4	80 ✓		$44	30 ✓

Written Work

Find the totals of each of the following sales:
Apply short methods when possible.

1.

4 doz. Men's Irish Linen Handkerchiefs	doz.	$4.43½
5 doz. Men's Hemstitched Handkerchiefs	doz.	5.38
14 doz. Men's White Initial Handkerchiefs	doz.	3.23
9 doz. Women's Imported Swiss Embroidered Handkerchiefs	doz.	2.16¾
22 doz. Children's Linen Handkerchiefs	doz.	1.23½
7 doz. Women's Swiss Lawn Handkerchiefs	doz.	.67½
13 doz. Children's Mull Handkerchiefs	doz.	.72½

2.

17 doz. Men's Leather Gauntlets	doz.	$ 7.68
11 doz. Men's Buckskin Gloves, sizes 8 to 10	doz.	9.49
16 pairs Men's Leather Automobile Mittens	pair	1.36½
4½ doz. Men's Suede Gloves, sizes 7¾ to 10	doz.	4.44
3½ doz. Men's No. J 264 Kid Gloves	doz.	10.50
5½ doz. Imported Kasan Cape Gloves	doz.	13.50

3.

16 Silk Taffeta Umbrellas	each	$2.15
28 8 Rib Pure Silk Taffeta Umbrellas	each	4.25
32 Men's Suit Case Umbrellas	each	2.25
12 Extra Size Umbrellas	each	.87½

4.

3 doz. pairs Walton Cotton Blankets	pair	$ .82½
1 doz. pairs Cotton Crib Blankets	pair	.57½
8 doz. pairs Wool Filled Gray Blankets	pair	3.85
7 doz. pairs Douglass Wool Finish Plaids	pair	1.89½
4 doz. pairs All Wool Plaids	pair	6.00
10 doz. pairs "Canada Camp" Blankets	pair	1.12½

5.

20 Assorted Pattern Smyrna Rugs, 36 × 72	each	$2.25
80 Scotch Jute Rugs	each	.37½
3 doz. Cotton Bathroom Rugs, 27 × 54	each	1.12½

6.

3 doz.	Size 6	Children's Fast Black Hose	doz.	$\$1.17\frac{1}{2}$
2 doz.	Size $6\frac{1}{2}$	Children's Fast Black Hose	doz.	$1.21\frac{1}{2}$
4 doz.	Size 7	Children's Fast Black Hose	doz.	$1.24\frac{1}{2}$
5 doz.	Size $7\frac{1}{2}$	Children's Fast Black Hose	doz.	$1.27\frac{1}{2}$
6 doz.	Size 8	Children's Fast Black Hose	doz.	$1.31\frac{1}{2}$
8 doz.	Size $8\frac{1}{2}$	Children's Fast Black Hose	doz.	$1.42\frac{1}{2}$
7 doz.	Size 9	Children's Fast Black Hose	doz.	$1.49\frac{1}{4}$
6 doz.	Size $9\frac{1}{2}$	Children's Fast Black Hose	doz.	1.61
5 doz.	Size 10	Children's Fast Black Hose	doz.	$1.66\frac{1}{2}$

7.

20 No. B 645	Silk Woven Waist Patterns (each pattern $3\frac{1}{2}$ yd.)	yd.	$\$.16\frac{2}{3}$
25 No. B 493	Dresden Silk Mull Waist Patterns ($3\frac{1}{2}$ yd. pieces)	yd.	$.21\frac{1}{2}$
18 bolts	Irish Linette, 10, 11^2, 12, 10^3, 11^1, 11^3, 12^1, 11^3, 10^3, 12, 10, 10^3, 10^1, 11^2, 12, 11^3, 10^2, 10^3	yd.	$.14\frac{1}{2}$
20 bolts	French Gauze Chiffon, 14, 13^3, 12^3, 14^1, 14, 13^2, 13, 12^3, 13, 13^1, 14^2, 13^3, 14^2, 14^1, 13^3, 12^3, 14, 13^3, 13^2, 12^3	yd.	$.37\frac{1}{2}$

NOTE. The small figures mean quarter yards. Thus 11^2 means $11\frac{2}{4}$ yards.

EXPLANATION OF GROCERY ORDERS

The sign # if placed before figures means "number"; if placed after figures it means pounds.

Articles sold in cases [cs.] are usually priced by the dozen. The total number of dozens in the cases is therefore given after the name of the commodity, and the price is the price per dozen.

Some articles sold in barrels and cases are priced by the pound; in such instances the total number of pounds in all the barrels or cases appears immediately before the price.

Sugar is sold by the hundred pounds (cwt.). The net weight of each barrel is given. Find the total number of pounds; point off two places to the left to find the number of hundred pounds, and multiply by the price per hundred.

8.

		Price
7 cases Acme Peas	14 doz.	$1.40
3 boxes Peona Soap		4.95
7 bbl. Northern Salt		2.10
15 bbl. Winter Wheat Flour ⅛ sacks		7.10
12 bbl. H. & E. Granulated Sugar 329, 335, 347, 351, 344, 347, 350, 331, 342, 355, 349, 333		6.15 per cwt.
5 sacks H. & E. Granulated Sugar	500#	6.10 per cwt.
9 cases Acme Peas	18 doz.	1.45
17 cases Acme Corn	34 doz.	1.70
6 boxes Dried Apples	150#	.08¾
2 bbl. 10# sacks Northern Salt		1.95
6 cases Algonac Tomatoes	12 doz.	1.37½

9.

1 cs. 25# Macaroni		$2.10
5# Cream of Tartar		.40
1 cs. Acme Peas	2 doz.	1.30
5 cases Hawthorne Pears	10 doz.	2.95
11 cases Hawthorne Peaches	22 doz.	2.50
12 cans Pimento		.14
15 cs. H.L. Shredded Pineapple		2.40
10 cases Tall Salmon	40 doz.	2.10
12 cases F. & B. Tomatoes	24 doz.	3.75
85 sacks Yellow Corn Meal	850#	.03
8 cases 25# Macaroni		2.10
5 cases 15# Spaghetti		2.10
7½ doz. pkgs. Seeded Raisins		1.40
125# Rice		.07
25 quarts Olive Oil		.75 gal.
18¾ gal. Sweet Pickles		1.06¼
6½ gal. Olives $\frac{70}{80}$		1.30
25 cases 4 oz. Grd. Pepper	125#	.30
6 cans Paprika		2.25 doz.
14 cases G. M. Soap		2.85

		PRICE
12 cases Soda Crackers	91¾#	$ .09
12 Cracker cans		.50
1 tub Peanut Butter	48½#	.12½
62½ bbl. ⅛ sacks Spring Wheat Flour		5.80
49¾ bbl. ⅛ sacks Winter Wheat Flour		6.25
16 cases #2 Algonac Lima Beans	32 doz.	1.40
14 cases #2 Algonac String Beans	28 doz.	1.37½

10.

10 doz. W/W 6 in. Plates	$ .82	doz.
13½ doz. W/W 4 in. Plates	.62½	doz.
5 doz. /S/ 2½ in. Butter Chips Tk.	.22	doz.
8¼ doz. #13 Cups	.87½	doz.
6 doz. W/W 4 in. Ice Creams	.44	doz.
15 doz. #64 /G/ Nappies	1.12½	doz.
8 doz. A.C. ½ Tk. 30's L/F Bowls	1.28	doz.
¾ doz. #211 Tea Pots	1.25	doz.
2½ doz. #0 /S/ Custard Pots	.98	doz.
17 doz. #1352 Tumblers	.45	doz.
12 doz. #1296 Finger Bowls	1.33⅓	doz.
1¾ doz. #3234 Oil Bottles	2.25	doz.
3½ doz. 12–199 Salts	1.50	doz.
3½ doz. 12–200 Peppers	1.50	doz.
2⅙ doz. 444 A Pitchers	5.00	doz.
⅓ doz. Punch Glasses	.75	each
12½ doz. 3404 Dessert Forks	2.35	doz.
12½ doz. 3405 Dessert Knives	2.30	doz.
16 doz. 3413 Tea Spoons	1.37½	doz.
8 doz. 3406 Dessert Spoons	2.25	doz.

Rule invoices and enter the following sales, assuming that you are the selling merchant.

11. To W. K. Sears,
Elgin, Illinois.
Sept. 10.

45 No. R 721 Body Brussels Rugs	6 × 9 ft. each	$13.25
32 No. R 731 Body Brussels Rugs	8 × 10 ft. each	19.75

		PRICE
36 No. R 741 Body Brussels Rugs	9 × 12 ft. each	$\$22.87\frac{1}{2}$
25 No. R 751 Body Brussels Rugs	11 × 13 ft. each	30.80

12. To C. B. Perkins,

Amherst, Minn.

Sept. 12.

20	H 20	Family Coffee Mills	each	\$1.35
16 sets	H 33	Sad Irons	set	.73
12 doz.	H 545	Flour Sifters	doz.	$1.83\frac{7}{8}$
25 doz.	H 463	Carpet Beaters	doz.	.89
8 doz.	H 6646	Galvanized Clothes Lines, 20 gauge wire,	doz.	$.92\frac{3}{8}$
10 doz.	H 6546	Galvanized Clothes Lines, 18 gauge wire,	doz.	$1.52\frac{1}{2}$

13. To Owen & Hendersen,

Cedar Rapids, Iowa.

Sept. 13.

35	F 384	Plush Library Chairs	each	\$14.75
28	F 376	Reed Rockers	each	3.10
18	F 394	Bedroom Rockers	each	4.85
26	F 465	Roman Chairs	each	7.88
19	F 414	Morris Chairs	each	18.95

CHAPTER XVI

COMMERCIAL DISCOUNTS

Cash Discount

144. Purpose of Cash Discount. In order to encourage prompt settlement of accounts, merchants frequently offer to deduct a certain per cent of the bill if it is paid within a fixed number of days. This deduction for prompt payment is called **cash discount.**

145. Terms. The terms state the discounts offered and the time when the bill is due. The terms are often expressed by an arrangement similar to the following:

2/10; 1/30; N/60.

The figures at the left of the line indicate the rate of discount offered; the figures at the right indicate the number of days within which payment must be made in order to obtain the discount. Thus, the terms stated above mean:

2 % discount from the face of the bill, if paid in 10 days;
1 % discount from the face of the bill, if paid in 30 days;
Net Amount (no discount is allowed, and bill is due) in 60 days.

146. Definitions. The amount of the purchase *before* subtracting the discounts is called the **list price.**

The amount of the purchase after subtracting the discounts is called the **net price.**

The **Rate of Discount** is stated as a per cent of the list price.

147. Cash Discount Illustrated. The terms of the invoice on page 176 were 2/10; N/30.

In order to obtain the discount, Scobey & Co.'s payment must be made on or before June 20, 1915.

The bill is due and payment is expected *in full* on July 10, 1915.

No discount is allowed if payment is made between June 20 and July 10, 1915.

148. Computing the Discount and the Net Price.

First Method:

$95.25	List price
.02	Rate of discount
$1.9050	Discount

$95.25	List price
1.91	Discount
$93.34	Net price

State a rule for finding discount and net price by this method. (Note that five mills or more in the discount are considered a cent. Thus, $1.905 is considered $1.91.)

Second Method:

100%	List price
2%	Rate of discount
98%	Per cent of bill to be paid

$95.25	List price
.98	
$93.345	Net price
or $93.34	

State a rule for computing the net price by this method.

149. The Advantages of Cash Discount. Cash Discounts may benefit both the purchaser and the seller. Merchants offer cash discounts because they encourage prompt payments, and thus decrease:

a. the loss from bad debts;
b. the cost of collecting accounts;
c. the amount of capital tied up in outstanding accounts.

It is usually good business for the purchaser to "take his discount" (pay the bill before it is due), even though the rate of discount may be small. An illustration will show this fact.

Suppose the terms of the sale are, Cash less 1%; net 30 days. If the purchaser pays cash, he receives 1% for the use of his money for one month. 1% a month is 12% a year; a high rate of interest.

The rates of cash discount are usually small, varying from 1% to 5%.

Small discounts are offered if the bill is due in a short time.

For example, 1/10; N/30.

Larger discounts are offered if the bill is due after a greater length of time.

For example, 5/30; N/4 months.

As a further means of insuring payment of invoices, merchants frequently charge interest on bills which are not paid when due.

Written Work

Find the list price, the discount, and the net price of each of the following purchases, and answer the following questions about each invoice.

a. What is the last day on which payment can be made and the discount secured?

b. When is the invoice due?

1. Invoice dated April 4. Terms 2/10; N/30.

7 cases Acme Peas	$1.40
3 cases Osea Soap	4.95
7 bbl. Northern Salt	2.10
15 bbl. $\frac{1}{8}$ sacks A. D. Flour	7.10

2. Invoice dated October 26. Terms 1/15; N/30.

12 bbl. H. E. Granulated Sugar, 329, 335, 347, 351, 344, 347, 350, 331, 342, 355, 349, 333 #	$6.15
5 sacks H. E. Granulated Sugar, 500 #	6.10

3. Invoice dated January 5. Terms 2/10; 1/20; N/30.

16 bolts White Dress Linen, 10, 12, 11^1, 12^2, 10^3, 11^3, 10^3, 12, 10^1, 11^2, 12, 11^1, 10^2, 11^3, 12^1, 10^2	yd.	$.44$\frac{1}{4}$
12 bolts Persian Lawn, 24^3, 25^1, 23^2, 25, 24^2, 24^3, 25^2, 24^1, 23^3, 24^3, 25^2, 25	yd.	.17$\frac{3}{4}$
48 bolts French Nainsook, 12-yard pieces	yd.	.18$\frac{1}{4}$
36 bolts Mercerized Lingerie Batiste, 24-yard pieces	yd.	.21
24 bolts Imported Linen Lawn, 10-yard pieces	yd.	.43$\frac{3}{4}$

NOTE. There will be two possible net prices for this invoice.

4. Complete the following table.

Combine the dollars in the column at the left, with the cents in the row near the top, to form the list price. Deduct the discount shown above the cents, and enter the net price in the table. The two results entered in the table show the method. Thus,

a. $26.10 the list price
.02 the discount
$ .52 the discount at 2 %

$26.10
.52
$25.58 Net price

b. $641.67
.025
$ 16.04 the discount at 2½ %

$641.67
16.04
$625.63 Net price

		Less 2 %		Less 5 %		Less 3 %		Less 2½ %	
			10		45		75		67
$26	00	$25	58						
14	00								
73	00								
19	00								
261	00								
112	00								
317	00								
216	00								
641	00							$625	63

In this exercise and in many which follow, it is not expected that each student will complete the entire table. The work may profitably be distributed among the students.

TRADE DISCOUNT

150. Purpose of Trade Discount. In some lines of business merchants sell both at wholesale and retail. They advertise their goods at a certain price, but when they sell to dealers, they frequently deduct a part of this price. The amount deducted is called a **Trade Discount.**

Trade discounts are most common in businesses which issue catalogues. When the catalogue is sent to a dealer, a discount sheet is inclosed.

Specimen Discount Sheet

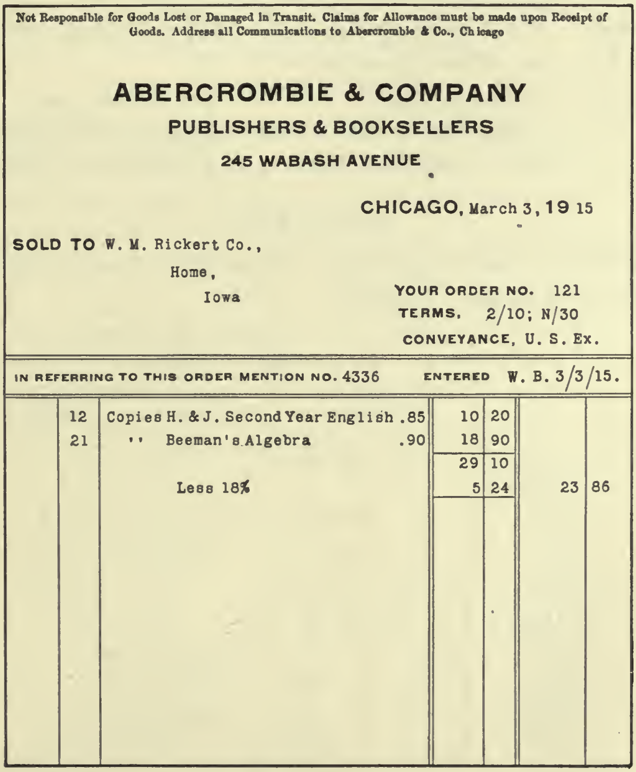
DISCOUNT SHEET

The following discounts are offered on articles listed in catalogue No. A 23.

Pages 1 to 15	20 %
Pages 16 to 39	28 %
Pages 39 to 67	40 %
Pages 68 to 90	Net
Pages 91 to 136	$14\frac{2}{7}$ %

Do not show this discount sheet to your customers.

NOTE. "Pages 68 to 90 Net" means, no trade discount is offered for goods listed on these pages.



Not Responsible for Goods Lost or Damaged in Transit. Claims for Allowance must be made upon Receipt of Goods. Address all Communications to Abercrombie & Co., Chicago

ABERCROMBIE & COMPANY
PUBLISHERS & BOOKSELLERS
245 WABASH AVENUE

CHICAGO, March 3, 19 15

SOLD TO W. M. Rickert Co.,
Home,
Iowa

YOUR ORDER NO. 121
TERMS. 2/10; N/30
CONVEYANCE, U. S. Ex.

IN REFERRING TO THIS ORDER MENTION NO. 4336 ENTERED W. B. 3/3/15.

	12	Copies H. & J. Second Year English .85	10	20		
	21	'' Beeman's Algebra .90	18	90		
			29	10		
		Less 18%	5	24	23	86

Class Discussion

1. Why do merchants offer trade discounts?

2. What is the catalogue price of the H. & J. Second Year English per copy?

3. How much does each copy of the English book actually cost the retail dealer?

4. If Mr. Rickert sells this book for the catalogue price, 85 cents, how much profit does he make on each book?

5. Trade discounts are usually larger than cash discounts. Why do you think this is the case?

151. Computing Trade Discounts. Trade discounts are computed in the same manner as cash discounts.

Example. What will goods cost a dealer if they are sold to him for $46, less a trade discount of 25%?

SOLUTION.	$46.00	List price	$46.00	List price
	.25	Rate of discount	11.50	Trade discount
	$11.50	Trade discount	$34.50	Price to dealer

Written Work

Complete the following table. Enter the price to dealer after deducting the Trade Discount.

TRADE DISCOUNT	CATALOGUE PRICE					
	$27	85	$126	39	$96	45
14²⁄₇%						
18%						
20%						
22%						
25%						
15%						
17%						
28%						
16⅔%						
10%						

152. Series of Two Discounts. Invoices are often subject to a cash discount in addition to the trade discount. Two or more discounts are called a "Discount Series."

Example. What is the smallest amount of money that will pay a bill of $236, subject to a trade discount of 20 % and a cash discount of 2 % ?

SOLUTION.

$236.00	List price	$188.80	Trade price
.20	Rate of trade discount	.02	Rate of cash discount
$ 47.20	Trade discount	$3.7760	Cash discount
$236.00	List price	$188.80	Trade price
47.20	Trade discount	3.78	Cash discount
$188.80	Price to the trade	$185.02	Net price

The cash discount is computed on the trade price, *not* on the list price. Therefore, 20 % trade discount and 2 % cash discount are not the same as a single discount of 22 %.

Written Work

Complete the following table. From each list price deduct the trade discount indicated above the list price; and from the trade price thus found, deduct the cash discount shown at the left.

CASH DISCOUNT	FACTS TO BE FOUND	TRADE DISCOUNT, 28 %		TRADE DISCOUNT, 25 %		TRADE DISCOUNT, 33⅓ %	
2 %	List Price	$36	45	$68	40	$239	64
	Trade Discount						
	Trade Price						
	Cash Discount						
	Net Price						
3 %	List Price	205	08	333	95	47	30
	Trade Discount						
	Trade Price						
	Cash Discount						
	Net Price						
5 %	List Price	35	00	713	20	609	49
	Trade Discount						
	Trade Price						
	Cash Discount						
	Net Price						

QUANTITY DISCOUNTS

153. Purpose of Quantity Discounts. In some lines of business, particularly in manufacturing, it is customary to give a larger rate of discount on a large order than on a small one.

A mail order firm advertises:

1 % discount on an invoice of $10; 2 % on $20; 5 % on $35.
What is the purpose of such quantity discounts?

A printer advertised the following rates:

$10 per thousand for the first thousand copies. On an order of more than 1000 copies, a discount of 15 %. This discount is offered because the work of typesetting, making up the forms, and making ready the press must all be done, although only one copy is to be printed. The cost of printing 2000 copies, therefore, is not twice as much as for 1000 copies.

Written Work

1. The following prices were quoted by a manufacturer of lockers: No. 6094. Each $6.30.

Discounts:

Orders of from 50 to 100 — Less 8 %.
Orders of from 101 to 300 — Less 10 %.
Orders of from 301 to 500 — Less $12\frac{1}{2}$ %.

Find the total cost and the cost per locker of an order of 75 lockers; 140; 475.

2. A paper manufacturer quoted the following prices:

No. 020 note paper, cut to size 8 by 10, $.55 per ream.

An extra charge of 3 cents per ream will be made for wrapping in packages of 500 sheets. (500 sheets considered 1 ream.)

On orders of 100 reams or more a discount of 4 % will be allowed.

Terms, 1/10; N/30.

Find the cost and state what discounts were allowed:

a. An order of 50 reams, unwrapped, payment made 20 days from date of sale.

b. An order of 150 reams, unwrapped, payment made 15 days from date of sale.

c. An order of 150 reams, wrapped, payment made 18 days from date of sale.

d. An order of 240 reams, wrapped, payment made 7 days from date of sale.

e. An order of 60 reams, wrapped, payment made 8 days from date of sale.

FLUCTUATION DISCOUNT

154. Purpose of Fluctuation Discounts.

Another important discount is that offered to change the catalogue price of an article to meet the changes, or fluctuations, in the market price. Fluctuation discounts are used by establishments which sell goods made from raw material the price of which frequently changes.

For example, suppose an article made from steel (the market price of which varies at frequent intervals) is quoted in the catalogue at $8.

When the market price of steel drops and the article can be sold for $7, a discount of 12½ % can be offered.

If raw steel should drop in price so that the article could be sold for $6, a discount of 25 % could be offered.

The fluctuation discount sheet is a means of economy to the manufacturer, because he can issue new discount sheets much more cheaply than complete catalogues.

155. Computing Fluctuation Discounts.

Example. What discount will change a catalogue price of $3 to a market price of $2.50?

SOLUTION. $3.00 Catalogue price
2.50 Price at which article is to be sold
$ .50 Amount to be deducted by discount
$.50 ÷ $3.00 = 16⅔ %, rate of discount to be offered

State a rule for finding the rate of discount by this method.

When the quotient obtained by the division involves a fractional per cent, the next lower whole number is sometimes taken as the per cent. Thus, a quotient of 15⅐ % may be regarded as 15 % ; a discount of 18.27 % may be regarded as 18 %.

Example. An article is listed at $24.50. The market conditions are such that it should sell for about $21.30. What rate of discount should be offered?

SOLUTION.		
	$24.50	List price
	21.30	Market price
	$ 3.20	Amount to be deducted by discount

$3.20 ÷ $24.50 = 13.06 %.
13 % would therefore be offered.
This discount would make the net price $.   ?

Written Work

Complete the following table, showing what rate of discount would be offered to reduce the list prices at the left to the market prices shown at the top of the table.

Express results to the nearest per cent.

LIST PRICES	MARKET PRICES				
	$8 50	$7 75	$8 00	$7 60	$6 00
$10 00					
12 00					
11 50					
9 80					
10 50					

DISCOUNT SERIES

156. Trade and Fluctuation Discounts Combined.

Some business concerns offer an unchanging trade discount and a changing fluctuation discount. This slightly increases the task of determining what fluctuation discount to offer.

Example. An article is listed at $12 less a trade discount of 25%. The market value drops so that the real net price should be $6. What additional discount must be offered?

SOLUTION.
$12 × .25 = $3, Trade discount.
$12 − $3 = $9, Price less trade discount.
$9 − $6 = $3, amount to be deducted by 2d discount.
$3 ÷ $9 = 33⅓ %, rate of fluctuation discount.

Written Work

Complete the following table.

The columns at the left show the list price and the trade discount offered. Find what fluctuation discount must be offered to reduce the price to market values.

Approximate results, correct to the nearest hundredth of a per cent.

List Prices		Trade Discount	Market Values							
			$24	00	$23	50	$22	00	$20	00
$28	00	10 %								
33	00	15								
30	50	12								
40	00	28								

Written Review

1. An implement dealer purchased the following invoice:

Dated April 17.

4 No. 364 Plows, $38.75, less 20 %.
7 Self Dump Hay Rakes, $ 20.50, less 18 %.
7 No. 264 Hay Stackers, $41.50, less 15 %.

Terms, 2 % for Cash in 20 days. Net 90 days.

On May 2, he paid the bill with a check for $——.

2. Which of the following prices is better for the purchaser:
$45, less 25 %, 18 %, and 2 %;
$60, less 28 %, 25 %, and 1 %?

3. A merchant's discounts were 25 % and 15 %. A clerk sold an invoice of $ 72 and gave a single discount of 40 %. How much did his error cost his employer?

4. A merchant lists a desk at $45 less 20 %. A competitor sells a similar desk for $48 less $33\frac{1}{3}$ %. In order to exactly meet his competitor's price, the first merchant decides to give an additional discount of — %.

5. Hewett paid an invoice in time to secure a discount of 3 %. If the check sent was for $208.55, what was the list price of the invoice?

6. Graff Brothers sent a check to a wholesale house to pay an invoice. The check was for $ 801.90. What was the list price of the invoice if the discounts taken were 10 % and 1 % ?

157. A Short Method of Finding a Single Discount Equivalent to a Discount Series. A single discount equivalent to two discounts may be found as follows:

From the sum of the two discounts subtract their product.

The rule may be easier to remember if stated thus:

Add the two discounts; multiply the two discounts; subtract the second result from the first.

Example. What single discount is equivalent to a trade discount of 20 % and a cash discount of 2 % ?

SOLUTION.

$$\begin{aligned} .20 + .02 &= .22 \\ .20 \times .02 &= \underline{.004} \\ \text{Single discount} &= .216 \text{ or } 21.6\,\% \end{aligned}$$

The buyers in some large business houses prepare an elaborate table similar to the one which follows. When manufacturers quote prices subject to a discount series, the buyers can tell by a glance at the table what single discount the series equals.

Written Work

Complete the following table, using the method just explained to find single discounts equivalent to the discount series.

TRADE DISCOUNT	CASH DISCOUNT, 3 %	CASH DISCOUNT, 6 %	CASH DISCOUNT, 5 %	CASH DISCOUNT, 7 %	CASH DISCOUNT, 10 %
18 %					
15					
20					
25					
$33\frac{1}{3}$					

With the aid of the table, find the net price of the following:

a. An invoice of $215, less 18 % and 5 %.

b. An invoice of $464.20 less 18 % and 10 %.

c. An invoice of $12.40 less 20 % and 6 %.

d. An invoice of $23.87 less $33\frac{1}{3}$ % and 3 %.

158. Series of Three Discounts.

It sometimes happens that a bill is subject to several discounts. Explain how this might be the case.

To find the net price of a bill subject to three discounts, find the discount equivalent to two of the discounts stated, then find the discount equivalent to this result and the third discount.

Example. What discount is equivalent to three discounts 25 %, 20 %, 10 %?

Solution. First find the discount equivalent to discounts of 25 % and 20 %; the result is 40 %.

Find the discount equivalent to discounts of 40 % and 10 %; the result is 46 %. Therefore a discount of 46 % is equivalent to discounts of 25 %, 20 %, 10 %.

Written Work

Find the net cost of the following bills:

1. $45 less discounts of 20 %, 15 %, and 10 %.
2. $28 less discounts of 25 %, 10 %, and 5 %.
3. $70 less discounts of 30 %, 20 %, and 2 %.

Find a single discount equivalent to each of the following series:

4. 20 %, $12\frac{1}{2}$ %, and 8 %.
5. 15 %, 10 %, and 4 %.
6. 28 %, 14 %, and 2 %.
7. 25 %, 20 %, and 10 %.
8. 40 %, 20 %, and 10 %.
9. $33\frac{1}{3}$ %, 20 %, and $12\frac{1}{2}$ %.
10. $37\frac{1}{2}$ %, 10 %, and 20 %.

CHAPTER XVII

RECORDING PURCHASES AND SALES

159. The Purchases Book. Merchants usually keep a record of purchases and sales. There are several different kinds of books used for this purpose. The following illustration shows a common form of the Purchases Book.

PURCHASES BOOK

Date of Invoice		From Whom Purchased	Amount		Terms	Discount term Expires		Cash Discount		Due Date		When & How Paid		
1915 March	3	Abercrombie & Co	23	86	2/10 : N/30	Mar	13		48	Apr	2	Mar	12	By Check $23.38

The entries in the purchases book are made from the invoice received at the time of the purchase. The model shows proper record of the purchase made from Abercrombie & Co., as shown by the invoice on page 187.

Notice that $23.86 is the amount of the invoice after deducting the trade discount of 18%. The cash discount is not deducted until the bill is paid.

The terms are taken from the invoice.

The discount term expires March 13. It is important to have this date recorded in the purchases book, as it is the last date on which payment may be made and the discount secured.

April 2 is the day when the bill is due and payment is expected. No discount is allowed when payment is made between March 13 and April 2.

160. How to find the Date when an Invoice is Due. If the terms are stated in days, count the actual number of days. Thus, 30 days from March 3 is April 2.

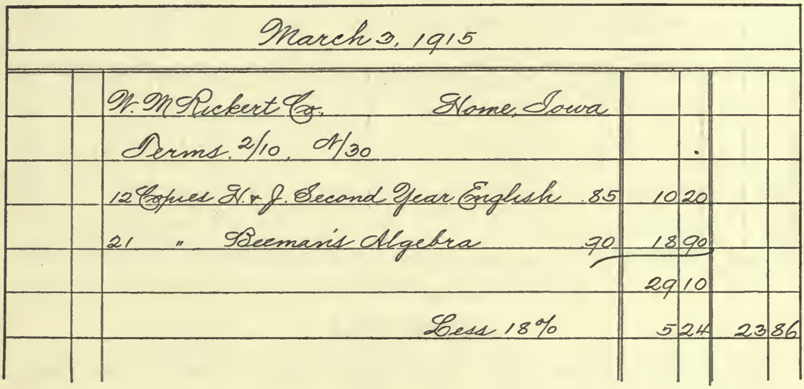
If the terms are stated in months, calendar months are counted. Thus, if the terms had been 2/10; N/1 month, the invoice would have been due on April 3.

No entry is made in the "When and How Paid" column until the invoice is paid.

It is not necessary to enter the items in the purchases book, because the invoice can be kept on file to supply this information.

161. The Sales Book. The following illustration shows a page from a commonly used form of sales book.

The entries were made from the invoice on page 187.



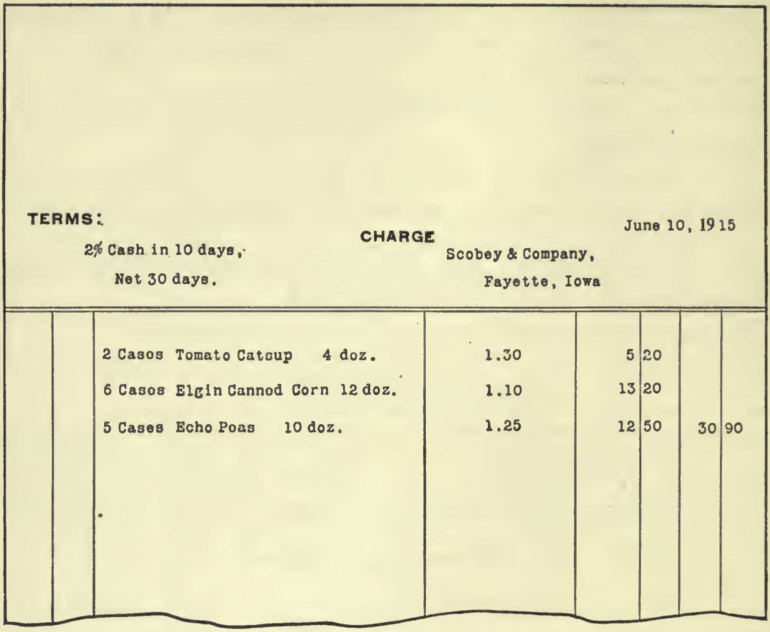
March 3, 1915

		W. M. Rickert Co. Home, Iowa				
		Terms 2/10, N/30				
		12 Copies H. & J. Second Year English .85	10	20		
		21 " Beeman's Algebra .90	18	90		
			29	10		
		Less 18%	5	24	23	86

162. Loose-leaf Sales Book. Many merchants record their sales in a loose-leaf sales book. At the time the invoice is made, a carbon copy is also made. This requires very little extra labor, and the carbon copies, called "Charge Sheets," having holes punched at the side, can be bound together in a binder.

The loose-leaf sales book has several advantages. Both the invoice and the charge sheet, which forms the sales book, can be made on the typewriter at the same time; and fraud is prevented because the invoices are numbered, and a clerk cannot sell goods and make an invoice without also making a charge slip. He cannot keep the money and destroy the charge slip because one of the numbered sheets would be missing, and the fraud would be apparent.

The following illustration shows the charge sheet made as a carbon copy of the invoice shown on page 187.



TERMS:

2% Cash in 10 days,

Net 30 days.

CHARGE Scobey & Company,

Fayette, Iowa

June 10, 1915

2 Cases Tomato Catsup 4 doz.	1.30	5	20		
6 Cases Elgin Canned Corn 12 doz.	1.10	13	20		
5 Cases Echo Peas 10 doz.	1.25	12	50	30	90

Written Work

Rule a Purchases Book and record the following purchases, making proper extensions.

1. From Eaton & Dunham, 213 Main Street, Indianapolis, Indiana.

Date, June 5, 1915.
Terms, 1/15; N/60.

6 doz. Silk Four-in-hand Ties	each	$ .37½
3 doz. White Lawn Ties	gross	2.94
38 doz. Assorted Style Amoryth Linen Collars	doz.	1.92
18 doz. Policeman's Suspenders	doz.	4.37½
9 doz. Khaki Overalls	doz.	8.97
6 doz. Boys' Flannelette Waists (ages 4 to 13 yr.)	doz.	2.16¼
10 doz. Men's Flannel Work Shirts (sizes 14 to 18)	doz.	8.76½
2¼ doz. Men's Negligee Shirts	doz.	16.40

2. From J. B. Clark, Aurora, Illinois.

Date, June 8, 1915.

Terms 2/20; N/3 months.

7 doz.	Assorted Sizes Men's Silk Half Hose	doz.	$4.29
19½ doz.	Mercerized Lisle Hose	doz.	4.78
3 doz.	Men's Black Overgaiters	doz.	6.93
18	Women's Taffeta Silk Waists (sizes 34 to 42)	each	2.17
24	Navy Blue Silk Chiffon Waists	each	2.98
5 doz.	Embroidered Lace Coat Sets	doz.	6.93¾
2 doz.	Amoskeag Gingham Aprons	doz.	4.69½
2¼ doz.	Lace Jabots	doz.	4.25

3. From Bishop & McGee, Independence, Iowa.

Date, June 10, 1915.

Terms, 1/5; N/20.

10 doz.	Defiance Food Choppers	each	$2.85
9 doz.	Climax Food Choppers	each	.72
14 doz.	No. 7 size Skillets	each	.22½
30 sets	Enamel-lined Iron Kettles, each set containing		
	3 2 quarts at .21		
	4 4 quarts at .29¾		
	6 6 quarts at .34⅔		
	4 8 quarts at .39¾		
	2 10 quarts at .46⅔		
2¾ doz.	H 921 Waffle Irons	doz.	$7.84⅞
6½ doz.	H 922 Waffle Irons	each	.93
25	H 464 Soapstone Griddles (round)	each	.55
12 doz.	H 465 Soapstone Griddles (oval)	each	.65⅜

4. From A. D. McHaughton, Fairchild, Missouri.

Date, July 14, 1915.

Terms, 3/30; N/90.

5 doz.	H 731 Wire Waste Baskets	doz.	$ .89
15 doz.	H 732 Wire Letter Baskets	doz.	1.23
28	H 881 Spring Seats	each	.88

15 sets	H 393 Cobblers' Outfits	set	$ .96
19 doz.	H 242 Curry Combs	doz.	.67
14	F 272 Upholstered Rockers	each	8.75
25	F 279 Oak Rockers	each	12.65
20	F 212 Turkish Rockers	each	12.75

Rule a Sales Book, and record the following sales, making proper extension.

5. To J. D. Preston, Monmouth, Illinois.
July 7, 1915.
Terms, 1/10; N/40.

47	No. L 601 Royal Worsted Wilton Rugs	8 × 10 ft.	each	$ 29.25
52	No. L 602 Royal Worsted Wilton Rugs	6 × 7 ft.	each	19.75
38	No. L 603 Royal Worsted Wilton Rugs	9 × 12 ft.	each	32.35
65	No. L 661 Worsted French Wiltons	9 × 12 ft.	each	44.60
50	No. L 662 Plain Color Wiltons	8 × 10 ft.	each	31.75

6. To R. J. Noble, Dubuque, Iowa.
July 8, 1915.
Terms, 1/15; N/60.

25 doz.	Half-bleached Cotton Towels	24 × 54	doz.	$ .98
16 doz.	Linen Monogram Towels	22 × 39	doz.	4.95
20 doz.	Bleached Turkish Bath Towels	23 × 52	doz.	2.37½
14 doz.	No. T 291 Cotton Face Cloths	10 × 13	doz.	.42½

7. To Oscar Hamilton, Reynolds, North Dakota.
July 9, 1915.
Terms, 2/5; N/2 months.

15 doz.	Palmetto Fiber Scrub Brushes	doz.	$ .86
22 doz.	H 221 Kitchen Spoons, 10 inch	doz.	.23
19 doz.	H 241 Kitchen Spoons, 12 inch	doz.	.27
26 doz.	H 251 Kitchen Spoons, 14 inch	doz.	.33
38 doz.	H 333 Kitchen Forks, 13 inch	doz.	.19
36 doz.	H 334 Kitchen Forks, 15 inch	gross	3.20
12 doz.	H 341 Perforated Steel Spoons	doz.	.84⅞
8 doz.	H 691 Kitchen Sets	doz.	8.75
16 doz.	H 692 Kitchen Sets	doz.	3.95

Marking Goods

163. Method of Marking Goods. When stock is placed on the shelves in the salesroom, the cost of each article should be marked either on the goods, on the package which contains them, on tags attached to the goods, or on card lists placed near the goods.

164. Advantage of Marking. In some lines of business it is necessary to have the selling price, or both the cost and the selling price, marked on the goods. When the stock becomes low and the buyer wishes to purchase a new supply, he can compare the price paid for the goods on the shelves, with quotations of prices made to him by the salesmen from the wholesale houses. It would also be a convenience to know the cost of an article if it proved to be a slow seller, and the manager determined to sell at a reduced price to unload the stock. One of the chief advantages of cost marking is in taking inventory of stock and finding its value at cost prices. In marking goods, the cost price is taken from the invoice.

165. "Blind Price Lists." It would be unwise to mark the cost in figures, as this would disclose the cost and the profit to purchasers. It is, therefore, customary for merchants to adopt a set of symbols, called a cipher, or blind price list. Any symbols may be chosen, but they will be more easily written if letters are used, and these letters will be more easily remembered if they form a word or phrase. The word or phrase selected must not contain the same letter twice. Otherwise the same letter will represent two different numbers.

The following will illustrate:

a d m o n i s h e r	mah
1 2 3 4 5 6 7 8 9 0	318
F i t z a u b r e y	izy
1 2 3 4 5 6 7 8 9 0	240

166. The Repeater. To further conceal the cost a "repeater" should be adopted. When the same figure is repeated, as in $1.55, the repeater sign is used for the repeated figure. "x" is frequently used as a repeater, but because it is so commonly used, some other repeater would perhaps be preferable. Words or phrases with eleven letters are frequently chosen as keys, one of the letters being used as a repeater.

Using "t" as the repeater, and "admonisher" as the key,

$3.88 would be written *mht.*

Written Work

Using "blacksmith" as the key word, and "d" as the repeater, indicate the following costs:

1. $9.82. **2.** $ .09. **3.** $ .25. **4.** $ 1.64.
5. $1.00. **6.** $6.20. **7.** $ .27. **8.** $ 7.47.
9. $2.45. **10.** $ .55. **11.** $1.33. **12.** $ .39.
13. $8.23. **14.** $5.00. **15.** $1.17. **16.** $12.47.
17. $6.62. **18.** $2.24. **19.** $ .36. **20.** $ 1.44.

Many articles are bought by the dozen and sold by the piece. In marking goods bought in this way it is necessary to divide the cost by 12. This division will be facilitated if the decimal equivalents of the 12ths, from $\frac{1}{12}$ to $\frac{11}{12}$ inclusive, are memorized.

Make such a table of equivalents and memorize it. Thus,

$$\tfrac{1}{12} = .08\tfrac{1}{3}\,; \qquad \tfrac{7}{12} = .58\tfrac{1}{3}.$$

Example. What is the cost of one hat at the rate of $29 a dozen?

SOLUTION. $\frac{1}{12}$ of \$29 $= \$2\frac{5}{12} = \$2.41\frac{2}{3}$.

Oral Work

What is the cost per article when the cost per dozen is:

1. $27? **2.** $34? **3.** $64? **4.** $11? **5.** $ 3.40?
6. $14.30? **7.** $7.50? **8.** $19.50? **9.** $46.60? **10.** $37.40?

167. Showing Cost and Selling Price. When both the cost price and the selling price are shown, it is customary to write the cost price above and the selling price below a line, thus:

Cost of an article, $6.25.
Selling price, $8.00.
Key word, Fitzaubrey.
Price mark, $\frac{\text{uia}}{8.00}$.

Key words or phrases are sometimes used to mark the selling price as well as the cost, thus:

Cost key, "purchased it"
123456789 0 "t" is the repeater.

Selling key, "studying her"
12345678 90 "r" is the repeater.

Cost, $3.14.
Selling price, $4.25.
Mark, $\frac{\text{rpc}}{\text{dty}}$.

Written Work

Using the above keys, show markings for the following:

	Cost	Selling Price
1.	$1.15	$1.50
2.	$ .65	$ .90
3.	$1.30	$1.75
4.	$2.18	$2.50
5.	$1.25	$2.60

What per cent of profit would be realized from selling goods marked as follows, using "purchased it" as the cost key and "studying her" as the selling key?

6. $\frac{\text{phi}}{1.75}$. **7.** $\frac{\text{ah}}{\text{ge}}$. **8.** $\frac{\text{dh}}{\text{sty}}$. **9.** $\frac{\text{at}}{\text{ny}}$.

Mark the cost of each of the following articles, using "Now Pay Quick" as the purchase key.

10. $3.00 per dozen.
11. $185.00 per hundred.
12. $11.52 per gross.
13. $8.64 per case of 4 doz.

Mark both the cost and the selling price of the following. Devise a cost key of your own, and mark the selling price by using the key "importance."

14. Cost $ 2.60, marked to gain 22 %.

15. Cost $ 1.75, marked to gain 18 %.

16. Cost $ 25.00, marked to gain 20 %.

The following is a list of key words and phrases:

Buy for Cash
The Big Four
Black Horse
Buckingham
Authorizes
United Cars
Our Last Key
Republican
Charleston
Bridgeport
Cumberland
Dozen Black
Equinoctial
No Suit Case
Frank Smith
Big Factory
Don't Be Lazy
Market Sign
Now Be Quick
You Mark His
Now Be Sharp
He Saw It Run
Hard Moneys
What Prices

168. Adding the Buying Expenses to the Cost of the Goods. Since the buying expenses are considered a part of the cost of the goods purchased, many merchants add a portion of the buying expenses in marking the cost of each article. Records of total purchases and buying expenses per year are kept for several years, and the average per cent of buying expenses is determined, as shown by the following illustration: To determine a rate per cent of buying expenses for purchases made in 1915.

1912, Buying Expenses,	$ 785.75	Purchases,	$ 16,240.00
1913, Buying Expenses,	835.50	Purchases,	19,360.00
1914, Buying Expenses,	923.90	Purchases,	21,365.00
Total Buying Expenses,	$ 2545.15	Purchases,	$ 56,965.00

$\$2545.15 \div \$56,965.00 = 4.46_{+}\,\%$, the per cent of buying expenses.

In marking costs, 4.4 % of the cost of each article should be added as buying expenses.

NOTE. Probably 5 % might be used for convenience.

Written Work

1. Compute the per cent of buying expenses to be added to the wholesale cost of goods purchased in 1916. Carry the result to the nearest whole per cent.

YEAR	PURCHASES	BUYING EXPENSES
1913	$10,246.80	$534.50
1914	12,726.95	592.90
1915	14,825.75	615.45

2. Find the marked cost of each of the following articles after adding the buying expenses. Use the per cent of buying expenses found in Problem 1.

a.	Wholesale cost,	$16.20.
b.	Wholesale cost,	15.75.
c.	Wholesale cost,	2.35.

3. At what price should each of these articles be sold to gain 23% on the total cost including buying expenses?

4. The purchases made by a store in 1915 were $3287.20; the buying expenses for the same year were $327.90. On the basis of these figures, what per cent should be added for buying expenses on purchases made in 1916? Approximate result to the nearest per cent.

5. What objection do you see to determining a per cent of buying expenses from the data of only one preceding year?

6. Find the marked cost of each of the following purchases, after adding the per cent of buying expenses. Use the per cent of buying expenses found in Problem 4.

a.	Wholesale cost,	$ 2.45.
b.	Wholesale cost,	1.60.
c.	Wholesale cost,	34.50.

7. At what selling price should each of the articles in Problem 6 be marked to gain $16\frac{2}{3}$% of the total cost?

CHAPTER XVIII

PAYING FOR GOODS

169. Making Change. The payment of debts between persons in the same community is usually made with either cash or checks. When cash is the medium of payment, it is often necessary to "make change."

Speed and accuracy in making change are very desirable. The following method is generally used by experienced tellers and cashiers. Beginning with the amount of the purchase, take from the cash drawer enough small coins to bring the total to even dollars, using as few coins as possible, then take out dollars or larger denominations until the total equals the payment made.

Example. A five-dollar bill was given in payment for a purchase of $.39. How should change be made?

SOLUTION. Take 1 penny,
1 dime,
1 fifty-cent piece,
4 dollars.

As a check on the accuracy of the change, say as you give the customer the money: "39 cents, 40, 50, $1, $5.00."

Oral Work

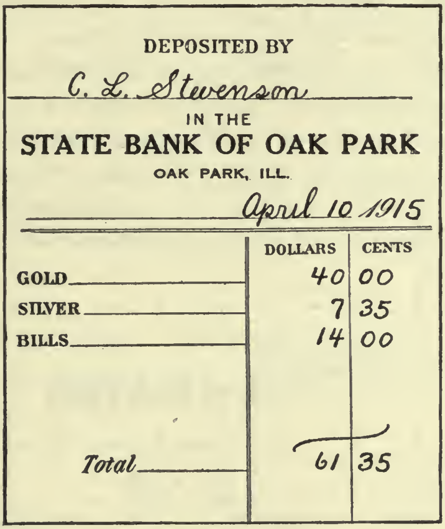
Following the method above, state what coins and bills should be given to make change for the following purchases, using the largest denominations possible:

	PURCHASE	PAYMENT		PURCHASE	PAYMENT
1.	$ .07	$ .50	**2.**	$ .21	$ .50
3.	$ .56	$ 1.00	**4.**	$ .63	$ 2.00
5.	$1.36	$ 5.00	**6.**	$2.79	$ 5.00
7.	$3.66	$ 5.00	**8.**	$4.24	$10.00
9.	$5.70	$10.00	**10.**	$6.32	$20.00
11.	$ .98	$ 5.00	**12.**	$2.77	$10.00

	Purchase	Payment		Purchase	Payment
13.	$ 3.46	$20.00	**14.**	$11.87	$15.00
15.	$14.42	$20.00	**16.**	$ 7.37	$ 8.00
17.	$11.13	$20.00	**18.**	$ 1.13	$20.00

170. Payments by Check. The payment of all bills by the actual transfer of money would be so inconvenient that the giving of checks has been substituted, and it is said that about 90% of all bills are now paid by checks.

171. Deposits and Withdrawals. Business men keep the greater part of their cash funds on deposit in banks or trust companies. When money is deposited, a deposit slip similar to the illustration below is filled out by the depositor, showing his name, the date, the item, and the amount of the deposit.



DEPOSITED BY

C. L. Stevenson

IN THE

STATE BANK OF OAK PARK

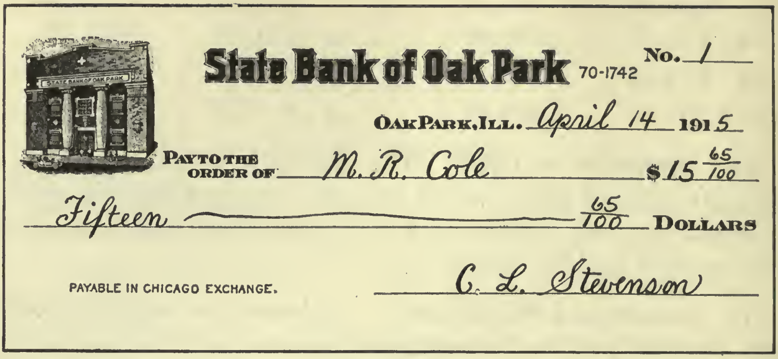
OAK PARK, ILL.

April 10 1915

	DOLLARS	CENTS
GOLD	40	00
SILVER	7	35
BILLS	14	00
Total	61	35

A Deposit Slip

The depositor, wishing to pay a bill, draws a check ordering the bank to pay from the funds on deposit the sum of money stated on the check to the person named thereon. If the depositor wishes to draw cash from his account, he may make the check payable to "Self" or to "Cash."



State Bank of Oak Park 70-1742 No. 1

Oak Park, Ill. April 14 1915

Pay to the order of M. R. Cole $15 65/100

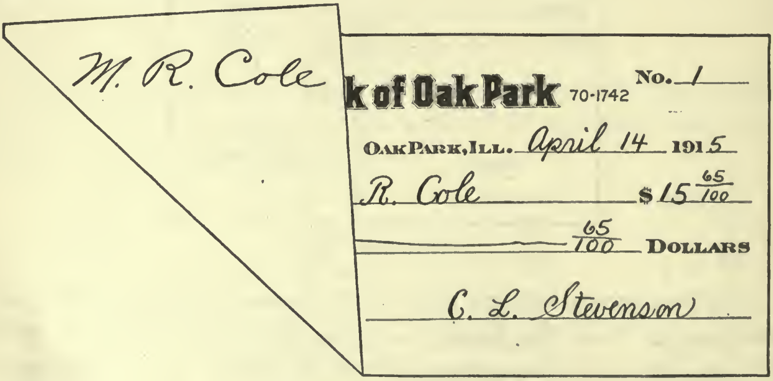
Fifteen 65/100 Dollars

PAYABLE IN CHICAGO EXCHANGE. C. L. Stevenson

A Check

C. L. Stevenson, the *drawer* of this check, is paying M. R. Cole, the *payee*, $15.65.

Depositors are credited by the bank with their deposits, and are charged with the checks drawn by them. Checks received from other people may be cashed at the bank, or they may be deposited the same as cash. All checks cashed or deposited must be indorsed; that is, they must be signed on the *back* by the person cashing or depositing them. Checks must also be indorsed when they are transferred to another person before being cashed at the bank. Indorsement should be made on the back of the left end of the check.



M. R. Cole

k of Oak Park 70-1742 No. 1

Oak Park, Ill. April 14 1915

R. Cole $15 65/100

65/100 Dollars

C. L. Stevenson

A Check Indorsed

M. R. Cole's indorsement before cashing the check.

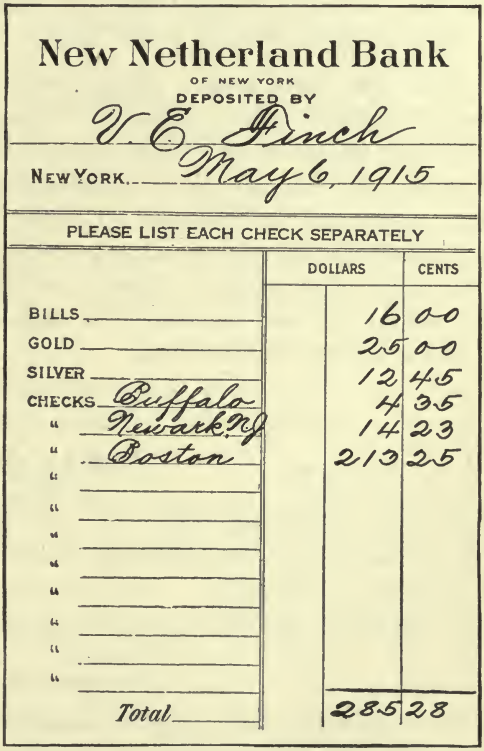
INDORSEMENTS

172. Blank Indorsement. This form of indorsement is illustrated on the back of the check on page 208. It makes the check payable to bearer.

173. Indorsement in Full. This form is illustrated below.

Pay to the order of
J. B. Banning
M. R. Cole

Checks indorsed in full must be indorsed by the new owner before being cashed; if lost, there is less danger of their being cashed by a dishonest finder than if they were indorsed in blank.



New Netherland Bank
OF NEW YORK
DEPOSITED BY
V. E. Finch

NEW YORK May 6, 1915

PLEASE LIST EACH CHECK SEPARATELY

	DOLLARS	CENTS
BILLS	16	00
GOLD	25	00
SILVER	12	45
CHECKS Buffalo	4	35
" Newark N.J.	14	23
" Boston	213	25
"		
"		
"		
"		
"		
"		
"		
"		
Total	285	28

DEPOSIT SLIP SHOWING CASH AND CHECK DEPOSIT

If a check, note, or draft is not paid by the party primarily liable each indorser in turn becomes responsible for its payment. An indorser may limit his responsibility by writing the words "without recourse" above his signature. When this is done, the indorsement is said to be *qualified*.

Written Work

1. You are opening an account by depositing to-day in some bank of your city, gold, $5; silver, $2.25; bills, $7; and the three following

checks: 1st, drawn by Henry Bailey on the Merchants' National Bank of Fargo, North Dakota, amount $26.30; 2d, drawn by W. A. Owen on the Corn Exchange National Bank of Chicago, amount $8.35; 3d, drawn by Frank Mitchell on Carter's Bank, Eldora, Missouri, amount $12.40.

Rule a deposit slip and enter this deposit properly.

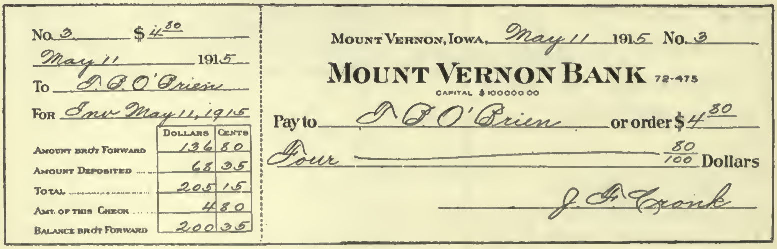
2. On a blank sheet of paper draw a check on the bank in which you have just made this deposit, paying F. G. Benton $23.25.

What is your balance in the bank after drawing this check?

3. Seven days ago you bought goods from F. G. Peterson, amounting to $28.50. The terms of the invoice were 2/10; N/30. To-day you pay the bill by check. Draw the check.

What is your balance in the bank?

174. The Check Book. Blank checks are furnished by the bank, bound together in a check book. Checks are usually attached to stubs, on which are recorded, at the time of drawing the check, its number, the date, the name of the payee, the purpose for which it was given, and the amount. On the stub there is also kept a record of the balance remaining in the bank. As shown by the following illustration, a balance of $136.80 was brought over from May 10, and on May 11 a deposit of $68.35 was made and a check for $4.80 was drawn.



No. 3 $4.80

May 11 1915

To T. P. O'Brien

For Inv May 11, 1915

	DOLLARS	CENTS
AMOUNT BRO'T FORWARD	136	80
AMOUNT DEPOSITED	68	35
TOTAL	205	15
AMT. OF THIS CHECK	4	80
BALANCE BRO'T FORWARD	200	35

MOUNT VERNON, IOWA, May 11 1915 No. 3

MOUNT VERNON BANK 72-475

CAPITAL $100000.00

Pay to T. P. O'Brien or order $4.80

Four 80/100 Dollars

J. F. Cronk

Written Work

1. Using the following information, rule a check and stub similar to the preceding illustration, enter the necessary facts, and draw the check.

Name of bank, Fourth National, your city.

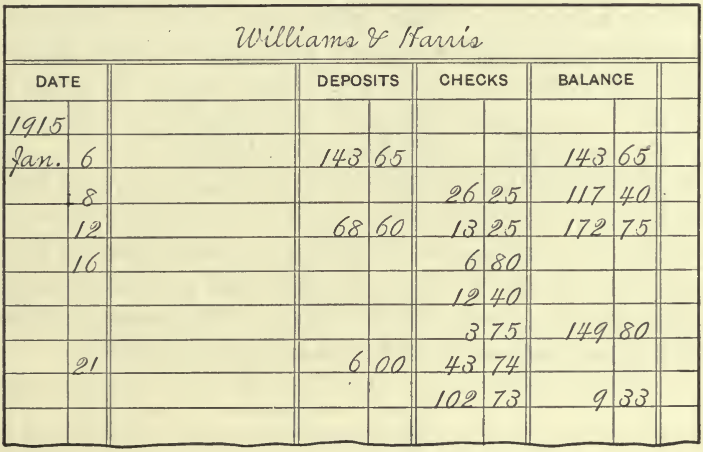
Balance from yesterday, $103.27. Deposit of $19.38.

Check No. 6, drawn by you to-day, payable to G. D. Fitzgerald and Co., in payment of the invoice purchased from them eight days ago. Amount of invoice, $73.28; terms, 2/10; N/60.

Write the check and stub.

What is the new balance?

175. The Bank's Accounts with its Depositors. All banks keep a depositors' ledger. Usually a page is devoted to the deposits and checks of each depositor.



Williams & Harris

DATE			DEPOSITS		CHECKS		BALANCE		
1915									
Jan.	6		143	65			143	65	
	8				26	25	117	40	
	12		68	60	13	25	172	75	
	16				6	80			
					12	40			
					3	75	149	80	
	21		6	00	43	74			
					102	73	9	33	

AN ACCOUNT IN A DEPOSITORS' LEDGER

Written Work

Rule a page of a depositors' ledger, similar to the preceding illustration, and enter the following deposits and checks made by yourself, computing the daily balances. In computing balances, use the method of adding and subtracting explained in Section 13.

March 2, Deposit $127.80. March 3, Check $26.14. March 4, Check $23.19. March 5, Deposit $114.32; Check $12.19. March 6, Checks $13.14, $12.17. March 10, Deposits $32.76, $22.14; Check $17.14. March 11, Checks $26.19, $127.14. March 13, Deposits $12.19, $76.27; Checks $32.15, $27.19,

$13.81. March 16, Deposit $27.94. March 18, Checks $23.42, $27.93.

176. The Clearing House. Any bank will cash the checks drawn on other banks, provided the person presenting them is known at the bank or is properly identified. For example, if you are given a check on the First National Bank, you can cash it or deposit it at the Second National Bank. The Second National then collects the amount of the check from the First National.

When there are several banks in the same city, the checks are collected through a Clearing House. The representatives of the various banks meet at the Clearing House, bringing the checks which they have paid for other banks. Each bank is credited with the amount of the checks which it has paid for other banks, and debited with the amount of the checks which all other banks have paid for it. The Clearing House collects from those having debit balances and pays those having credit balances.

Let us suppose that there are three banks in a city: the First National, the First State, and the City National. The representatives will meet, each bringing the checks paid by his bank for the other banks the preceding day, and a sheet showing the amount paid for each of the other banks. The statements brought by the representatives of the banks are shown below:

FIRST NATIONAL

PAID FOR	AMOUNT
First State	$16,265.00
City National	12,146.80
Total	$28,411.80

FIRST STATE

PAID FOR	AMOUNT
First National	$ 8,368.28
City National	9,208.48
Total	$17,576.76

CITY NATIONAL

PAID FOR	AMOUNT
First National	$12,875.00
First State	4,738.80
Total	$17,613.80

At the Clearing House a sheet similar to the following is prepared. In the debit column is placed the total value of checks paid *for* each bank; in the credit column is placed the total value of checks paid *by* each bank. The balance columns show the balances payable by or to each bank.

MANAGER'S SHEET

BANK	DEBITS	CREDITS	DEBIT BALANCE	CREDIT BALANCE
First National	$21,243.28	$28,411.80		$7168.52
First State	21,003.80	17,576.76	$3427.04	
City National	21,355.28	17,613.80	3741.48	
Totals	$63,602.36	$63,602.36	$7168.52	$7168.52

The First State and the City National each turns over to the Clearing House the amount of its debit balance; the Clearing House pays this amount to the First National. In this way payment of over sixty-three thousand dollars is made by the actual transfer of only a little more than seven thousand dollars.

Written Work

Prepare a Manager's Sheet, and find the debit or credit balance of each bank in the following Clearing House Association. The amount of the checks paid by each bank is shown by the debit sheets which follow.

HARPER'S STATE BANK

PAID FOR	AMOUNT
Harper's State	
First National	$23,756.27
Traders'	14,259.95
Merchants' National	25,726.87
Farmers' & Mechanics	9,243.65
First State	12,060.26

FIRST NATIONAL BANK

PAID FOR	AMOUNT
Harper's State	$13,263.37
First National	
Traders'	8,836.39
Merchants' National	14,856.74
Farmers' & Mechanics	8,345.93
First State	8,325.57

TRADERS' BANK

PAID FOR	AMOUNT
Harper's State	$5,273.85
First National	12,573.82
Traders'	
Merchants' National	7,252.55
Farmers' & Mechanics	11,257.37
First State	10,456.25

MERCHANTS' NATIONAL

PAID FOR	AMOUNT
Harper's State	$3,123.64
First National	6,027.30
Traders'	10,936.40
Merchants' National	
Farmers' & Mechanics	7,238.37
First State	9,235.35

FARMERS' & MECHANICS

PAID FOR	AMOUNT
Harpers' State	$2,346.75
First National	5,236.73
Traders'	4,232.25
Merchants' National	7,727.86
Farmers' & Mechanics	
First State	2,005.39

FIRST STATE

PAID FOR	AMOUNT
Harper's State	$8,946.27
First National	1,856.23
Traders'	11,472.75
Merchants' National	923.88
Farmers' & Mechanics	2,423.73
First State	

Which banks will pay to the Clearing House, and how much will each pay?

Which banks will receive from the Clearing House, and how much will each receive?

177. Exchange on Checks. Checks are frequently used to send money to people in cities other than the one in which the drawer of the check resides. For example, if you have money deposited in the Home National Bank of your city, you may draw a check against this deposit and send it to Mr. Perkins of Omaha. Mr. Perkins will take it to his bank in Omaha, which will pay the check, even though it has no knowledge of you or your deposit in the Home National Bank. The fact that Mr. Perkins is known at his bank, and that he agrees to refund the money if the check proves worthless, is sufficient protection for his bank.

When banks pay out-of-town checks, they incur a certain amount of expense in collecting them. To cover this expense they often require persons for whom they cash checks to pay them "Exchange." When exchange is charged, a common rate is $\frac{1}{10}$ of one per cent of the face of the check, with a minimum charge of from 10 cents to 25 cents.

Example. Mr. Black, who lives in Bloomington, Illinois, received two checks in his mail. One was from Wm. Harris, of Lincoln, Nebraska, for $365.85; the other from H. B. Felter, of Cincinnati, Ohio, for $28.25. The bank at which Mr. Black cashed these checks charges for exchange $\frac{1}{10}$ % of the face of each check, or a minimum of 10 cents. What was the exchange on the checks?

Solution. $\frac{1}{10}\%$ of \$365.85 = \$.37.
$\frac{1}{10}\%$ of \$28.25 = \$.028.

Since \$.028 is below the minimum, 10 cents exchange is charged. The total exchange is \$.47.

Written Work

Prepare deposit tickets for the following:

1. Deposited by Wm. Frend, of Peoria, Illinois, in the Peoria State Bank, on August 7, 1915:

Gold, \$25; silver, \$37.50; currency, \$17; and checks as follows:

Check drawn by D. P. Snyder of Amboy, Illinois, \$65.75.
Check drawn by Hammond & Davis of Lima, Iowa, \$236.57.
Check drawn by B. J. Harris of Springfield, Illinois, \$235.

The bank charges $\frac{1}{10}\%$ exchange. Minimum, 15 cents.

Find the total of the items and deduct the charge for collection.

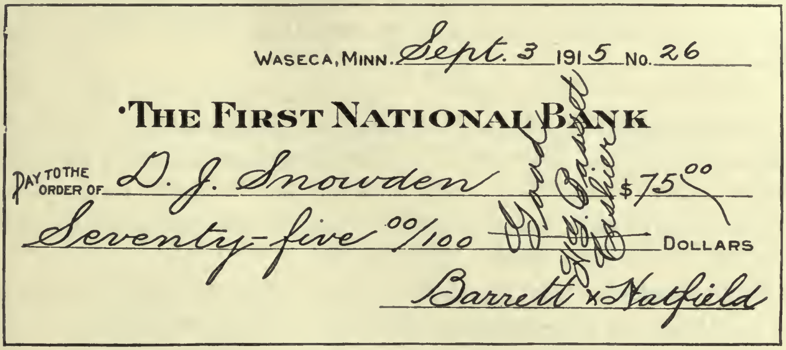
2. Deposited by G. D. Bernham of Hawkeye, Iowa, in the Hawkeye State Bank, on August 20, 1915.

Check drawn by S. Y. Benton of Clarinda, Iowa, \$75.88.
Check drawn by D. W. Galbreath of Morris, Illinois, \$57.93.
Check drawn by F. J. Ressler of Pasadena, California, \$346.49.
Check drawn by D. T. Bailey of Buffalo, New York, \$48.95.

The bank charges exchange on the California and New York checks; rate, $\frac{1}{10}\%$, minimum 25 cents each.

Note. It is the custom of some banks to cash checks on banks in near-by states without charging exchange.

178. Certified Checks. If you wish to assure the person to whom you send a check that you have sufficient money in the bank to pay the check, you may have the check certified. Draw the check in the usual way and take it to your bank. The cashier will write "Accepted" or "Certified" or "Good" across its face and will sign his name. He will then set aside from your deposit the amount of the check, holding it in the bank's funds until the check is presented for payment. Certifying the check reduces the receiver's risk of cashing a worthless check, but it does not relieve him from a possible exchange fee.



WASECA, MINN. Sept. 3 1915 No. 26

THE FIRST NATIONAL BANK

PAY TO THE ORDER OF D. J. Snowden $75.00

Seventy-five 00/100 DOLLARS

Barrett & Hatfield

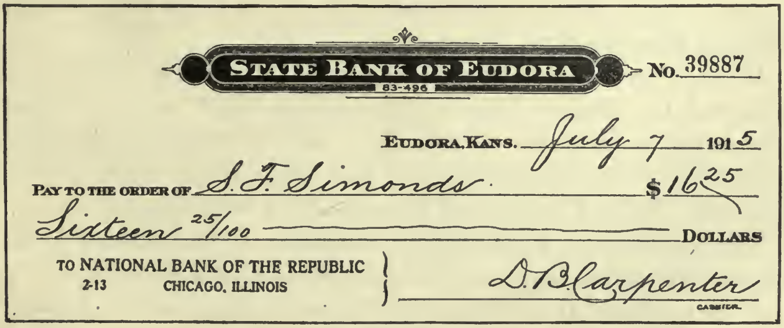
Good
H. G. Barrett
Cashier

A CERTIFIED CHECK

179. Bank Drafts. Banks keep funds on deposit in the banks of the larger cities, and draw checks on these city banks just as you may draw a check on the bank in which you have deposited money. An order drawn by one bank against its deposit in another bank is called a "Bank Draft."

180. Advantages of Bank Drafts. A bank draft is preferable to a check when sending money from one town to another, for two reasons:

a. The draft is drawn by a bank, while the check is drawn by an individual. There is therefore a greater certainty that the bank draft is genuine, and will be paid when presented.



STATE BANK OF EUDORA
83-496
No. 39887

EUDORA, KANS. July 7 1915

PAY TO THE ORDER OF S. F. Simonds $16.25

Sixteen 25/100 DOLLARS

TO NATIONAL BANK OF THE REPUBLIC
2-13 CHICAGO, ILLINOIS

D. B. Carpenter
CASHIER

A BANK DRAFT

b. Bank drafts are drawn on banks in large cities, while checks may be drawn on banks in small towns. There is less expense, therefore, incurred in collecting a bank draft than in collecting a check, and banks seldom charge exchange when cashing bank drafts.

Explanation. D. B. Carpenter is the cashier of the State Bank of Eudora. This bank has funds on deposit in the National Bank of the Republic, Chicago. When the National Bank of the Republic pays this draft, it will deduct the amount from the balance of the Eudora Bank.

S. F. Simonds lived in Eudora and wished to send $16.25 to F. E. Craig of Bloomington, Illinois. When he purchased the draft from his bank, he had it made payable to himself. He might have had it made payable to F. E. Craig, but if he had done so, his (Simonds's) name would not have appeared on the draft, and if it had become separated from the letter accompanying it, Craig might have had difficulty in telling from whom he received it.

After receiving the draft, Simonds indorsed it as follows:

Pay to the order of
F. E. Craig.
S. F. Simonds.

and sent it to Mr. Craig.

Mr. Craig indorsed it and cashed it at the Corn Belt National Bank of Bloomington.

We will now follow the steps by which the Corn Belt National collected the draft. This bank deposits funds with the Continental National Bank of Chicago, which is called its "correspondent," and it therefore sent this draft, with others paid the same day, to the Continental National bank as a deposit.

The Continental National sent the draft to the Clearing House, which collected it from the National Bank of the Republic.

After paying the draft, the National Bank of the Republic charged the amount to the State Bank of Eudora.

The preceding statement may be summarized as follows:

State Bank of Eudora sells the draft to

↓

S. F. Simonds, who paid for it either with cash or a check; and sent it to

↓

F. E. Craig, who gave Simonds credit for $16.25; and deposited the draft at

↓

Bloomington Bank, which gave Craig $16.25; and sent the draft to

↓

Continental National, which gave the Bloomington Bank credit for $16.25; and collected the draft through the Clearing House from

↓

National Bank of the Republic, which paid the Continental National $16.25, and charged Eudora Bank $16.25.

INDORSEMENTS ON THE MODEL BANK DRAFT

Pay to the order of
F. E. Craig
S. F. Simonds

Pay to the order of the
Corn Belt National Bank
F. E. Craig

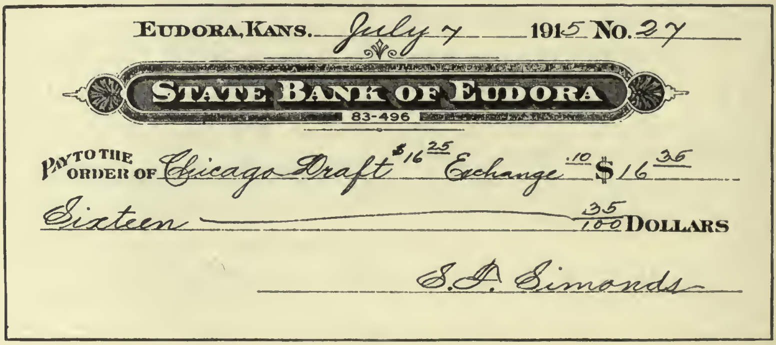
Pay to the order of
Continental National Bank.
All prior indorsements guaranteed.
CORN BELT NATIONAL BANK,
BLOOMINGTON, ILL.,
F. A. FREND, Cashier.

Pay to the order of
Any Bank, Banker, or Trust Co.
Prior indorsements guaranteed.
July 12, 1915.
CONTINENTAL NATIONAL BANK,
M. B. JONES, Cashier.

These indorsements indicate the successive owners of the draft. The last two indorsements are made with rubber stamps. The last one is made payable to "any Bank, Banker, or Trust Company," for convenience in collecting through the Clearing House.

181. The Cost of Bank Drafts. When a bank sells a draft, it gives the purchaser the benefit of its deposit relations with a large city bank. For this convenience it may make a charge called "Exchange." It will be noted that exchange on a check is paid by the person receiving and cashing it, while the exchange on a draft is paid by the person purchasing and sending it. When exchange is charged, $\frac{1}{10}\%$ with a minimum of 10 cents is a customary fee.

The following is a copy of the check given by S. F. Simonds to the State Bank of Eudora to pay for the draft purchased. There was a charge of 10 cents for exchange.



EUDORA, KANS. July 7 1915 No. 27

STATE BANK OF EUDORA

83-496

PAY TO THE ORDER OF Chicago Draft $16.25 Exchange .10 $16 36

Sixteen 35/100 DOLLARS

S. F. Simonds

Written Work

1. You owe H. J. Palmer, of Toledo, Ohio, $27.35. You buy a draft, payable to yourself for this amount, from the Merchants' State Bank of your city, of which A. R. Burton is the cashier. The draft is drawn on the Home National Bank of New York City. Exchange, 10 cents. Draft is made payable to you.

Write a draft similar to the one which the bank would give you, and the check which you would give the bank in payment.

Indorse the draft. How soon can Mr. Palmer get his money?

2. Mr. Palmer cashed the draft at the High Street National Bank of Toledo, B. F. Ohren, Cashier. The High Street National sent it to the Bankers' National Bank of New York, G. B. Martin, Cashier.

The Bankers' National collected it through the Clearing House. Show the indorsements on the draft.

3. Find the cost of a draft sent to pay an invoice of $246.50, less a cash discount of 2%. Exchange, $\frac{1}{10}$% of the face of the draft; minimum, 15 cents.

4. On October 8, you purchased from Benedict & Meredith, Pittsburgh, Pennsylvania, an invoice amounting to $794.86. Terms, 2/15; N/60. On October 16, you paid the invoice by a draft purchased at the Claim Street State Bank of your city, J. D. Haines, Cashier. The draft was drawn on the First National Bank of New York. Exchange, $\frac{1}{10}$%.

Write the draft which you received, the indorsement, and the check you gave to pay for the draft.

182. Postal Money Orders. A postal money order is an order drawn by one postmaster on the postmaster at some other office, calling for the payment of a stated sum of money to the person named on the order. Postal money orders are commonly used to make payments by mail. They are issued in any amount from $.01 to $100.00. As a means of protection, no order is issued for more than $100.00. If it is desired to send more than this amount, additional orders may be purchased.

Fees for money orders payable in the United States (which includes Hawaii and Porto Rico) and its possessions, comprising the Canal Zone, Guam, the Philippines, as well as in Bermuda, British Guiana, British Honduras, Canada, Cuba, Mexico, Newfoundland, at the United States Postal Agency in Shanghai (China), in the Bahama Islands, and in certain other islands in the West Indies, are as follows:

For orders from	$ 0.01 to	$ 2.50	3 cents
from	2.51 to	5.00	5 cents
from	5.01 to	10.00	8 cents
from	10.01 to	20.00	10 cents
from	20.01 to	30.00	12 cents
from	30.01 to	40.00	15 cents
from	40.01 to	50.00	18 cents
from	50.01 to	60.00	20 cents
from	60.01 to	75.00	25 cents
from	75.01 to	100.00	30 cents

Examples. 1. What is the cost of an order for $27.35?

SOLUTION. $27.35 lies between $20.01 and $30.00, and the rate is 12 cents.
Total cost of order, $27.35 + .12 = $27.47.

2. What is the cost of sending $267.95 by postal money orders?

SOLUTION. Two orders for $100.00 each, and one order for $67.95, will be purchased.

Fees of two $100.00 orders, 60 cents.
Fee of $67.95 order, 25 cents.
Total fees, 85 cents.
Total cost of orders, $267.95 + .85 = $268.80.

183. Bank Drafts and Postal Money Orders Contrasted. Note the following differences between a bank draft and a postal money order:

A bank draft may be indorsed as many times as desired.

A postal money order may be indorsed only once.

A bank draft may be cashed at *any* bank.

A postal money order must be presented to the post office on which it is drawn, or to a bank which can cash it at that post office.

A bank draft is payable as soon as it is presented to a bank.

At the time of issuing a money order the issuing postmaster sends a notice to the paying postmaster. A postal money order will not be paid until the paying office has received this notice. However, this does not usually cause any delay.

184. Express Money Orders. Express money orders are similar in many respects to postal money orders. They can be purchased from the agent of the express company, and are payable from the funds of the express company on deposit in various banks specified in the order.

No order is issued for an amount larger than $50. If it is desired to send a larger amount, additional orders must be purchased.

Express Money Order Rates. The rates for express money orders are the same as for postal money orders, although no order is issued for more than $50.

Example. What will be the total fee for the transfer of $241.75 by express money orders?

Solution. Four $50 orders at the rate of 30 cents per hundred will cost 60 cents.

One order for $41.75 will cost 18 cents. Total, 78 cents.

185. Telegraph Money Transfers. Money may be transferred by telegraph when there is an urgent necessity for immediate payment. The rates for this service between points in the United States are determined as follows:

To the tolls for a fifteen-word message between the office of deposit and the office of payment add the following charges:

For $25.00 or less	25 cents
25.01 up to $50	35 cents
50.01 up to 75	60 cents
75.01 up to 100	85 cents

For amounts above $100 add (to the $100 rate) 25 cents per hundred (or any part of $100) up to $3000.

For amounts above $3000 add (to the $3000 rate) 20 cents per hundred (or any part of $100).

Examples. 1. What is the charge for sending $25 or less to a point where the fifteen-word rate is 65 cents?

Solution.

$.25	Minimum charge
.65	Tolls on 15-word message
$.90	Total charge

2. What is the total charge for transfer of $105 if the tolls on a fifteen-word message are 65 cents?

Solution.

$.85	Charge for first $100
.25	Charge for fraction of second $100
.65	Tolls on 15-word message
$1.75	Total rate

Written Work

1. Assuming that you owe the following bills:

F. G. Young, $39.40;
P. S. Sanborn, $112.75;
H. L. Colwell, $416.25, less 3%;

determine the fees for the purchase of either postal or express money orders for transferring money to make payments. Complete the following blank form.

Name	Net Amount of Bills	Fees for Money Orders	Total Cost	Denominations of Orders Received from Ex. Co.	Denominations of Orders Received from P. O.

2. What is the charge for a telegraph money transfer of $23.15 to a point where the fifteen-word message rate is 85 cents?

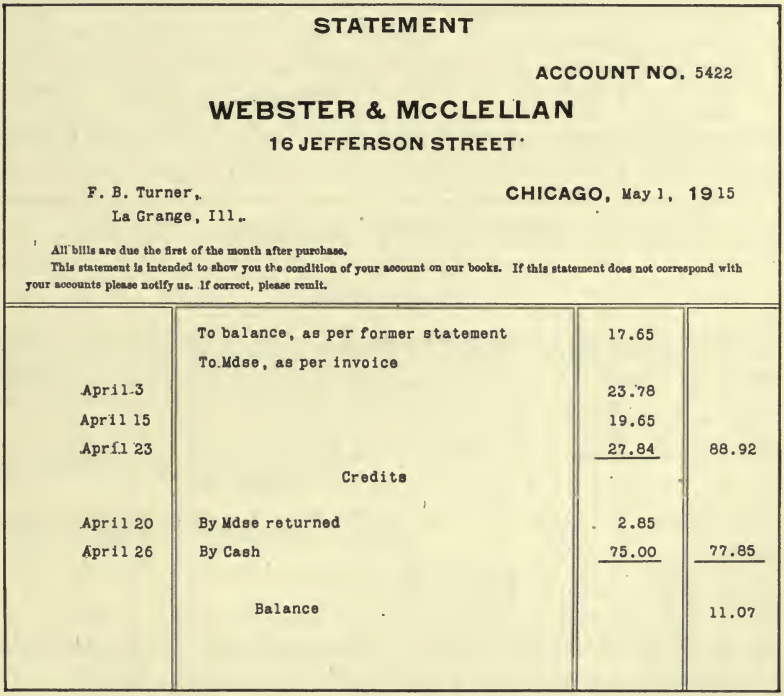
3. What is the charge for a telegraph money transfer of $52.85 to a point where the fifteen-word message rate is $1.30?

4. What is the total cost of a telegraph money transfer for $215 to a point where the fifteen-word rate is 65 cents?

CHAPTER XIX

COLLECTING BILLS

186. Statements. At periodical intervals, usually on the first day of each month, merchants send a statement to their customers.



STATEMENT

ACCOUNT NO. 5422

WEBSTER & McCLELLAN

16 JEFFERSON STREET

F. B. Turner,
La Grange, Ill.

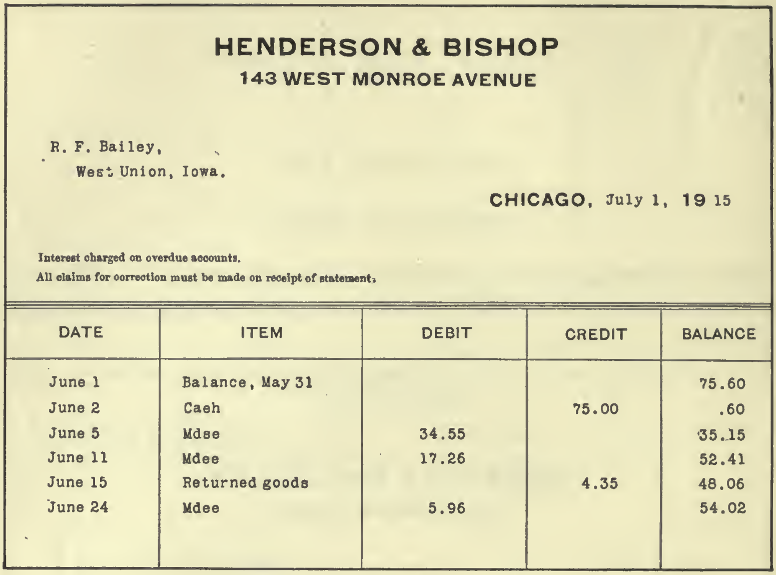
CHICAGO, May 1, 1915

All bills are due the first of the month after purchase.

This statement is intended to show you the condition of your account on our books. If this statement does not correspond with your accounts please notify us. If correct, please remit.

	To balance, as per former statement	17.65	
	To Mdse, as per invoice		
April 3		23.78	
April 15		19.65	
April 23		27.84	88.92
	Credits		
April 20	By Mdse returned	2.85	
April 26	By Cash	75.00	77.85
	Balance		11.07

This statement shows the date and amount of each purchase, the date and amount of each payment, and the balance due. The items sold are not enumerated in the above statement because the invoices may be consulted to obtain this information.



HENDERSON & BISHOP
143 WEST MONROE AVENUE

R. F. Bailey,
West Union, Iowa.

CHICAGO, July 1, 1915

Interest charged on overdue accounts.
All claims for correction must be made on receipt of statement.

DATE	ITEM	DEBIT	CREDIT	BALANCE
June 1	Balance, May 31			75.60
June 2	Cash		75.00	.60
June 5	Mdse	34.55		35.15
June 11	Mdse	17.26		52.41
June 15	Returned goods		4.35	48.06
June 24	Mdse	5.96		54.02

Another Form of Statement

Written Work

1. Rule a statement similar to the first illustration, and enter the following: Your transactions with S. J. Smith, Fayette, Missouri.

Balance, August 1, $127.86.

Purchases, August 3, $72.85; August 11, $23.89; August 14, $75.23; August 20, $14.76; August 28, $38.97.

Payments, August 2, Cash $126.00; August 13, Returned merchandise, $15.00.

Statement rendered September 1.

2. Rule a statement similar to the second illustration, and enter the following transactions with Beardsley & Russel, Montgomery, Alabama.

Balance, October 1, 1915, $32.11.

Purchases, October 5, $45.89; October 9, $23.87; October 12, $243.87; October 15, $65.00; October 21, $52.57; October 26, $40.50; October 29, $50.25.

Payments, October 7, $50.00; October 21, $125.00; October 24, $40.00.

Returned merchandise, October 20, $5.50.

Statement rendered November 1.

187. Commercial Drafts. Commercial drafts offer an effective method of collecting accounts. A commercial draft is an order drawn by the party to whom money is due, requesting the debtor to pay a stated sum of money either to the drawer of the draft or to a third party mentioned in the draft.

Two and Three Party Drafts. If a draft requests the debtor to pay money to the drawer, it is called a **two-party draft.** Such drafts are collected through a bank.

If a draft requests the debtor to pay money to a third party, it is called a **three-party draft.**

Sight and Time Drafts. If a draft requests immediate payment, it is called a **sight draft.** Such a draft is payable at once, without acceptance.

If a draft is to be paid after a stated time, it is called a **time draft.**

188. Parties to a Draft. The **drawer** is the person who draws the draft. The **drawee** is the person on whom the draft is drawn, and who is requested to pay the money. The **payee** is the person to whom the money is to be paid. In case of a two-party draft, the drawer is also the payee.

189. Reasons for Drawing Drafts.

(*a*) To effect prompt payment of invoices.

1. How a sight draft is used: Wholesale houses often make terms similar to the following: "Sight draft in ten days, less 2%." At the expiration of ten days from the date of sale, the selling merchant sends a sight draft for the net amount of the bill, to the purchasing merchant's bank. The bank presents it to the purchaser for collection.

2. How a time draft is used: If the terms of the sale were "Thirty-day draft," the selling merchant would, at the time of

making the sale, draw a thirty-day draft on the purchaser and send it to him for acceptance. After the draft has been accepted, it has the same value as a note because the purchaser has agreed to pay it when due. The seller may borrow money from a bank, giving the accepted draft as security.

(*b*) As a means of collection. When a bill is overdue, and the debtor is slow in making settlement, a draft, to be collected by his bank, will often bring about a settlement.

(*c*) To make C. O. D. shipments by freight. When a merchant sells goods to be shipped by freight, he receives a bill of lading from the railroad company. This bill of lading is the railroad's receipt for the goods. To make a C. O. D. shipment the seller obtains an Order Bill of Lading from the railroad. The purchaser cannot get the goods from the railroad without surrendering the bill of lading. Therefore, a selling merchant, instead of sending the bill of lading direct to the purchaser, attaches it to a sight draft and sends the draft and the bill of lading to a bank in the purchaser's town.

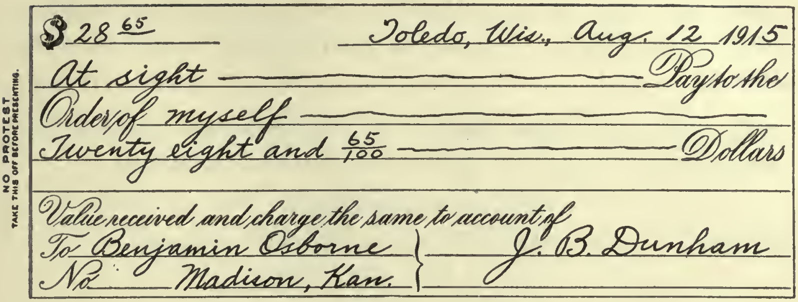
In order to get the goods the purchaser must have the bill of lading ; in order to get the bill of lading he must pay the draft at the bank.

(*d*) To avoid the transfer of money. If

Jones owes Smith, and
Smith owes Brown,

Smith may collect the debt from Jones and pay his own debt to Brown, or he may request Jones to send the money direct to Brown. If Jones and Brown live in the same town, both debts can be paid without sending the funds through the mail, even though Smith lives in a distant city. A three-party draft is used for this purpose, Smith requesting Jones to pay Brown.

190. Two-party Sight Draft. Benjamin Osborne of Madison, Kansas, owes J. B. Dunham of Toledo, Wisconsin, $28.65. In order to collect this bill, Mr. Dunham draws the following draft on Mr. Osborne.



NO PROTEST
TAKE THIS OFF BEFORE PRESENTING.

$28 65 Toledo, Wis., Aug. 12 1915

At sight Pay to the

Order of myself

Twenty eight and $\frac{65}{100}$ Dollars

Value received and charge the same to account of

To Benjamin Osborne
No. Madison, Kan.

J. B. Dunham

Mr. Dunham then indorses the draft as follows:

Pay to the order of
The First State Bank of Toledo
for collection only.
J. B. Dunham.

and gives it to his bank to collect.

The bank, in turn, indorses it as follows :

Pay to the order of
The Farmers National Bank of Madison
for collection only.
First State Bank,
Toledo, Wisconsin,
E. J. Baxter, Cashier.

and forwards it to the bank at Madison to collect. The Madison bank presents it to Mr. Osborne for payment. Mr. Osborne is not obliged to pay this draft, but if he owes the bill, he will probably do so in order to keep his standing good at his bank.

If Osborne pays the draft, the bank stamps a receipt thereon, and gives the draft to Osborne. The bank keeps a certain amount as a collection fee, and returns the balance to the bank at Toledo, where Mr. Dunham receives the proceeds in cash or as a deposit credit.

If Osborne does not pay the draft, it is returned to the bank at Toledo with a statement of the reason for non-payment.

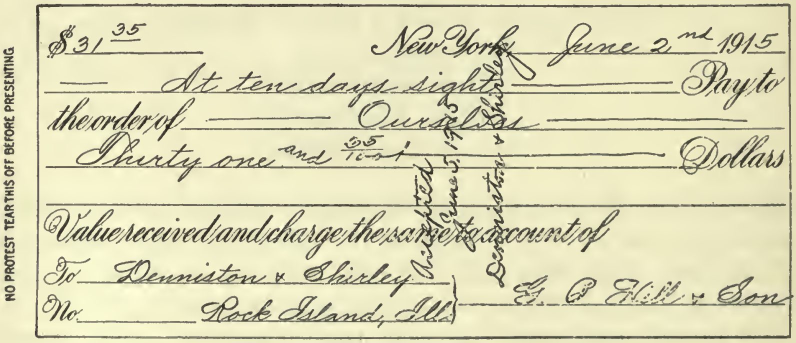
191. Rates for Collecting Drafts. "Collection" rates, like exchange rates, differ. A common rate is $\frac{1}{10}$ %, with a fixed minimum.

192. Two-party Time Draft. By substituting for the words "At sight" in the preceding illustration, such words as

Ten days after date,
At thirty days' sight, or
Thirty days after sight

the draft becomes a time draft. Instead of paying this draft when it is first presented, Mr. Osborne will accept it; that is, he will agree to pay it when it is due. Acceptance consists of writing across the face of the draft the word "Accepted" and signature.

193. Maturity of Time Drafts. Drafts payable "after sight" or "at — days' sight" are payable a stated number of days after acceptance, and the acceptance should therefore be dated. Drafts payable "after date" become due a stated number of days after the date of the draft. The acceptance, therefore, does not need to be dated, but it usually is.



NO PROTEST TEAR THIS OFF BEFORE PRESENTING

$\$31^{35}$ New York June 2nd 1915
At ten days sight Pay to
the order of Ourselves
Thirty one and $\frac{35}{100}$ Dollars
Value received and charge the same to account of
To Denniston & Shirley
No. Rock Island, Ill. G. P. Hill & Son

Accepted June 5, 1915
Denniston & Shirley

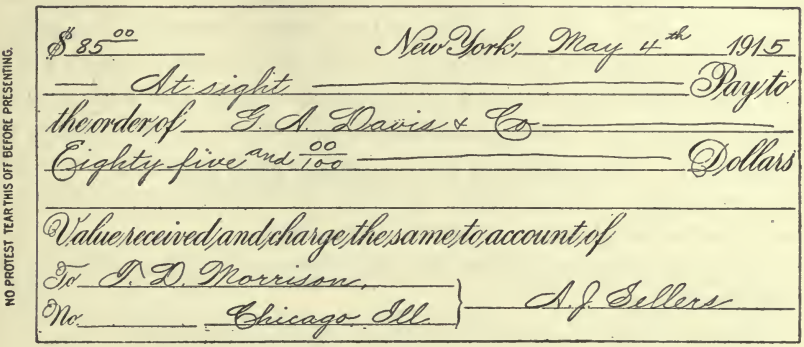
AN ACCEPTED TIME DRAFT

This draft is due ten days after June 5, or June 15.

194. Three-party Drafts. T. D. Morrison of Chicago owes A. J. Sellers of New York $85.00.

Mr. Sellers owes G. A. Davis & Co. of Chicago $150.00.

Mr. Sellers draws the following draft, asking Mr. Morrison to pay G. A. Davis & Co. $85.00.



NO PROTEST TEAR THIS OFF BEFORE PRESENTING.

$85.00 New York, May 4th 1915

At sight Pay to

the order of G. A. Davis & Co

Eighty five and 00/100 Dollars

Value received and charge the same to account of

To T. D. Morrison,

No. Chicago Ill. } A. J. Sellers

THREE-PARTY SIGHT DRAFT

Oral Work

What two parties make a payment when this draft is used?

Explain how one transfer of money is saved.

How much will Sellers owe G. A. Davis & Co. after they have received this draft?

Three-party drafts may also be made payable after a stated time by using the terms explained on page 230.

Written Work

1. On July 13, you purchased from A. B. Butler of Minneapolis, Minnesota, an invoice of goods amounting to $123.60; terms, sight draft in 10 days, less 3%. On July 20, Mr. Butler drew a sight draft on you for the net amount of the bill. He indorsed the draft, and gave it to the Fourth Street Bank of Minneapolis for collection. The Minneapolis bank indorsed it and sent it to the First National Bank of your city. The draft was presented to you by the bank on the 23d of July.

Write the draft with indorsements.

If the First National Bank charged 15 cents for collection, how much did Mr. Butler receive in payment of the draft?

How much did you pay to the bank?

2. You purchased an invoice of goods from Winthrop and Monroe of Toledo, Ohio. Amount of invoice, $365.00. It was your first transaction with them and consequently you were unknown to them. As you wanted the goods immediately and did not wish to delay while inquiries were made regarding your credit, you instructed them to send the goods by freight and send a sight draft, with bill of lading attached, to the First National Bank of your city. The sight draft, which was dated yesterday, arrived this morning. It was made payable to Winthrop and Monroe and was indorsed to the order of the bank.

Write the draft and show the indorsement made by Winthrop and Monroe.

Write a check in favor of the bank to pay the draft.

What will you receive in exchange for the check?

How will Winthrop and Monroe get their money?

3. On September 3, you purchased an invoice of goods from S. L. Norton of Springfield, Illinois. Terms, thirty-day draft from date of sale, less 1%. Amount of invoice, $75.00. On September 5, you received the draft in the mail; it was dated September 3, and was payable 30 days after date. You immediately accepted the draft and returned it to Mr. Norton.

Write the draft, and show your acceptance.

When will you be expected to pay the draft?

4. You owed S. D. Briggs of Winona, Minnesota, $75.00; Mr. Briggs owed F. R. Tuttle of your city, $136.00. On June 19, Mr. Briggs drew a sight draft on you for the amount of your debt, and sent it to Mr. Tuttle as part payment of his debt. Mr. Tuttle presented it to you. You gave Mr. Tuttle cash in payment of the draft, and received the draft as a receipt.

Write the draft.

How much did Briggs owe Tuttle after the draft was paid?

CHAPTER XX

FOREIGN MONEY AND EXCHANGE

FOREIGN MONEY

195. The trade between the United States and foreign countries necessitates changing money values of the United States coinage into that of foreign countries, and also changing values stated in the coinage of foreign countries into that of the United States. In order to provide a standard of comparative values, the Director of the United States Mint at frequent intervals estimates the values of foreign coins, and these values are proclaimed by the secretary of the treasury. This proclamation is then used as the standard in computing the value of foreign merchandise imported into this country, until the succeeding proclamation is issued. The data in the table on the following page are copied from a recent proclamation.

196. To reduce a value in foreign coinage to United States money.

Multiply the value of the coin in United States money by the number of the coins.

Example. Change 2500 francs to dollars.

SOLUTION.

$$1 \text{ franc} = \$.193.$$
$$2500 \times \$.193 = \$482.50.$$

197. To reduce a value in United States money to foreign coinage.

Divide the value in United States money by the value of the foreign unit.

Example. Change $86.85 to francs.

SOLUTION. 1 franc = $.193
$86.85 ÷ $.193 = 450, the number of francs.

VALUES OF FOREIGN COINS

COUNTRY	LEGAL STANDARD	MONETARY UNIT	VALUE IN TERMS OF U.S. MONEY	REMARKS
Argentine Republic	Gold	Peso . .	$0.9647	Currency: depreciated paper, convertible at 44 per cent of face value.
Austria-Hungary .	Gold	Crown .	.203	
Belgium	Gold and silver	Franc .	.193	Member of Latin Union; gold is the actual standard.
Canada	Gold	Dollar .	1.000	
France	Gold and silver	Franc .	.193	
German Empire .	Gold	Mark .	.238	
Great Britain . .	Gold	Pound sterling	4.8665	
Greece	Gold and silver	Drachma	.193	Member of Latin Union; gold is the actual standard.
Italy	Gold and silver	Lira . .	.193	
Japan	Gold	Yen . .	.498	
Mexico	Gold	Peso . .	.498	
Netherlands . . .	Gold	Florin .	.402	
Newfoundland . .	Gold	Dollar .	1.014	
Norway	Gold	Crown .	.268	
Panama	Gold	Balboa .	1.000	
Peru	Gold	Libra .	4.8665	
Philippine Islands	Gold	Peso . .	.500	
Roumania . . .	Gold	Leu . .	.193	
Russia	Gold	Ruble .	.515	
Sweden	Gold	Crown .	.268	
Switzerland . . .	Gold	Franc .	.193	
Turkey	Gold	Piaster .	.044	
Uruguay	Gold	Peso . .	1.034	
Venezuela . . .	Gold	Bolivar .	.193	

Written Work

1. The following table gives the value of the United States imports from, and exports to, various European countries in a recent year. Imports are stated in the coinage of the nation from which they came; use the preceding table of values and change the values to United States coinage. Exports are stated

in the coinage of this country; change these values into the coinage of the nation to which they were shipped.

Country	Imports from	Exports to
Great Britain	56,085,622 pounds sterling	$564,372,186
Germany	720,085,605 marks	306,959,021
France	645,328,797 francs	135,388,851
Belgium	215,945,179 francs	51,387,618
Netherlands	88,478,696 florins	103,702,859
Italy	248,852,481 liras	65,261,268
Switzerland	124,138,326 francs	855,355
Sweden	35,528,936 crowns	9,451,011
Norway	30,789,992 crowns	8,331,723
Austria-Hungary	82,333,960 crowns	22,388,930
Russia	40,110,530 rubles	21,515,660
Turkey	9,852,709 piasters	2,597,239
Greece	19,810,186 drachmas	966,641

Rule a form and enter these statistics, showing the value of imports in dollars, and the value of exports in terms of the coinage of the country to which the merchandise was exported.

2. Half a dozen pairs of gloves were bought in England for £1 2*s*. What was the cost of 1 pair in United States money?

Foreign Exchange

Most of the methods used for the transfer of money and the collection of accounts between persons in the same country are also available when the persons live in different countries.

198. Postal Money Orders. Domestic money orders were discussed on page 221. The following table of rates shows the cost of international postal money orders:

When payable in Apia, Austria, Belgium, Bolivia, Cape Colony, Costa Rica, Denmark, Egypt, Germany, Great Britain, Honduras, Hongkong, Hungary, Italy, Japan, Liberia, Luxemburg, New South Wales, New Zealand, Orange River Colony, Peru, Portugal, Queensland, Russia, Salvador, South Australia, Switzerland, Tasmania, the Transvaal, Uruguay, and Victoria.

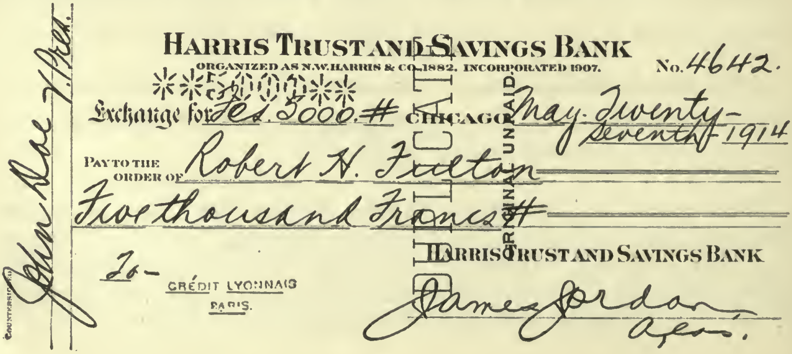
For Orders from		For Orders from	
$00.01 to $2.50	10 cents	$30.01 to $40.00	45 cents
2.51 to 5.00	15 cents	40.01 to 50.00	50 cents
5.01 to 7.50	20 cents	50.01 to 60.00	60 cents
7.51 to 10.00	25 cents	60.01 to 70.00	70 cents
10.01 to 15.00	30 cents	70.01 to 80.00	80 cents
15.01 to 20.00	35 cents	80.01 to 90.00	90 cents
20.01 to 30.00	40 cents	90.01 to 100.00	1 dollar

When payable in any other foreign country.

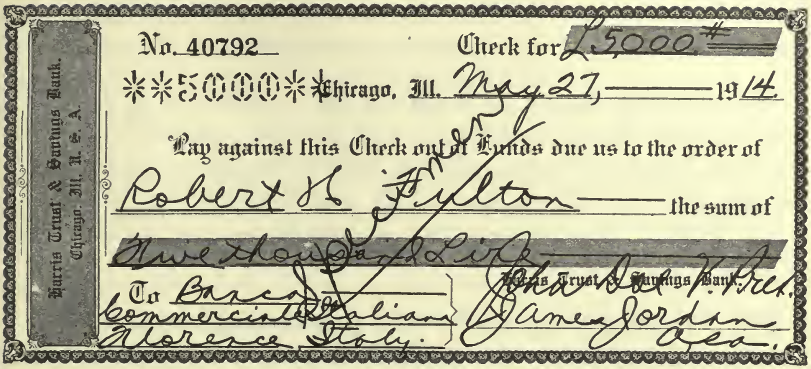
For Orders from		For Orders from	
$00.01 to $10.00	10 cents	$50.01 to $60.00	60 cents
10.01 to 20.00	20 cents	60.01 to 70.00	70 cents
20.01 to 30.00	30 cents	70.01 to 80.00	80 cents
30.01 to 40.00	40 cents	80.01 to 90.00	90 cents
40.01 to 50.00	50 cents	90.01 to 100.00	1 dollar

199. Bills of Exchange. Bills of exchange are drafts of a person or a bank in one country on a person or a bank in another country. They are of three kinds: Bankers' Bills, Commercial Bills, and Letters of Credit. Formerly, when the danger of long delays and possible loss in the foreign mails was greater than it is at present, foreign bills of exchange were issued in sets of three; the first to arrive at its destination was paid and the others became worthless automatically. Later, the number was reduced to two, and it is now customary for only one of the set to be mailed, and the other filed for use in case the first is lost.

200. Bankers' Bills. A banker's bill of exchange is a draft drawn by a bank in one country on a bank in a foreign country. It corresponds to the bank draft of domestic exchange.



HARRIS TRUST AND SAVINGS BANK
ORGANIZED AS N.W. HARRIS & CO. 1882. INCORPORATED 1907.
No. 4642.
5000
Exchange for Fcs. 5000.# CHICAGO May Twenty-seventh 1914
PAY TO THE ORDER OF Robert H. Fulton
Five thousand Francs#
HARRIS TRUST AND SAVINGS BANK
To- CRÉDIT LYONNAIS
PARIS.
James Jordan
Afcs.
DUPLICATE
ORIGINAL UNPAID.
COUNTERSIGNED
John Doe V. Pres.



Harris Trust & Savings Bank. Chicago, Ill. U. S. A.

No. 40792

Check for £5000#

5000 Chicago, Ill. May 27, 1914.

Pay against this Check out of Funds due us to the order of

Robert H. Fulton the sum of

Five thousand Lire

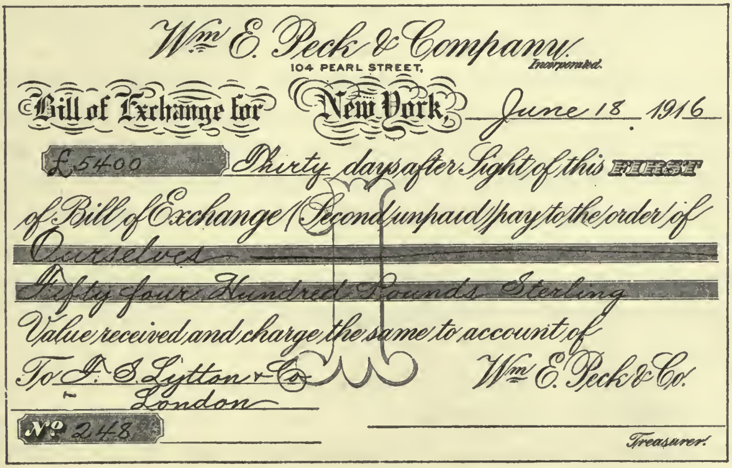
To Banco Commerciale Italiana Florence Italy.

Harris Trust & Savings Bank.
John Doe V. Pres.
James Jordan Sec.

BANKER'S BILL

201. Commercial Bills. A commercial bill of exchange is a draft drawn by a shipper of merchandise upon a foreign buyer or his representative. Commercial bills are made payable either at sight or after a certain time. If such a bill is payable in less than thirty days, it is known as a *short bill;* if in thirty days or more, it is known as a *long bill.*

If the bill is accompanied by a bill of lading and other shipping papers, it is known as a *documentary bill;* if no papers accompany it, it is known as a *clean bill.*



Wm E. Peck & Company.
104 PEARL STREET,
Incorporated.

Bill of Exchange for New York, June 18 1916

£5400 Thirty days after Sight of this FIRST of Bill of Exchange (Second unpaid) pay to the order of

Ourselves

Fifty four Hundred Pounds Sterling

Value received and charge the same to account of

To J. S. Lytton & Co.
London

Wm E. Peck & Co.

No. 248

Treasurer.

COMMERCIAL BILL

A *documentary bill* is accompanied by the bill of lading, the invoice of the goods, and usually by the insurance certificate.

202. Sending Money by Banker's Bill. When it is desired to send money abroad, a banker's bill may be purchased from the bank in much the same manner that a domestic bank draft is purchased. In fact, bankers' bills are frequently called foreign bank drafts. The banker's bill is sent abroad by the purchaser in payment of his debt.

203. Collecting Accounts by Using Commercial Bills. When it is desired to collect an account from a foreign debtor, a merchant may draw a commercial bill and leave it with his bank for collection. The method of collecting the bill is similar to that employed by banks in the collection of domestic sight or time drafts.

204. Securing Immediate Payment for Goods by Using Documentary Bills. Exporters often obtain immediate payment for goods sent abroad by the following method. The goods are delivered to the transportation company and a bill of lading is received by the shipper. The goods are insured against loss in transit, and a certificate of insurance is received from the insurance company. The shipper draws a bill of exchange on the foreign importer, and attaches the bill of lading and the certificate of insurance to the bill of exchange, thus creating a documentary bill. The three papers are indorsed to the order of a bank which purchases the draft. The exporter thus receives payment for his goods at the time of shipment. If the bill cannot be collected, the goods are taken in payment. If the goods are lost, the certificate of insurance guarantees that the bank will receive payment from the insurance company. The bank, in turn, indorses the documents and sends them to some foreign bank, receiving credit for the bill of exchange. The foreign bank collects the bill from the exporter.

The following summary illustrates the method of collecting a documentary bill of exchange:

American Exporter sells the documentary bill to an American Bank, and receives cash.

American Bank sends the documentary bill to a foreign bank, and receives credit against which it can draw bankers' bills.

Foreign Bank collects the documentary bill from the foreign importer.

205. Rates of Exchange. The amount which a bank will pay for a commercial bill and the amount which it will charge for a banker's bill, depend upon the rate of exchange existing between the countries involved in the transaction.

The **mint par of exchange**, as shown by the table on page 234, is the actual value of the coin of one country stated in terms of the coin of another country. It remains comparatively constant.

The **rate of exchange** is the market value of a bill of exchange. These values or rates of exchange are constantly fluctuating, because the value of a bill of exchange, like the value of other property, varies with the supply and demand.

As we have seen, bankers buy documentary bills to send abroad to create a deposit against which they can draw bankers' bills. At a time when American exporters are shipping large quantities of goods abroad, documentary bills on London will be plentiful. American bankers can easily procure all of these bills they desire in order to keep up their balance in foreign banks. Supply will be relatively much greater than demand, and the price will therefore be low. Exchange on England will be below par, or at a discount.

Par is $4.8665. If the rate of exchange is below par, say at $4.84, the man who sells a documentary bill on London may receive for it less than its face; while the man who wishes to buy a banker's bill payable in London may buy it for less than its face.

On the other hand, let us assume a condition when American imports greatly exceed exports. While imports are being made in large quantities, American banks will have frequent demands for bankers' bills to send abroad in payment for goods. In order to issue these bills, American banks must maintain a large deposit in the foreign banks. To keep up this large balance, they will require numerous documentary bills payable in London. But since exports are low, the supply of these documentary bills will be small, and bankers will bid against each other to obtain them. This will cause the rate of exchange to go above par. This means that the American exporter can obtain more than $4.8665 per pound sterling for documentary bills on London which he has for sale; while the importer who wishes to send money abroad will have to pay more than $4.8665 per pound sterling for bankers' bills.

206. Quotations of Rates of Exchange. Exchange on Great Britain is quoted at the number of dollars to the pound sterling. 4.87 means that a pound bill on London will cost $4.87.

Exchange on the Latin Countries (France, Spain, Switzerland, and Italy) is quoted at the number of foreign coins per dollar. Exchange on France, quoted at 5.15$\frac{7}{8}$, means that 5.15$\frac{7}{8}$ francs can be purchased for $1.00.

Exchange on Germany is quoted at the number of cents per four marks. 95 means a banker's bill for four marks can be purchased for 95 cents.

Many banks receive daily the quotations of foreign exchange.

The following is an extract from such a quotation:

London Cables	4.8915
London Demand	4.8865
Sixty Day Grain	4.8625
Paris Checks	5.15
Berlin	95$\frac{3}{8}$

To find the cost of a banker's bill on London.

Example. What is the cost of a £600 draft on London, purchased at 4.8675?

SOLUTION.

$ 4.8675	Cost per pound
600	Number of pounds
$2920.50	Cost of £600 draft

To find the cost of a draft on Paris.

Example. What is the cost of a 300-franc draft on Paris, purchased at 5.15?

SOLUTION. 5.15 francs cost $1.00.

300 francs will cost as many dollars as 5.15 is contained in 300; therefore the cost is $58.25.

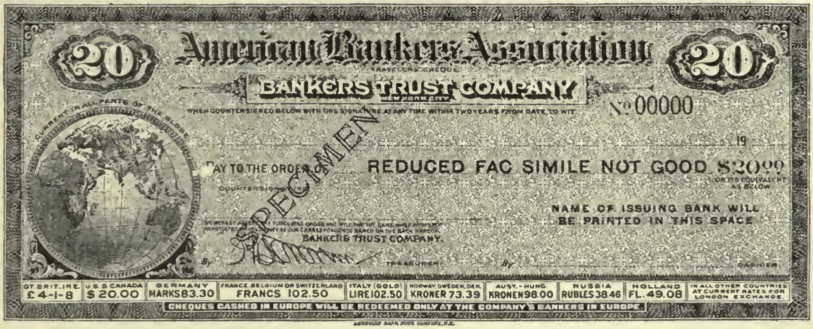
To find the cost of a draft on Berlin.

Example. What is the cost of a 1000-mark draft purchased at $95\frac{1}{4}$?

Solution. $1000 \div 4 = 250$, number of lots of 4 marks purchased.
$\$.95\frac{1}{4} \times 250 = \$238.13$.

Reverse these processes to find the value in foreign coinage of drafts purchasable with a specified amount of United States money.

207. Travelers' Checks. Travelers' checks are issued by the express companies and by the American Bankers Association. These checks are issued in denominations of United States money, but show on their face their cashable value in the coinage of foreign countries. The purchaser signs the checks when purchased, and again when cashed, as a means of identification. The cost of the checks is the face plus $\frac{1}{2}\%$ commission.



20 American Bankers Association 20

TRAVELERS CHEQUE

BANKERS TRUST COMPANY

NEW YORK CITY

WHEN COUNTERSIGNED BELOW WITH THIS SIGNATURE AT ANY TIME WITHIN TWO YEARS FROM DATE, TO WIT

№ 00000

19

PAY TO THE ORDER OF REDUCED FAC SIMILE NOT GOOD $\$20^{00}$ OR ITS EQUIVALENT AS BELOW

SPECIMEN

NAME OF ISSUING BANK WILL BE PRINTED IN THIS SPACE

BANKERS TRUST COMPANY.

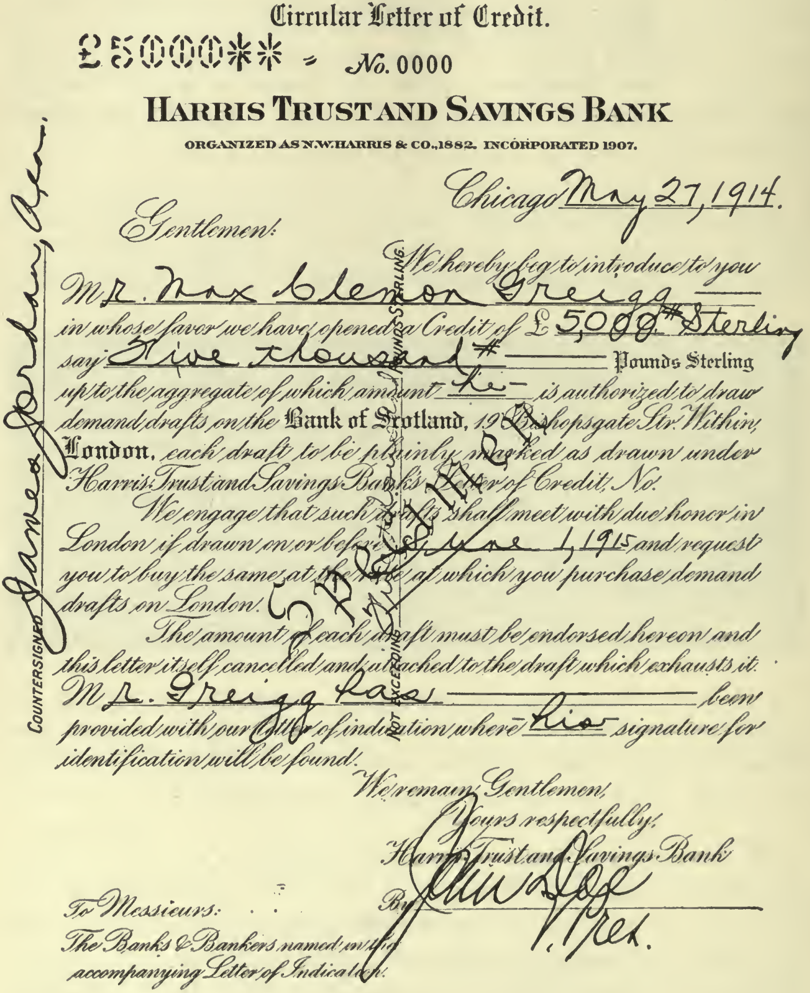
By TREASURER By CASHIER

CURRENT IN ALL PARTS OF THE GLOBE

GT. BRIT., IRE.	U. S. & CANADA	GERMANY	FRANCE BELGIUM OR SWITZERLAND	ITALY (GOLD)	NORWAY, SWEDEN, DEN.	AUST.-HUNG.	RUSSIA	HOLLAND	IN ALL OTHER COUNTRIES AT CURRENT RATES FOR LONDON EXCHANGE.
£4-1-8	$20.00	MARKS 83.30	FRANCS 102.50	LIRE 102.50	KRONER 73.39	KRONEN 98.00	RUBLES 38.46	FL. 49.08	

CHEQUES CASHED IN EUROPE WILL BE REDEEMED ONLY AT THE COMPANY'S BANKERS IN EUROPE.

208. Letters of Credit. A letter of credit is a circular letter addressed to the correspondents of the issuing bank, introducing the holder, certifying that he is authorized to draw a certain sum of money, and requesting that his drafts be honored up to that amount. By depositing cash or securities with a bank, a traveler can obtain a letter of credit. This document is drawn by the bank in which the cash has been deposited. When a bank issues a letter of credit, it thereby authorizes its correspondents (banks in which it has money on deposit) to pay drafts drawn by the traveler up to a specified sum. The bank also furnishes the purchaser a Letter of Indication containing his signature; when the



Circular Letter of Credit.

£5000** No. 0000

HARRIS TRUST AND SAVINGS BANK

ORGANIZED AS N.W. HARRIS & CO., 1882. INCORPORATED 1907.

Chicago May 27, 1914.

Gentlemen:

We hereby beg to introduce to you Mr. Max Glenon Greigg in whose favor we have opened a Credit of £5,000# Sterling say Five thousand# Pounds Sterling up to the aggregate of which amount he is authorized to draw demand drafts on the Bank of Scotland, 19 Bishopsgate Str. Within, London, each draft to be plainly marked as drawn under Harris Trust and Savings Bank's Letter of Credit No.

We engage that such drafts shall meet with due honor in London if drawn on or before June 1, 1915 and request you to buy the same at the rate at which you purchase demand drafts on London.

The amount of each draft must be endorsed hereon and this letter itself cancelled and attached to the draft which exhausts it. Mr. Greigg has been provided with our letter of indication where his signature for identification will be found.

We remain, Gentlemen,

Yours respectfully,

Harris Trust and Savings Bank

By John Doe

V. Pres.

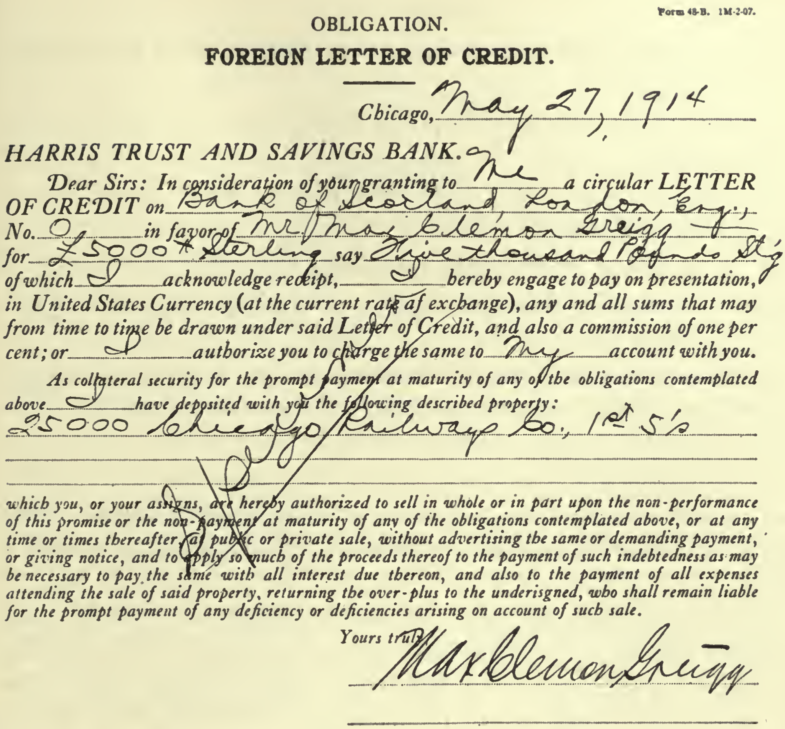
To Messieurs:

The Banks & Bankers named in the accompanying Letter of Indication.

COUNTERSIGNED James Jordan,

NOT EXCEEDING POUNDS STERLING.

traveler presents a draft to one of the foreign banks specified in the letter of credit, the signature on the draft is compared with that on the letter of indication. If the signatures are identical,



Form 48-B. 1M-2-07.

OBLIGATION.

FOREIGN LETTER OF CREDIT.

Chicago, May 27, 1914

HARRIS TRUST AND SAVINGS BANK.

Dear Sirs: In consideration of your granting to me *a circular LETTER OF CREDIT on* Bank of Scotland, London, Eng., *No.* 0 *in favor of* Mr. Max Clemon Greigg Jr. *for* £5000 Sterling *say* Five thousand Pounds Stg., *of which* I *acknowledge receipt,* I *hereby engage to pay on presentation, in United States Currency (at the current rate af exchange), any and all sums that may from time to time be drawn under said Letter of Credit, and also a commission of one per cent; or* I *authorize you to charge the same to* my *account with you.*

As collateral security for the prompt payment at maturity of any of the obligations contemplated above I *have deposited with you the following described property:*

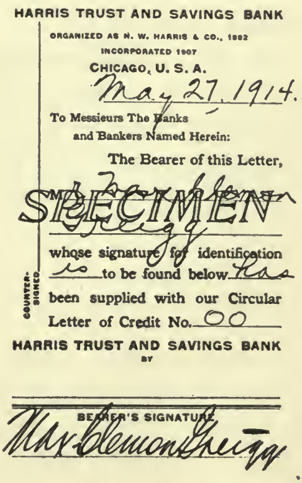
25000 Chicago Railway Co. 1st 5's

which you, or your assigns, are hereby authorized to sell in whole or in part upon the non-performance of this promise or the non-payment at maturity of any of the obligations contemplated above, or at any time or times thereafter, at public or private sale, without advertising the same or demanding payment, or giving notice, and to apply so much of the proceeds thereof to the payment of such indebtedness as may be necessary to pay the same with all interest due thereon, and also to the payment of all expenses attending the sale of said property, returning the over-plus to the underisgned, who shall remain liable for the prompt payment of any deficiency or deficiencies arising on account of such sale.

Yours truly,

Max Clemon Greigg

the foreign bank pays the draft, charges the amount to the bank issuing the letter, and indorses the amount of the draft on the letter. Thus, the letter of credit shows at all times the balance remaining subject to draft. When a draft is drawn which exhausts the balance, the letter is surrendered, and sent, together with the draft, to the issuing bank.



HARRIS TRUST AND SAVINGS BANK
ORGANIZED AS N. W. HARRIS & CO., 1882
INCORPORATED 1907
CHICAGO, U. S. A.

May 27, 1914.

To Messieurs The Banks
and Bankers Named Herein:

The Bearer of this Letter,

Mr. Max Clemon Greigg

SPECIMEN

whose signature for identification is to be found below has been supplied with our Circular Letter of Credit No. 00

COUNTER-SIGNED

HARRIS TRUST AND SAVINGS BANK
BY

BEARER'S SIGNATURE

Max Clemon Greigg

Letters of credit are issued in an amount equivalent to the exchange value of the cash or securities deposited, minus a percentage charged for the accommodation.

Written Work

1. Find the total cost of a postal money order for $85, payable in Hongkong.

2. Find the total cost of a postal money order for $215.65, payable in Paris.

3. What would be the cost of a London draft for £115, exchange being quoted at 4.8675?

4. A man had $500 which he wished to remit to his mother in Paris. How many francs did she receive, the rate of exchange being 5.15?

5. A man residing in this country wished to send his nephew in Berlin a birthday present of $10. What was the amount of the draft in marks, exchange being at 95¼?

6. M. Le Count, traveling in America, presented to a Chicago bank a draft on the Credit Lyonnais, Paris, for 2000 francs. How much American money did he receive, exchange being quoted at 5.16?

7. The Western Milling Co. sold a Glasgow customer three hundred barrels of XXXX flour at 19*s.* per barrel. The Milling Co. sold its sixty-day time bill, with bill of lading attached, to its banker for 4.86⅔ per pound sterling. What was the amount of the draft in English currency, and how much per barrel United States currency did the company receive for its flour?

8. The Lincoln Grain Co. sold three cars of wheat to a Liverpool miller at 4*s.* per bushel. The net weight of the cars was: 58,950#, 59,040#, and 64,360#. The bank, as agent for the grain company, sold the draft with bill of lading attached at 4.8555. How much did the Grain Company receive?

CHAPTER XXI

ACCOUNTS

An account is an orderly record of the transactions pertaining to any one person or thing.

209. The Cash Account. In keeping a record of cash,

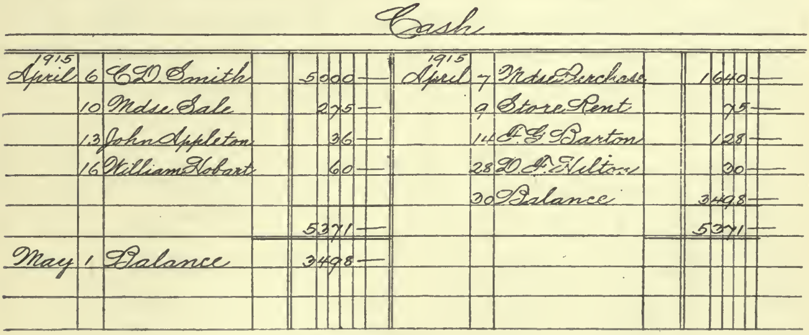
All cash receipts are entered on the left, or debit side;

All cash payments are entered on the right, or credit side;

The difference between the total receipts and total payments is the balance of cash on hand.

The following transactions are recorded in the model cash account in the illustration.

April 6, 1915, C. D. Smith invested $5000 in business; April 7, he paid $1640 for goods; April 9, he paid his store rent, $75; April 10, he received $275 for merchandise; April 13, he received $36 from John Appleton; April 14, he paid F. G. Barton $128; April 16, William Hobart paid him $60 on account; April 28, he paid D. F. Hilton $30 on account.



Cash

1915 April	6	C. D. Smith	5000 —	1915 April	7	Mdse. Purchase	1640 —
	10	Mdse. Sale	275 —		9	Store Rent	75 —
	13	John Appleton	36 —		14	F. G. Barton	128 —
	16	William Hobart	60 —		28	D. F. Hilton	30 —
					30	Balance	3498 —
			5371 —				5371 —
May	1	Balance	3498 —				

210. Personal Accounts. Transactions with persons are recorded in personal accounts. Persons to whom goods are sold are debited for all goods sold to them on account; they are credited for payments made by them on account.

Persons from whom goods are bought, are credited for goods purchased from them on account; they are debited for payments made to them on account.

The two illustrations which follow will show the two common rulings for personal accounts.

May 1, 1915, purchased from Harold Booth, merchandise, $260.

May 7, purchased merchandise from him, $37.90; May 10, paid him for the invoice of May 1, less 1% discount, Cash $257.40, Discount $2.60; May 17, paid him for the invoice of May 7, less 1% discount, Cash $36.52, Discount $.38; May 20, purchased merchandise, $271.25.

Harold Booth

1915 May	10	Dis 2.60 Cash 257 40		260 —	1915 May	1	Mdse		260 —
	17	Dis. 38 Cash 36.52		37 90		7	Mdse		37 90
						20	Mdse		271 25

June 1, 1915, sold to B. A. Newcomber, goods amounting to $345.70; June 4, sold to Newcomber merchandise amounting to $38.45; June 11, received payment for invoice of June 1, less 2%, Cash $338.79, Discount $6.91; June 12, sold Newcomber an invoice amounting to $83.92; June 14, received payment for the invoice of the 4th, less 1%, Cash $38.07, Discount $.38.

B. A. Newcomber

Date		Sale		Cash		Discount		Balance		
1915 June	1	345	70					345	70	
"	4	38	45					384	15	
"	11			338	79	6	91	38	45	
"	12	83	92					122	97	
"	14			38	07		38	83	92	

211. Accounts Receivable and Accounts Payable. If a person's account is larger on the debit side, it shows a balance owed *by* that person; it is therefore an account receivable, and the balance is a resource.

If a person's account is larger on the credit side, it shows a balance owed *to* that person; it is therefore an account payable, and the balance is a liability.

Written Work

1. Rule a cash account and enter the following transactions; find the balance, and rule the account.

August 2, you invest $5000 in a grocery business; August 3, purchase merchandise, paying cash for the same, $1345.75; August 6, receive cash for merchandise sold, $127.50; August 10, receive cash for goods sold, $50.25; August 12, pay for advertising, $5.60; August 17, receive cash for merchandise sold, $23.74; August 18, pay cash for an invoice of merchandise, $56.35; August 21, receive cash from Henry Belmont on account, $45.80; August 23, pay cash to Oscar Haines on account, $54.85; August 31, pay store rent, $30, and clerk hire, $35.

2. Rule a personal account similar to the illustration of Harold Booth's account, page 246, and enter the following transactions:

September 3, you purchase from R. G. Henderson, on account, 2/10; N/60, an invoice of goods amounting to $237.40; September 9, purchase an invoice of goods from Henderson, $58.35, terms 1/15; N/2 months; September 11, purchase an invoice of merchandise from Henderson, $123.60, terms 2/5; N/30; September 12, pay the invoice of the 3d, less the discount; September 14, purchase goods amounting to $25; September 16, pay the invoice of the 11th, less the discount. What is the balance of Mr. Henderson's account? Is this an account receivable or an account payable? Is the balance a resource or a liability?

3. Rule an account similar to the illustration of B. A. Newcomber's account, page 246, and enter the following transactions:

July 2, sold R. S. Clark, terms 1/5; N/30, merchandise amounting to $57.82; July 5, sold Clark goods to the value of $23.95, terms 2/10; N/60; July 7, received payment for the invoice of the 2d, less the discount; July 9, sold Clark an invoice of $75, less a trade discount of 10%, terms 1/10; N/30; July 11, sold him a bill of goods amounting to $40; July 15, received cash for the invoice of the 5th, less the discount.

Is this account an account receivable or an account payable?

Is the balance a resource or a liability?

CHAPTER XXII

TAKING INVENTORY

At certain regular intervals an inventory is taken to determine the value of the stock on hand. Two clerks usually work together in taking the inventory. One counts the number of items of each kind, and reads aloud the cost price marked on the goods. The second clerk records these facts. The inventory is then sent to the office, where the value of each item is extended and the total value of the stock on hand is determined.

Inventories are entered in various forms, depending upon the details of information desired.

212. Periodic Inventories. A simple form like the following may be used:

QUANTITY	NAME OF ITEM	COST		COST EXTENSION	
19 bbl.	Herald Flour	5	60	106	40
345 #	Java Coffee		26	89	70
28½ #	Ceylon Tea		43	12	26

Written Work

Rule an inventory similar to the preceding form, enter the following items, find the cost of each item, and the total cost.

This is one of several inventory sheets used in taking stock in a grocery.

87	lb.	Mexican Java Coffee	lb.	$ .23½
136	lb.	Ceylon Tea	lb.	.42
167	pkg.	Half-pounds Ceylon Tea	lb.	.47
2	cases	Boneless Herring (4 doz.)	doz.	1.10

83	lb.	Salt Mackerel	lb.	$ .22
$6\frac{5}{12}$	doz.	Cans Corn	doz.	.85
$4\frac{11}{12}$	doz.	Cans Tomatoes	doz.	.95
$8\frac{3}{4}$	doz.	Cans Peas	doz.	1.25
$9\frac{1}{6}$	doz.	Cans Beans	doz.	1.35
$3\frac{5}{6}$	doz.	Bottles Olives	doz.	1.00
$4\frac{1}{3}$	doz.	Bottles Olives (18 oz.)	doz.	3.20
$\frac{7}{12}$	doz.	Bottles Stuffed Olives (10 oz.)	doz.	$.82\frac{1}{2}$
345	lb.	Evaporated Apricots	lb.	$.08\frac{3}{4}$
163	lb.	Evaporated Apples	lb.	$.07\frac{1}{4}$
109	lb.	Dried Prunes	lb.	$.05\frac{1}{2}$
230	lb.	Dried Peaches	lb.	$.07\frac{3}{4}$
67	lb.	Seedless Raisins	lb.	$.09\frac{1}{2}$
1	case	Currants (36 lb.)	lb.	$.08\frac{1}{2}$

A more elaborate form of inventory similar to the following may be used when desired.

Add	Deduct	Lot No.	Size	Name and Quantity	Cost Price	Cost Extension	% Dep.	Loss Assumed
		A 42	6	3 Pairs Mens Gun Metal Oxfords	2 80	8 40		
	1	A 42	6½	4 Pairs do	2 80	8 40		
4		A 42	7	5 Pairs do	2 80	25 20		
		A 42	7½	6 Pairs do	2 80	16 80		
		A 42	8	2 Pairs do	2 80	5 60		
		C 9	$\frac{1}{8}$ $\frac{2}{8\frac{1}{2}}$	3 Pairs Mens Blucher Oxfords	3 10	9 30	10%	.93

213. Explanation of Form. The Lot Number is a number given an article by either the manufacturer or the merchant. Since different styles of goods are given different lot numbers, this number may be entered in the inventory in place of a description.

The "size" column is used to show the different sizes of an article in stock. As illustrated in the model inventory, each size of an article may appear on a separate line, or all sizes may be entered on the same line. When the latter method is followed, the quantity of a certain size is indicated by writing the size below a short line, and the number of articles of that size above the line. Thus, $\frac{3}{7}$, $\frac{5}{8}$, means 3 No. 7's, 5 No. 8's.

When an error is made in the count, this error may be corrected by using the "Add" or "Deduct" columns. These columns are also used when goods are purchased or sold while the inventory is being taken. By adding goods purchased, and deducting goods sold during the taking of inventory, the value of stock at the end of the inventory may be determined.

To find the cost extension, multiply the cost by the number of items plus the number of items in the "Add" column or minus those in the "Deduct" column.

If goods become shopworn or out of style, if the market price has decreased, or if, for any other reason, the value has become less than the cost marked on the goods and entered in the inventory, it is necessary to make some allowance for this depreciation in value.

Perhaps an article is worth only half its cost price. In that case a 50% depreciation should be entered in the "% Depreciation" column. Then 50% of the cost extension for that item is entered in the "Loss Assumed" column, and the total of this column is subtracted from the total of the "Cost Extension" column to determine the actual value of the stock on hand.

Oral Work

1. What does 25% depreciation mean? 10% depreciation? 20% depreciation?

2. If stock is damaged so that it is worth only $\frac{3}{4}$ of its original cost, what per cent depreciation should be entered?

3. Goods are worth $\frac{2}{3}$ of their original cost. What is the per cent depreciation?

4. Goods are worth $\frac{4}{5}$ of their original cost. What is the per cent depreciation?

5. Goods are worth $\frac{7}{8}$ of their original cost. What is the per cent depreciation?

6. An article purchased for $3.20 can now be bought for $2.80. What per cent depreciation should be entered in the "per cent depreciation" column?

7. What would be the amount of the loss assumed on 27 of the articles mentioned in problem 6?

Written Work

Rule an inventory form similar to the one on page 250 and enter the following items which form a portion of the inventory of a shoe stock. Enter one size to a line, find the value of each item listed, and the total value of the stock shown on the page after making corrections for stock returned, for sales, and for depreciation.

Lot No. 219 B; $\frac{2}{6}$, $\frac{1}{6\frac{1}{2}}$, $\frac{3}{7}$, $\frac{4}{7\frac{1}{2}}$, $\frac{2}{8}$, $\frac{2}{8\frac{1}{2}}$. Men's Tan Oxfords, $3.15 per pair.

Lot No. 712; $\frac{2}{6}$, $\frac{3}{6\frac{1}{2}}$, $\frac{4}{7}$, $\frac{4}{7\frac{1}{2}}$, $\frac{3}{8}$, $\frac{4}{8\frac{1}{2}}$, $\frac{2}{9}$. Men's Vici Kid Bluchers, $2.90 per pair.

Lot No. 322 A; $\frac{1}{6}$, $\frac{1}{6\frac{1}{2}}$, $\frac{2}{7}$. Romeos at $1.90 per pair.

Lot No. 618 D1; $\frac{2}{7}$, $\frac{3}{7\frac{1}{2}}$, $\frac{3}{8}$, $\frac{2}{8\frac{1}{2}}$, $\frac{2}{9}$, $\frac{1}{9\frac{1}{2}}$. Men's Russian Calf Boots, $3.20 per pair.

One pair of Lot No. 712, Size 7, taken home by a customer before taking inventory, was returned after Lot No. 712 was listed.

During stock taking, the following sales were made:

1 pair Lot No. 712, size $7\frac{1}{2}$.
1 pair Lot No. 618 D1, size $7\frac{1}{2}$.
1 pair Lot No. 618 D1, size $9\frac{1}{2}$.

Lot No. 322 A can now be purchased for $1.75.

214. Perpetual Inventory. In some lines of business a perpetual inventory is kept. It is also called a Stock Record. When such a record is kept, a book is required for the purpose, one page being devoted to each item carried in stock. Both purchases and sales are recorded at cost price. The following model illustrates a form of perpetual inventory.

No. A 603 Brass Castings

Danger Point 15

Date		Purchases			Sales		Balance	
1915		No.	Cost	Value	No.	Value	No.	Value
July	8	60	18	10 80			60	10 80
"	17				14	2 52	46	8 28
"	22				26	4 68	20	3 60
"	30	80	18	14 40				18 00
Aug.	3				25	4 50		13 50

215. Value of Perpetual Inventory. Considerable labor is required to maintain a perpetual inventory, but it is valuable for several reasons.

When stock is kept in warehouses or in storerooms at some distance from the salesrooms, the perpetual inventory is a great convenience since it shows without delay the amount of stock on hand.

A "danger point" is fixed for each item. When the balance on hand has decreased to this danger point, a new supply is purchased.

An inventory may be taken at periodical intervals, and the quantities shown compared with the stock record. If the inventory and the stock record do not agree, an investigation may be made to discover the cause of the discrepancy.

Written Work

Rule a page of a stock record similar to the model on page 253, and enter the following facts, making a perpetual inventory of the stock of Solvay Coke.

Purchases

October 22, 54,200#	at $4.45 per ton
November 18, 57,600#	at $4.45 per ton
December 7, 52,800#	at $4.45 per ton

Sales

October 29, 2 tons; October 30, 5 tons; October 31, 7500#; November 2, 9500#; November 5, 7 tons; November 17, 2 tons; November 19, 12 tons; November 20, 7000#; November 23, 4 tons; November 24, 9000#; November 30, 2 tons; December 9, 13,000#; December 11, 5 tons.

CHAPTER XXIII

GROSS TRADING PROFIT

216. Definitions. Gross Trading Profit is the difference between the selling price and the cost price.

Net Profit is found by subtracting the expenses and losses of the business from the gross profit.

217. The Per Cent of Gross Trading Profit is found by dividing the gross profit by the net sales.

If goods are sold at less than their cost, a loss results. The per cent of loss is found by dividing the loss by the net sales.

Oral Work

1. A hat which cost $2 is sold for $3. What is the profit? What is the per cent of profit?

2. A set of books cost a dealer $20. In order to sell them the dealer lost $2.00. What was the per cent of loss?

3. Through an error in an advertisement a merchant was obliged to sell some parasols at $16\frac{2}{3}\%$ less than their cost. He lost 30 cents on each parasol. What did they cost, and what was the selling price? At what price should they have been sold to gain 10% of the cost?

Written Work

1. Mr. Fisher bought a carriage for $55 and sold it for $75. What was the gross profit? What was the per cent of gross profit?

2. During a certain year a merchant purchased goods which cost him $14,000. At the close of the year this stock inventoried $3000. What was the cost of the goods sold?

His sales for the year were $14,080. What was his gross profit? What was the per cent of gross profit?

3. The inventory of a dry goods merchant's stock at the beginning of the year was $3560. The purchases during the year were $28,265. At the close of the year the inventory showed a stock of $3940 unsold. What was the cost of the goods sold?

His sales for the year were $31,673. What was the gross profit? What was the per cent of gross profit?

218. Buying Expenses. In computing the cost of goods purchased, it is customary to add to the wholesale price, the freight, drayage, and all other expenses incurred in getting the goods on the shelves ready to sell. These expenses are called *buying expenses.*

Written Work

1. Mr. Tracey bought the lumber business formerly conducted by Mr. Boyce. The stock on hand inventoried $5280. Mr. Tracey's purchases during the year were $16,385. The freight was $236, the cost of labor in placing lumber in the yards and all other buying expenses $385. At the end of the year the stock inventoried $6125. What was the cost of the goods sold during the year?

If the sales during the year were $19,245, what was the gross profit and per cent of gross profit?

2. Complete the following table:

INVENTORY, JAN. 1, 19—		PURCHASES		BUYING EXPENSES		TOTAL COST OF PURCHASES		INVENTORY, DEC. 31, 19—		COST OF GOODS SOLD		SALES		PROFIT OR LOSS	PER CENT PROFIT OR LOSS
3125	75	16,294	90	370	25			4685	60			18,495	40		
5428	80	23,926	56	627	90			5626	35			33,756	20		
1290	00	6,754	00	271	25			2140	00			8,325	75		
4728	50	12,398	80	1417	25			1265	70			13,215	75		
3680	00	15,927	00	485	00			2726	00			14,850	00		

Enter profit and per cent of profit in black ink.
Enter loss and per cent of loss in red ink.

BORROWING AND LOANING

CHAPTER XXIV

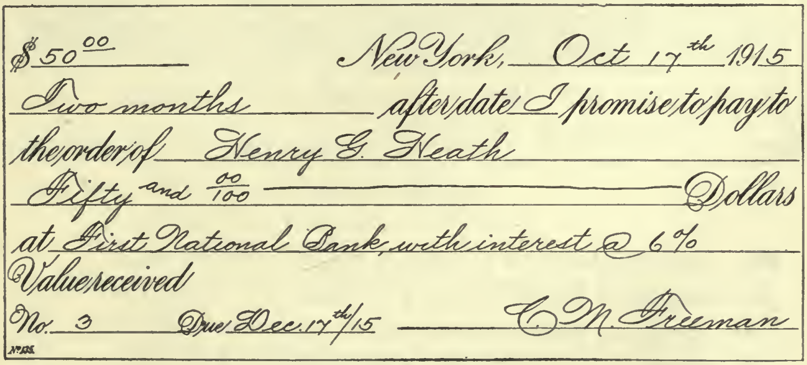
INTEREST

If you should rent a house, you would agree to pay a certain sum for the use of the house and you would probably sign an agreement to pay this rent. If you should borrow a sum of money, you would probably agree to pay the lender a certain sum for the use of the money and you would probably be required to sign an agreement to repay with interest the sum borrowed.

219. Terms Used. Money paid for the use of money is called **interest.** Interest is usually computed as a certain per cent of the amount borrowed. The per cent of interest is called the **rate** and the amount borrowed the **principal.** The statement that a man borrowed $3000 at 6 %, means that he must pay 6 % of $3000, or $180, each year for the use of the $3000. At the end of a year he would owe the lender $3000 plus the interest ($180), or $3180. The sum of the principal and the interest is called the **amount.**

220. Promissory Notes.

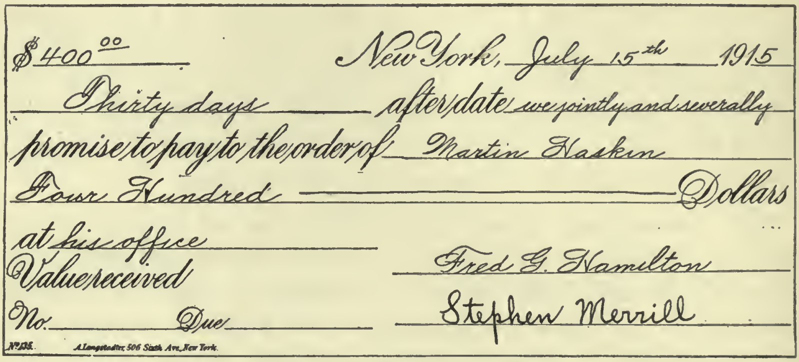
It is the custom in business for the man who borrows money to give a written promise to repay the sum borrowed. Such a promise is called a **promissory note.**



$\$50^{00}$ New York, Oct 17^{th} 1915
Two months after date I promise to pay to
the order of Henry G. Heath
Fifty and $\frac{00}{100}$ Dollars
at First National Bank, with interest @ 6%
Value received
No. 3 Due Dec. 17^{th}/15 C. M. Freeman

Promissory Note

If the borrower is a person of small means, or if, for any other reason, it may be unsafe to loan him money, he may be requested to have some person of recognized financial standing sign the note with him. Such notes are either **joint notes**, or **joint and several notes**, depending upon the wording.



$400.00 New York, July 15th 1915

Thirty days after date we jointly and severally promise to pay to the order of Martin Haskin

Four Hundred Dollars

at his office

Value received

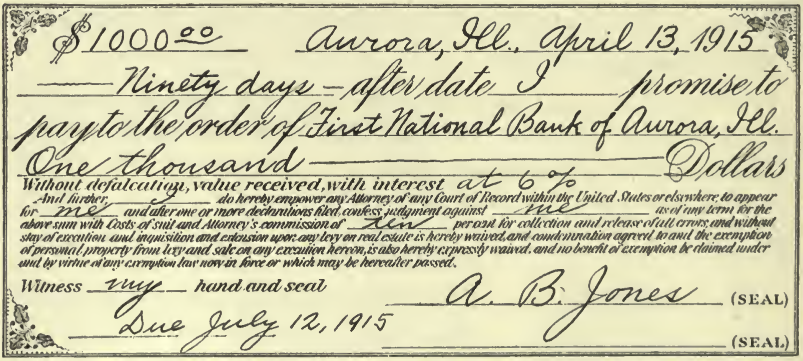
Fred G. Hamilton

No. Due

Stephen Merrill

No. 135. A. Langstadter, 506 Sixth Ave., New York.

JOINT AND SEVERAL NOTE



$1000.00 Aurora, Ill., April 13, 1915

Ninety days after date I promise to pay to the order of First National Bank of Aurora, Ill.

One thousand Dollars

Without defalcation, value received, with interest at 6 %

And further, I do hereby empower any Attorney of any Court of Record within the United States or elsewhere, to appear for me and after one or more declarations filed, confess judgment against me as of any term for the above sum with Costs of suit and Attorney's commission of ten per cent for collection and release of all errors, and without stay of execution and inquisition and extension upon any levy on real estate is hereby waived, and condemnation agreed to and the exemption of personal property from levy and sale on any execution hereon, is also hereby expressly waived, and no benefit of exemption be claimed under and by virtue of any exemption law now in force or which may be hereafter passed.

Witness my hand and seal

A. B. Jones (SEAL)

Due July 12, 1915

(SEAL)

JUDGMENT NOTE

221. To find interest at 6 % for 6, 60, 600, or 6000 days.

The interest on $1 for 1 year at 6 % is	$.06
The interest on $1 for 2 months (60 da.) is	.01
The interest on $1 for 6 days ($\frac{1}{10}$ of 60 da.) is	.001

Thus, to find the interest on $1 for 60 days at 6 %, move the decimal point two places to the left; to find the interest for 6 days, move the point three places to the left. Since this is true if the principal is $1, it will be true of any principal, and we have the following rule:

Given any principal, to find the interest at 6 %.

For 6 *days, move the decimal point three places to the left; for* 60 *days, two places to the left; and for* 600 *days, one place to the left. For* 6000 *days the interest will be the same as the principal.*

Oral Work

State the interest on each of the following principals at 6 %, for 6, 60, 600, and 6000 days:

1. $1000. **2.** $350. **3.** $125.60. **4.** $3620.
5. $12.20. **6.** $325.50. **7.** $6.

Find the interest at 6 % on:

8. $245 for 6 days. **9.** $27 for 6 days.
10. $37.50 for 60 days. **11.** $12.50 for 6 days.

222. To find interest at 6 % for fractions or multiples of 6, 60, 600, or 6000 days.

The time is not always 6, 60, or 600 days, yet the above method can be easily used when the time is an easy fraction or an exact multiple of 6, 60, 600, or 6000 days.

Examples. 1. Find the interest on $500 for 12 days at 6 %.

SOLUTION. $.50 = the interest for 6 days.
$2 \times \$.50 = \$1$, the interest for 12 days.

2. Find the interest on $700 for 40 days at 6 %.

SOLUTION. $7 = the interest for 60 days.
$\frac{2}{3}$ of $7 = $4.67, the interest for 40 days.

3. Find the interest on $300 for 3 months at 6 %.

SOLUTION. $3 = the interest for 2 months (60 days).
$\$3 \div 2 = \$1.50$, the interest for 1 month.
$\$1.50 \times 3 = \$4.50$, the interest for 3 months.

Written Work

Rule a form similar to the following model; compute the interest at 6 % on the various principals for the different times.

PRINCIPALS		24 DA.	30 DA. (1 MO.)	40 DA.	80 DA.	90 DA. (3 MO.)	18 DA.	50 DA.	70 DA.	4 MO.	6 MO.
$ 125	60										
28	95										
378	45										
500	00										
960	00										
96	00										
9	60										
475	00										
4	75										
30	00										
146	00										
419	50										
395	00										
25,000	00										

When the time is not an easy fraction or an exact multiple of 6, 60, or 600 days, an adaptation of the method shown in the last exercise may be used.

Examples. 1. Find the interest on $500 for 99 days at 6 %.

SOLUTION. $5.00 = the interest for 60 days.
2.50 = the interest for 30 days.
.50 = the interest for 6 days.
.25 = the interest for 3 days.
$8.25 = the interest for 99 days.

2. Find the interest on $1000 for 82 days at 6 %.

SOLUTION. $10.00 = the interest for 60 days.
$3.33\frac{1}{3}$ = the interest for 20 days.
$.33\frac{1}{3}$ = the interest for 2 days.
$13.67 = the interest for 82 days.

Oral Work

How would you find the interest on any principal for the following number of days at 6 %:

3, 4, 5, 7, 8, 9, 36, 66, 69?

Written Work

Enter the interest at 6% in the proper place on a form ruled like the following model:

	$845.00	$632.00	$1296.00	$5483.00	$74.60	$143.40	$746.95	$25.00	$130.00
10 da.									
66 da.									
63 da.									
68 da.									
75 da.									
80 da.									
85 da.									
96 da.									
98 da.									
45 da.									
82 da.									
28 da.									

223. To find interest at 6% for any number of days.

Sometimes the method of adding the interest for different fractions or multiples of 6 and 60 days cannot conveniently be used. In such cases

Point off three places in the principal; this will give the interest for six days. Multiply by the number of days; this will give the interest for six times the number of days. Divide by six; this will give the interest for the required number of days.

Example. Find the interest on $500 for 19 days at 6%.

SOLUTION: $.50 = the interest for 6 days.
$.50 × 19 = $9.50, the interest for 19 × 6 days.
$9.50 ÷ 6 = $1.58, the interest for 19 days.

Written Work

Enter the interest at 6% on the various principals for the different times in the proper place on a form ruled like the following model:

	$ 435.00	$ 127.90	$ 213.65	$ 472.90	$ 123.50	$ 12.35
23 da.						
29 da.						
37 da.						
19 da.						
25 da.						
73 da.						
47 da.						
81 da.						
114 da.						
95 da.						
61 da.						
77 da.						
183 da.						
211 da.						

224. Interchanging Principal and Time. There is a short method which may sometimes be used when the days are not exact multiples of 6, 60, 600, or 6000.

The interest is the same in each of the following cases:

$ 600.00 for 29 days at 6 %, and
$ 29.00 for 600 days at 6 %.

It is much easier to compute the interest in the latter case, because all that is necessary is to point off one place in the principal, giving the amount of interest immediately, $ 2.90.

Therefore, when the days are not exact multiples or easy fractions of 6, 60, 600, or 6000, and when the principal *is* an exact multiple or easy fraction of one of these numbers, interchange the number of dollars and days, and proceed as before.

Examples. 1. Find the interest on $ 360.00 for 83 days at 6 %.

Solution. Interchanging, we have $ 83.00 for 360 days at 6 %.
Pointing off two places, $ .83 interest for 60 days.
Multiply by 6 for 360 days, and the result is $ 4.98.

2. Find the interest on $ 200.00 for 47 days at 6 %.

Solution. Interchanging, we have $ 47.00 for 200 days at 6 %.
Point off one place for 600 days, $ 4.70.
Divide by 3 for 200 days, $ 1.57.

Oral Work

How would you find the interest on the following at 6 %?

1. $700.00 for 71 days.
2. $150.00 for 59 days.
3. $420.00 for 113 days.
4. $30.00 for 57 days.
5. $66.00 for 171 days.
6. $1200 for 37 days.

Written Work

Compute and enter the interest at 6 % on a form ruled like the following model:

		117 Da.		34 Da.		161 Da.		59 Da.		74 Da.		3 Mo. 7 Da.	
$ 300	00												
1200	00												
240	00												
4200	00												
150	00												
72	00												
450	00												
54	00												
6000	00												
5000	00												

225. To find the interest when the rate is other than 6 %.

First find the interest at 6 %, then take $\frac{1}{6}$ of the result; this will give the interest at 1 %. Multiply this quotient by the given rate.

The following short methods are of value in computing interest:
If the rate is 7 %, add $\frac{1}{6}$ of the interest at 6 %.
If the rate is 5 %, subtract $\frac{1}{6}$ of the interest at 6 %.
If the rate is $7\frac{1}{2}$ %, add $\frac{1}{4}$ of the interest at 6 %.
If the rate is $4\frac{1}{2}$ %, subtract $\frac{1}{4}$ of the interest at 6 %.

Oral Work

How would you find the interest at 4 %? $5\frac{1}{2}$ %? $6\frac{1}{2}$ %? 8 %? 9 %?

Written Work

Complete the following blank:

SIMPLE INTEREST

RATE	36 DA.	45 DA.	63 DA.	19 DA.	47 DA.	184 DA.	2 MO. 7 DA.
	$ 436.50	$ 75.80	$ 174.30	$ 66.00	$ 1748.60	$ 73.90	$ 400.00
6 %							
8 %							
5 %							
4½ %							
3½ %							

226. To find the interest when the time is stated in years, months, and days.

In the preceding problems the time has been stated in days and months. When the time has been stated in months, we have reduced it to days, but when the time is stated in years, months, and days, it is too laborious to reduce it to days, and the following method should be used.

Find the interest for the number of years at the given per cent. To this result add the interest for the given number of months and days.

Example Find the interest on $ 375 for 2 yr. 3 mo. 21 da. at 5 %.

SOLUTION. The interest for 1 yr. is 5 % of the principal.
The interest for 2 yr. is 10 % of the principal.
10 % of $ 375 = $ 37.50.
The interest on $ 375 for 3 mo. 21 da. is $ 5.78.
Therefore, the interest for 2 yr. 3 mo. 21 da. is $ 43.28.

Written Work

Rule a blank like the following model. Compute and enter the interest.

Complete the blank:

SIMPLE INTEREST

TIME			$124.00	$37.62	$1245.75	$463.80	$2000.00
Years	Months	Days	6%	5%	3½%	4%	5½%
1	5	24					
1	8	12					
2	4	18					
3	8	6					
4	10	12					
10	2	7					
8	6	2					
5	7	9					
3	4	15					
2	7	28					
3	3	20					
4	7	27					
9	1	19					

227. To compute the time between two dates. In all the problems given thus far, the time has been stated. You will often be called upon to compute the time. In such cases you will be given the date when the note was made and the date when it is due (date of maturity).

There are two methods of computing the time between dates:

First Method. Counting the actual number of days. (Not used if the interval is more than one year.)

Second Method. Compound subtraction. (May be used for any interval.)

a. **To find the time by counting the actual number of days.**

Example. Find the number of days between February 15, 1915, and May 24, 1915.

SOLUTION.

February has 28 days, of which 15 have expired, leaving in February	13 days
March has	31 days
April has	30 days
The note matures on May	24
Total	98 days

From this problem we derive the following rule:

Add the number of days remaining in the month after the day the note was made, the exact number of days in each complete month, and the number of days through which the note is to run in the last month.

Written Work

Find the exact number of days between the dates in the table:

Made	Due					
	Oct. 26	Nov. 13	Dec. 27	Jan. 3	Jan. 17	Feb. 26
Oct. 13						
Sept. 27						
Aug. 18						
July 16						
July 3						
May 17						
May 1						

b. **To find the interest period by compound subtraction of dates.**

Example. Find the time between May 18, 1910, and February 16, 1913.

$$\text{Solution.}\quad \begin{array}{ccc} 1913 & 2 & 16 \\ \underline{1910} & \underline{5} & \underline{18} \end{array}$$

Since 18 cannot be subtracted from 16, and since 5 cannot be subtracted from 2, reduce 1 month to days, and 1 year to months.

$$\begin{array}{ccc} 1912 & 13 & 46 \\ \underline{1910} & \underline{5} & \underline{18} \\ 2 \text{ yr.} & 8 \text{ mo.} & 28 \text{ da.} \end{array}$$

Written Work

Find the time by compound subtraction. The figures in the left-hand column are the dates on which the notes were given. The dates at the top are the dates of maturity.

From	To			
	Maturity 10–22–1913	Maturity 6–15–1914	Maturity 11–14–1914	Maturity 5–20–1915
6–17–1913				
8– 5–1913				
9–25–1912				
11– 6–1912				
4–19–1910				
2–26–1909				
4–30–1912				
7–14–1913				
10–11–1906				
12–31–1912				

The following table may be used in finding the exact number of days between any two dates:

For illustration, the time from Apr. 7 to Nov. 7 is found to be 214 da. From Sept. 3 to Jan. 3 is found to be 122 da. To find the time from Sept. 3 to Jan. 18, add 15 da. to 122 da.

From Any Day of	To the Same Day of the Next											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
January . .	365	31	59	90	120	151	181	212	243	273	304	334
February . .	334	365	28	59	89	120	150	181	212	242	273	303
March . . .	306	337	365	31	61	92	122	153	184	214	245	275
April . . .	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June . . .	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
August . . .	153	184	212	243	273	304	334	365	31	61	92	122
September . .	122	153	181	212	242	273	303	334	365	30	61	91
October . . .	92	123	151	182	212	243	273	304	335	365	31	61
November . .	61	92	120	151	181	212	242	273	304	334	365	30
December . .	31	62	90	121	151	182	212	243	274	304	335	365

Review

1. In the following exercise, enter the time (in red ink[1]) on the upper of the two lines, and the interest (in black ink) on the lower line.

---

[1] At teacher's discretion.

If the time is less than a year, compute the actual number of days.

If the time is more than a year, find the interval by compound subtraction, and state it in years, months, and days.

From	Rate 6%			
	$216.80	$625.00	$923.40	$87.86
	To			
	9–29–1914	11–26–1914	12–24–1914	1–26–1915
9–22–1914				
6–24–1914				
4– 2–1914				
2–11–1914				
11–13–1913				
8– 6–1913				
5–22–1913				
4–25–1912				
7–13–1910				

2. Complete the following, entering the time on the upper of the two lines, and the interest on the lower.

From	5%	4½%	7%	7½%
	To			
	May 26, 1914	Aug. 17, 1914	Oct. 9, 1914	Feb. 10, 1915
Principal, $219.80 April 22, 1914				
Principal, $326.00 May 17, 1914				
Principal, $80.00 September 19, 1913				
Principal, $260.00 May 14, 1913				

Written Review

Use the simplest method to find the interest on the following:

1. Interest at 6 % on $321.65 for 24 da.
2. Interest at 6 % on $523.19 for 20 da.
3. Interest at 6 % on $413.00 for 75 da.
4. Interest at 6 % on $300.00 for 57 da.
5. Interest at 5 % on $450.00 for 35 da.
6. Interest at $7\frac{1}{2}$ % on $216.80 for 42 da.
7. Interest at 4 % on $625.00 for 117 da.
8. Interest at 6 % on $80.00 for 3 yr. 8 mo. 15 da.
9. Interest at 6 % on $50.00 from March 24, 1914, to May 26, 1914.
10. Interest at 7 % on $125.00 from July 19, 1911, to January 14, 1915.

228. Accurate Interest. The methods of computing interest already explained are based on the assumption that a year contains 360 days. The slight error is ignored because of the convenience of the method. However, the United States government and some business houses prefer to use exact interest.

When we compute interest on a basis of 360 days to the year, we ignore 5 days. Five days are $\frac{5}{365}$ or $\frac{1}{73}$ of a year. When, for example, we call 180 days, $\frac{180}{360}$ or $\frac{1}{2}$ a year, we obtain a result which is $\frac{1}{73}$ too large. Since accurate interest is $\frac{1}{73}$ smaller than the interest computed by the simple methods, we can find accurate interest by the following rule:

To find accurate interest.

Find the interest by one of the common methods; diminish this result by $\frac{1}{73}$ *of itself.*

Example. What is the accurate interest on $3500.00 for 15 days at 6 %?

Solution. $35.00 is the interest for 60 days.
$35.00 ÷ 4 = $8.75, the interest for 15 days.
$\frac{1}{73}$ of $8.75 = $.12.
$8.75 − $.12 = $8.63, the accurate interest.

Written Work

Find the accurate interest on the following:

1. $584.00 for 24 days at 6%.
2. $275.00 for 84 days at 6%.
3. $23.75 for 113 days at 5%.
4. $65.80 for 2 months 12 days at 6%.
5. $138.50 for 9 months 16 days at 7%.

229. To find the time. Situations sometimes arise in which it is necessary to determine the time, when the principal, the rate, and the interest are known.

Oral Work

The interest on $250.00 for 6 days, at 6%, is $.25. $1.00 is the interest for how many days at 6%?

The interest on $1200 for 1 day at 6% is $.20. $4.40 is the interest for what length of time at 6%?

From these problems the following rule will be understood.

To find the time when the principle, rate, and interest are known. *Find the interest on the given principal at the given rate for one day. Divide the total interest by the interest for one day to determine the number of days.*

Example. The interest on $4500.00 at 6% for a certain time is $10.50. What is the time?

Solution. The interest on $4500.00 for 1 day at 6% is $.75.
$10.50 ÷ $.75 = 14, the number of days.

Written Work

Find the time in each of the following:

	Principal	Rate	Interest	Time
1.	$ 360.00	6%	$ 1.62	
2.	$1500.00	6%	$ 9.75	
3.	$1473.00	6%	$22.59	
4.	$ 279.80	6%	$ 1.91	
5.	$ 176.90	5%	$ 2.82	

230. To find the principal.

The interest on \$1.00 for 72 days at 6 % is \$ .012. \$ .24 is the interest on how many dollars for 72 days at 6 %?

To find the principal when the rate, the time, and the interest are known.

Find the interest on \$1.00 *for the given time at the given rate. Divide the given interest by the interest on* \$1.00 *to determine the principal.*

Example. The interest on a certain principal at 6 % for 54 days is \$2.48. What is the principal?

Solution. The interest on \$1.00 for 54 days at 6 % is \$ .009.
\$2.48 ÷ \$ .009 = 275.56, the number of dollars in the principal

Written Work

Find the principal in each of the following:

	Time	Rate	Interest	Principal
1.	75 days	6 %	\$ 3.36	
2.	35 days	6 %	\$ 1.67	
3.	92 days	5 %	\$ .52	
4.	72 days	6 %	\$21.60	
5.	90 days	5 %	\$45.00	
6.	60 days	5 %	\$ 1.00	
7.	30 days	7 %	\$ 1.40	
8.	90 days	6 %	\$13.50	

231. To find the rate when the principal, the interest, and the time are given.

Find the interest on the given principal for the given time at 1 %. *Divide the total interest by the interest at* 1 % *to find the rate.*

Example. The interest on \$1500.00 for 30 days at a certain rate is \$6.25. What is the rate?

Solution. The interest on \$1500.00 for 30 days at 1 % is \$1.25.
\$6.25 ÷ \$1.25 = 5, the rate per cent.

Written Work

Find the rate in each of the following:

	Principal	Time	Interest	Rate
1.	$1375.00	42 days	$ 9.63	
2.	$ 876.25	95 days	$11.56	
3.	$ 694.75	27 days	$ 3.65	
4.	$ 940.00	2 yr. 6 mo.	$94.00	

232. Periodic Interest. Periodic interest is the simple interest on the principal, plus the simple interest on each installment of interest which was not paid when due. The interest on the principal may be due at any stated interval, usually quarterly, semiannually, or annually.

Periodic interest is not legal in some states. Special contracts are sometimes made which provide for its payment.

Example. What is the periodic interest on a loan of $5000.00 at 6% for 1 yr. 6 mo., interest due quarterly?

SOLUTION. The simple interest on $5000.00 for 1 yr. 6 mo. at 6% is $450.00.

The first interest payment of $75 is due at the end of 3 mo., and remains unpaid for	1 yr. 3 mo.
The second interest payment is due in 6 mo. and is unpaid for	1 yr.
The third interest payment is due in 9 mo. and is unpaid for	9 mo.
Fourth interest payment is unpaid for	6 mo.
Fifth interest payment is unpaid for	3 mo.
The sixth interest payment is due at the maturity of the note, and is paid when due.	
The total time during which interest payments remain unpaid is	3 yr. 9 mo.

The interest on $75.00 for 3 yr. 9 mo. at 6% is $16.88

Simple interest on the principal = $450.00

Interest on unpaid interest = 16.88

Periodic interest = $466.88

The amount due at the end of 1 yr. 6 mo. (principal and interest) is $5466.88.

Written Work

Complete a form ruled like the following model. The results in the illustration are entered in the first column to illustrate how to fill in the various blanks.

	Principal $5000 Rate 6% Time 1 Yr. 6 Mo. Int. Due Quarterly	Principal $2000 Rate 6% Time 2 Yr. 8 Mo. Int. Due Quarterly	Principal $1200 Rate 6% Time 4 Yr. Int. Due Semi-annually	Principal $1700 Rate 6% Time 6 Yr. 6 Mo. Int. Due Semi-annually	Principal $6500 Rate 7% Time 1 Yr. 9 Mo. Int. Due Quarterly	Principal $1500 Rate 5% Time 2 Yr. Int. Due Annually	Principal $3000 Rate 8% Time 4 Yr. Int. Due Quarterly
Simple interest . . .	$ 450.00						
Amount of each interest payment . . .	75.00						
Sum of periods interest is unpaid	3 yr. 9 mo.						
Interest on unpaid interest	$ 16.88						
Total interest . . .	466.88						
Amount of note at maturity	5466.88						

CHAPTER XXV

PARTIAL PAYMENTS

PART payment of an interest-bearing debt is known as a "partial payment." Such payments should be recorded on the back of the note.

One of the following methods is usually employed in computing the balance remaining after partial payments have been made.

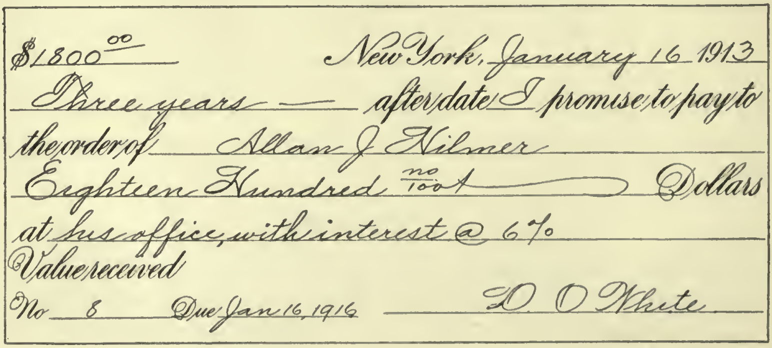
233. The United States Rule. The United States Rule has been approved by the Supreme Court of the United States, and is usually employed when the time of the interest-bearing debt exceeds one year. This method provides that:

a. Payments must be applied to pay accrued interest before any deduction can be made from the principal.

b. Payments which do not pay the accrued interest leave the principal undiminished until other payments are made which are sufficient to cover *all* accrued interest.

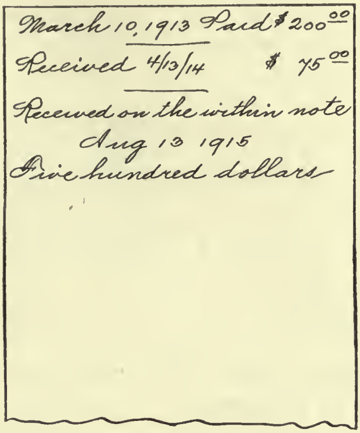
c. Any surplus remaining after accrued interest is paid in full is applied toward the reduction of the principal.

Partial payments may be made only by agreement between the maker and the payee. This agreement may appear in the note.



$1800 00 New York, January 16 1913
Three years — after date I promise to pay to
the order of Allan J Hilmer
Eighteen Hundred no/100 Dollars
at his office, with interest @ 6%
Value received
No 8 Due Jan 16, 1916 D. O White

The above note had the following indorsements, showing payments made with the consent of the payee.



March 10, 1913 Paid $\$200^{00}$

Received 4/13/14 $\$\ 75^{00}$

Received on the within note
Aug 13 1915
Five hundred dollars

Example. What amount was necessary to pay the note and interest at maturity?

Solution. Time from January 16, 1913, to March 10, 1913, 1 mo. 24 da.

Interest on $1800.00 for 1 mo. 24 da. is $16.20.

$200.00 (first payment) – $16.20 (accrued interest) = $183.80, surplus to apply on principal.

$1800.00 – $183.80 = $1616.20, unpaid principal.

Time from March 10, 1913, to April 13, 1914, is 1 yr. 1 mo. 3 da.

Interest on $1616.20 for 1 yr. 1 mo. 3 da. is $105.86.

The payment of $75.00 does not cover the accrued interest, and the principal therefore remains undiminished.

Time from April 13, 1914, to August 13, 1915, is 1 yr. 4 mo.

Interest on $1616.20 for 1 yr. 4 mo. is $129.30.

$105.86 + $129.30 = $235.16, total accrued interest.

$75.00 + $500.00 = $575.00, total payments.

$575.00 – $235.16 = $339.84, surplus remaining to apply on principal.

$1616.20 – $339.84 = $1276.36, unpaid principal.

Time from August 13, 1915, to January 16, 1916, is 5 mo. 3 da.

Interest on $1276.36 for 5 mo. 3 da. is $32.55.

$1276.36 + $32.55 = $1308.91, amount required to pay principal and accrued interest at maturity.

The work may be arranged in tabular form, as shown by the following illustration :

PARTIAL PAYMENTS ON A NOTE COMPUTED BY THE UNITED STATES RULE

Date	Accrued Interest					Payment		Surplus to apply on Principal		Principal	
	Time			Amount							
	Yr.	Mo.	Da.								
Of Note, 1913–1–16										1800	00
1st Payment, 1913–3–10		1	24	16	20	200	00	183	80	1616	20
2d Payment, 1914–4–13	1	1	3	105	86	75	00				
3d Payment, 1915–8–13	1	4	0	129	30	500	00				
				235	16	575	00	339	84	1276	36
Maturity, 1916–1–16		5	3	32	55					1308	91
										Amount at matu	due rity

234. The Merchants' Rule. The Merchants' Rule is generally used by banks and by business men to find the balance due when partial payments are made on interest-bearing notes in which the time is one year or less. It provides that:

a. The principal shall draw interest from the date the loan is made until the date of the final settlement.

b. Each payment shall draw interest from the date the payment is made until the final settlement.

Example. On a note of $500.00, dated July 8, 1914, payable in one year with interest at 6%, the following payments were made: Sept. 15, 1914, $200.00; Jan. 8, 1915, $150.00; May 8, 1915, $60.00. What amount was necessary to make final settlement on July 8, 1915?

SOLUTION. Principal		$500.00	
Int. on $500.00 for 1 yr. at 6%		30.00	
Total			$530.00
First payment	$200.00		
Int. on first payment from 9–15–14 to 7–8–15	9.78		
Total		$209.78	
Second payment	$150.00		
Int. on second payment from 1–8–14 to 7–8–15	4.50		
Total		154.50	
Third payment	$60.00		
Int. on third payment from 5–8–14 to 7–8–15	.60		
Total		60.60	
Sum of payments and interest			424.88
Balance due July 8, 1915			$105.12

This work may be arranged in tabular form as shown in the following illustration:

TABULAR FORM FOR INDICATING RESULTS OF PARTIAL PAYMENTS COMPUTED BY THE MERCHANTS' RULE

DATES		LOAN						PARTIAL PAYMENTS							
		Principal		Interest		Amount		Payment		Interest				Amount of Payment and Interest	
										Time mo.	da.	Dollars			
Of note	7–8–1914	500	00												
Maturity	7–8–1915			30	00	530	00								
First payment	9–15–1914							200	00	9	25	9	78	209	78
Second payment	1–8–1915							150	00	6	0	4	50	154	50
Third payment	5–8–1915							60	00	2	0		60	60	60
														424	88
								Bal.	due	at fi	nal	settl'	m't	105	12
						530	00							530	00

Written Work

The first two problems, being for long-time loans, should be computed by the United States Rule. Prepare a ruled form similar to the model and show your results thereon. In each problem, find the balance due at maturity.

1. Principal, $4000.00.

Date of paper, November 16, 1916.
Time, 4 yr.
Rate, 6%.

Payments:

Jan. 20, 1917,	$ 75.00
Sept. 26, 1917,	125.00
April 5, 1918,	500.00
Oct. 21, 1918,	100.00
July 10, 1919,	250.00
Oct. 21, 1919,	1500.00

2. Principal, $1200.00.
Date of paper, October 29, 1917.
Time, 2 yr. 6 mo.
Rate, 6%.
Payments:

Jan. 16, 1918,	$ 60.00
Nov. 6, 1918,	25.00
March 12, 1919,	165.00
July 10, 1919,	15.00
Sept. 28, 1919,	150.00
Dec. 3, 1919,	275.00

The following problems, being for short-time loans, should be computed by the Merchants' Rule. Rule a form similar to the model.

3. Principal, $600.00.
Date of paper, September 15, 1917.
Time, 1 yr.
Rate, 6%.
Payments:

Oct. 20, 1918,	$ 75.00
Nov. 11, 1918,	60.00
Feb. 16, 1919,	115.00

4. Principal, $1000.00.
Date of paper, March 19, 1917.
Time, 6 mo.
Rate, 5%.
Payments:

April 21,	$ 65.00
May 27,	125.00
June 24,	180.00
Aug. 19,	200.00

CHAPTER XXVI

COMPOUND INTEREST

235. Compound Interest. Compound interest is interest computed, at regular intervals, on the sum of the principal and any unpaid interest. In other words, as soon as interest becomes due and is unpaid, it begins to draw interest at the same rate as the principal. Compound interest is generally paid on the deposits in savings banks and is used in calculating sinking funds.

Interest may be compounded quarterly, semi-annually, annually, or at the end of any other period agreed upon. In some states the collection of compound interest is not permitted.

Example. Find the amount and the compound interest of \$1200 at 6% for two years, interest compounded semi-annually.

SOLUTION.	\$1200.00	First principal
	36.	Interest for 6 months
	1236.	Principal at beginning of second 6-months period
	37.08	Interest for second 6 months
	1273.08	Principal at beginning of third period
	38.19	Interest for third period
	1311.27	Principal at beginning of fourth period
	39.34	Interest for fourth period
	\$1350.61	Amount at end of two years
	\$1350.61	Amount at end of two years
	1200.00	Principal
	\$150.61	Compound interest

Written Work

1. Find the compound interest on \$500.00 for 5 years at 6%, interest compounded annually.

2. What is the amount and the compound interest on \$7500.00 loaned for 3 years at 6%, interest compounded semi-annually?

3. Find the interest on \$1200.00 loaned for 2 years at 5% compound interest, interest compounded quarterly.

Compound interest is computed with much less work by the use of tables showing the accumulation of interest and principal when $1.00 is loaned at compound interest.

236. Compound Interest Table.

Showing the amount of $1.00 compounded annually at various rates

YEARS	1 %	1½ %	2 %	2½ %	3 %	3½ %	4 %	4½ %	5 %	6 %	YEARS
1	0.010000	1.005000	1.020000	1.025000	1.030000	1.035000	1.040000	1.045000	1.050000	1.060000	1
2	1.020100	1.030225	1.040400	1.050625	1.060900	1.071225	1.081600	1.092025	1.102500	1.123600	2
3	1.030301	1.045678	1.061208	1.076891	1.092727	1.108718	1.124864	1.141166	1.157625	1.191016	3
4	1.040604	1.061364	1.082432	1.103813	1.125509	1.147523	1.169859	1.192519	1.215506	1.262477	4
5	1.051010	1.077284	1.104081	1.131408	1.159274	1.187686	1.216653	1.246182	1.276282	1.338226	5
6	1.061520	1.093443	1.126162	1.159693	1.194052	1.229255	1.265319	1.302260	1.340096	1.418519	6
7	1.072135	1.109845	1.148686	1.188686	1.229874	1.272279	1.315932	1.360862	1.407100	1.503630	7
8	1.082857	1.126493	1.171659	1.218403	1.266770	1.316809	1.368569	1.422101	1.477455	1.593848	8
9	1.093685	1.143390	1.195093	1.248863	1.304773	1.362897	1.423312	1.486095	1.551328	1.689479	9
10	1.104622	1.160541	1.218994	1.280085	1.343916	1.410599	1.480244	1.552969	1.628895	1.790848	10
11	1.115668	1.177949	1.243374	1.312087	1.384234	1.459970	1.539454	1 622853	1.710339	1.898299	11
12	1.126825	1.195618	1.268242	1.344889	1.425761	1.511069	1.601032	1.695881	1.795856	2.012197	12
13	1.138093	1.213552	1.293607	1.378511	1.468534	1.563956	1.665074	1.772196	1.885649	2.132928	13
14	1.149474	1.231756	1.319479	1.412974	1.512590	1.618695	1.731676	1.851945	1.979932	2.260904	14
15	1.160969	1.250232	1.345868	1.448298	1.557967	1.675349	1.800944	1.935282	2.078928	2.396558	15
16	1.172579	1.268986	1.372786	1.484506	1.604706	1.733986	1.872981	2.022370	2.182875	2.540352	16
17	1.184304	1.288020	1.400241	1.521618	1.652848	1.794676	1.947901	2.113377	2.292018	2.692773	17
18	1.196148	1.307341	1.428246	1.559659	1.702433	1.857489	2.025817	2.208479	2.406619	2.854339	18
19	1.208109	1.326951	1.456811	1.598650	1.753506	1.922501	2.106849	2.307860	2.526950	3.025600	19
20	1.220190	1.346855	1.485947	1.638616	1.806111	1.989789	2.191123	2.411714	2.653298	3.207136	20
21	1.232392	1.367058	1.515666	1.679582	1.860295	2.059431	2.278768	2.520241	2.785963	3.399564	21
22	1.244716	1.387564	1.545980	1.721571	1.916103	2.131512	2.369919	2.633652	2.925261	3.603537	22
23	1.257163	1.408377	1.576899	1.764611	1.973587	2.206114	2 464716	2.752166	3.071524	3.819750	23
24	1.269735	1.429503	1.608437	1.808726	1.032794	2.283328	2.563304	2.876014	3.225100	4.048935	24
25	1.282432	1.450945	1.640606	1.853944	1.093778	2.363245	2.665836	3.005434	3.386355	4.291871	25

Example. Find the compound interest on $3562.80 for 4 years at 6 %, interest compounded annually.

SOLUTION. $1.00 compounded annually at 6 % for 4 years amounts to $1.262477, as shown by the table.

3562.80 × $1.262477 = $4497.95, amount of $3562.80 compounded annually for 4 years at 6 %.

$4497.95 − $3562.80 = $935.15, compound interest.

When interest is compounded semi-annually, take ½ the rate for twice the time.

When interest is compounded quarterly, take $\frac{1}{4}$ the rate for 4 times the time.

Example. What is the compound interest on $5000.00 at 8% for 3 years 6 months, interest compounded quarterly?

Solution. $\frac{1}{4}$ of 8% = 2%.
4 times 3 years 6 mo. = 14 years.
The amount of $1.00 compounded at 2% for 14 years is $1.319479.
5000 × $1.319479 = $6597.40.
$6597.40 − $5000.00 = $1597.40, compound interest.

Written Work

1. Find the compound interest on $2500.00 for 5 years at 5%, interest compounded annually.

2. $5000.00 was invested at 4%, interest compounded semi-annually. To what sum did this investment amount in 7 years?

3. What was the compound interest on a loan of $3750.00, made June 1, 1909, and due December 1, 1914, at 6%, interest compounded quarterly?

4. What sum must be invested at 5% compound interest to amount, in 7 years, to $3249.84, if the interest is compounded semi-annually?

CHAPTER XXVII

SAVINGS BANKS

237. Checking and Savings Accounts. Most banks receive deposits under two classes of accounts, checking accounts and savings accounts.

Money deposited in checking accounts can be drawn out by checks payable to the depositor or to any other party. These checks are payable on presentation at the bank. The cash on deposit usually does not bear interest.

Money deposited in a savings account cannot be drawn out by a check payable to any person other than the depositor himself. The law usually provides that the bank may require notice of from 10 to 60 days before paying money from a savings account. Banks rarely take advantage of this privilege, however. The cash on deposit bears interest at some rate fixed by the bank. 3%, $3\frac{1}{2}$%, and 4% are common rates.

238. Computing Interest on Savings Accounts. Interest is computed on the **smallest balance** which the depositor leaves in the bank during the entire time between fixed days. These fixed days are called **interest days**, and the time between the interest days is called the **interest term**.

Some banks pay interest on dollars only, and ignore the cents. Interest which is not withdrawn by the depositor is added to his account, and draws interest the same as a deposit. Thus, savings banks really pay compound interest.

Example. The interest days of the Snowden Savings Bank are January 1, April 1, July 1, and October 1. On each of these interest days, interest at the rate of 4% per annum is computed on the smallest quarterly balance. The account of W. M. Scott is shown on the following page.

W. M. Scott

Date		Deposits		Interest		Withdrawals		Balance	
1915									
Feb.	7	120	00					120	00
April	20	300	00					420	00
April	30					100	00	320	00
July	1			1	20			321	20
July	15	50	00					371	20
Sept.	17	75	00					446	20
Oct.	1			3	21			449	41
Nov.	16					80	00	369	41
1916									
Jan.	1			3	69			373	10

Explanation. The first interest term was from Jan. 1 to April 1, but since Mr. Scott made no deposit until Feb. 7, the smallest balance on deposit during the entire interest term was $0.00, and, therefore, no interest was added on April 1.

The second interest term was from April 1 to July 1. The smallest balance on deposit during the entire term was $120.00 (the balance on April 1). Interest, $1.20.

The third interest term was from July 1 to Oct. 1. Smallest balance, $321.20. Interest, $3.21.

The fourth interest term was from Oct. 1 to Jan. 1. Smallest balance, $369.41. Interest, $3.69.

239. Interest Terms and Dates of Adding Interest. Interest terms are not of uniform length among the various banks of the country. Some banks compute interest on monthly balances, some on quarterly balances, and some on semi-annual balances.

Some banks add interest quarterly and some add it semi-annually. The rules applying to the interest computations of any bank may usually be found in the by-laws of the bank, printed in the depositor's pass book.

Examples. 1. Suppose the Snowden Savings Bank computed interest on quarterly balances, but added interest semi-annually. W. M. Scott's account would appear as follows:

W. M. SCOTT

DATE		DEPOSITS		INTEREST		WITHDRAWALS		BALANCE	
1915									
Feb.	7	120	00					20	00
April	20	300	00					420	00
April	30					100	00	320	00
July	1			1	20			321	20
July	15	50	00					371	20
Sept.	17	75	00					446	20
Nov.	16					80	00	366	20
1916									
Jan.	1			6	87			373	07

		SMALLEST BALANCE	QUARTERLY INTEREST	SEMI-ANNUAL DIVIDEND OF INT.
EXPLANATION.	First Quarter	$ 0.00	$0.00	
	Second Quarter	120.00	1.20	$1.20
	Third Quarter	321.20	3.21	
	Fourth Quarter	366.20	3.66	6.87

2. Suppose the Snowden Savings Bank computed interest on the monthly balance and added interest quarterly. Scott's account would appear as follows:

EXPLANATION.

MONTH	MONTHLY BALANCE		1 MONTH INTEREST $\frac{1}{3}$ OF 1%		QUARTERLY DIVIDEND OF INTEREST	
Feb.	$000	00	$0	00		
March	120	00		40	$0	40
April	120	40		40		
May	320	40	1	06		
June	320	40	1	06	2	52
July	322	92	1	07		
Aug.	372	92	1	24		
Sept.	372	92	1	24	3	55
Oct.	451	47	1	50		
Nov.	371	47	1	23		
Dec.	371	47	1	23	3	96

In computing interest on savings accounts, cents in the principal are dropped. Fractions of cents in interest are dropped.

Written Work

1. Check the computations in each of the illustrations above.

2. Prepare W. M. Scott's account on the supposition that the bank computes interest on monthly balances and adds interest semi-annually, Jan. 1 and July 1.

3. Prepare W. M. Scott's account on the supposition that the bank computes interest on semi-annual balances and adds interest semi-annually. Remember that interest is paid on the smallest balance on deposit during the *entire* interest term.

4. Which of the methods employed in this section is most profitable for the depositor?

5. Suppose you deposited $30 in a savings bank and left it to draw compound interest for 5 years at 4%, interest compounded semi-annually. How much could you withdraw at the end of the fifth year?

6. Suppose you had loaned the $30 for 5 years at 4% simple interest. What would the interest have been for five years?

7. Compare the simple and compound interest in Problems 5 and 6.

8. A man deposits $1200 in a savings bank to be used to send his boy to college. The savings bank pays 4% interest, compounded semi-annually. The deposit was made on the boy's tenth birthday. What amount will he have to his credit on the boy's eighteenth birthday if no withdrawals and no other deposits are made?

9. Make accounts with D. O. Dorman; one for each of the following methods of declaring interest. Rate 4%.

a. Interest computed on monthly balances. Dividends added Jan. 1, April 1, July 1, and Oct. 1.

b. Interest computed on quarterly balances. Interest days same as above.

c. Interest computed on quarterly balances. Dividends added Jan. 1 and July 1.

Date	Deposits	Withdrawals
1915		
Jan. 16	$500.00	
Feb. 19	700.00	
Mar. 1		$ 25.00
Mar. 8	60.00	
June 10	125.00	
July 19		60.00
Aug. 5	200.00	
Oct. 19	145.00	
Nov. 20		125.00

240. Postal Savings Banks. The United States Government has provided a savings bank system operated in conjunction with the postal service, whereby savings may be deposited at interest with the security of the United States Government.

Deposits. Any person ten years of age or over may become a depositor. Sums less than $1.00 cannot be deposited. No person can deposit more than $100.00 in any one calendar month, nor have a balance at any time of more than $500.00, exclusive of interest. Depositors receive a postal savings certificate for the amount of each deposit. Interest is allowed on these certificates at the rate of two per cent for each *full year* that the money remains on deposit, *beginning with the first day of the month following the one in which it was deposited.*

Withdrawals. Money may be withdrawn by surrendering to the officer where the deposit was made the savings certificates covering the desired amount.

Bonds. Under certain conditions a depositor may surrender certificates in amounts of $20.00 or any multiple of $20.00, up to and including $500.00 and receive in return government bonds bearing interest at $2\frac{1}{2}\%$ per year. These postal savings bonds may be held in addition to the $500.00 deposit allowed to one depositor.

Written Work

1. How much interest would a man receive if he deposited $15.00 in a postal savings bank on March 1, 1915, and withdrew it three months later?

2. How much interest would he receive if he withdrew the money on March 11, 1916?

3. A man made the following deposits in a postal savings bank:

May 20, 1915	$15.00
June 17, 1915	6.00
August 11, 1915	26.00
October 29, 1915	17.00

When would he be entitled to the full year's interest on each of his deposits, and how much interest would he receive all together?

CHAPTER XXVIII

CONTRACT PURCHASES AND INSTALLMENT PAYMENTS

REAL estate and other forms of property are often purchased and sold on contracts. Such contracts usually specify that the purchaser shall make periodic payments of a stated sum in payment of accrued interest and principal. The contract may permit the purchaser to pay, if he desires, a larger sum than that specified by the contract.

241. Applying the Payment to reduce the Principal. Contracts vary widely in the provisions for reducing the principal. The following typical cases will illustrate :

a. C borrows $1000.00 from D on May 1, 1915 at 6 %, under a contract to pay $25.00 on the first day of each month. Each payment of $25.00 to be applied as follows :

1. To pay the interest on the unpaid principal for the preceding month ;

2. The balance of the payment to be applied immediately to the reduction of the principal.

A record of the payments might be kept in the following manner :

DATE	PAYMENT	ACCRUED INTEREST	BALANCE OF PAYMENT TO APPLY ON PRINCIPAL	PRINCIPAL
May 1, 1915				$1000 00
June 1, 1915	$25 00	$5 00	$20 00	980 00
July 1, 1915	25 00	4 90	20 10	959 90
Aug. 1, 1915	25 00	4 80	20 20	939 70

b. On September 1, 1915, M sells a house to N for $6400.00. N pays cash $2500.00 and signs a contract agreeing to pay installments of $35.00 monthly, with the privilege of paying larger amounts. Each payment is to be applied as follows:

At the expiration of every quarter (three months) the total accrued interest at 6% is to be deducted from the total payments, and the balance applied to the reduction of the principal.

A record of the payments might be kept in the following manner:

Date	Payments	Total Payments	Accrued Interest	Balance to Apply on Principal	Principal
Sept. 1, 1915					$3900.00
Oct. 1, 1915	$35.00				
Nov. 2, 1915	45.00				
Dec. 1, 1915	35.00	$115.00	$58.50	$56.50	3843.50
Jan. 2, 1916	60.00				
Feb. 1, 1916	35.00				
March 1, 1916	40.00	135.00	57.65	77.35	3766.15

Written Work

1. Check the items in the above form.

2. On May 1, 1917, G. A. Sanders sold a house and lot to A. R. Crain; purchase price, $4850.00. Mr. Crain paid $2900.00 cash and gave the following note for the balance.

$1950.00 BARTLETT, IOWA, May 1, 1917.

FOR VALUE RECEIVED, I promise to pay to the order of G. A. Sanders, the sum of Nineteen Hundred and Fifty Dollars, with interest thereon at the rate of six per cent per annum, principal and interest payable in monthly installments in the sum of twenty-five dollars or more, each, the first of which is due on June 1, 1917, and one on the first day of each and every month thereafter until said principal sum of Nineteen Hundred and Fifty Dollars and interest thereon are fully paid.

Each and every of said installments is to be applied as follows:

1st. In payment of accrued interest on said sum of $1950.00 or on the unpaid portion thereof.

2d. In monthly reduction of said principal sum of $1950.00.

A. R. CRAIN.

Payments were made as follows:

June 2, 1917	$25.00	January 2, 1918	$40.00
July 1, 1917	25.00	February 2, 1918	25.00
August 1, 1917	30.00	March 2, 1918	32.50
September 1, 1917	25.00	April 1, 1918	25.00
October 1, 1917	27.50	May 1, 1918	45.00
November 1, 1917	35.00	June 1, 1918	25.00
December 1, 1917	25.00		

Notice that some payments are made on the second day of the month, due to the fact that the first day of the month was either a Sunday or a holiday.

Prepare a statement similar to the illustration on page 288, showing a record of the payments made.

3. Assume that the contract between Mr. Sanders and Mr. Crain called for a monthly payment of $25.00 or more, and that the principal was to be decreased semi-annually by the balance of the payments remaining after paying all accrued interest. Prepare a statement similar to the illustration on page 289, showing a record of the payments made.

4. Which of the two propositions is better for the purchaser? Explain.

CHAPTER XXIX

DISCOUNTING NOTES AND OTHER COMMERCIAL PAPER

242. Borrowing from a Bank. Banks prefer to have the borrower pay the interest in advance. The interest may be paid in cash, or the bank may subtract it from the face of the note, giving the borrower the difference, which is called the **proceeds.**

If you gave a bank the following note:

$100.00 CHICAGO, ILL., March, 2, 1915.

Ninety days after date, I promise to pay to the order of the Bowen National Bank One Hundred Dollars.

Your Signature

the bank would deduct ninety days' interest, $1.50, and you would receive the proceeds, $98.50.

NOTE. There is no promise to pay interest, because the interest is prepaid. Banks charge any rate they choose, within the limits set by the state law. In this text, when no rate is mentioned use 6 %.

The above illustration shows the method of **discounting a note.**

243. Terms Used in Bank Discount. The following terms are frequently used, and you should become familiar with them:

Maturity means the date on which the note is due.

The **value** of the note is the amount due at maturity.

If the note draws interest, the *value* is the sum of the principal and the interest. Otherwise, the *value* is the face of the note.

The **discount period** is the exact number of days between the date of discounting the paper and its maturity.

The **bank discount** is the simple interest on the value for the discount period.

The difference between the value and the bank discount is called the *proceeds.*

Oral Work

In the illustration on the previous page what is:

The value? The maturity? The discount period?
The bank discount? The proceeds?

244. To find the bank discount and the proceeds.

To compute the bank discount, compute the simple interest on the value of the note for the discount period.

To find the proceeds, subtract the bank discount from the value of the note.

Written Work

How much would you receive from the bank if you discounted the following notes?

1. A note for $500, without interest, due in 30 days, discounted at 6 %.

2. A note for $725, without interest, due in two months, discounted at 6 %.

3. A note for $85.50, without interest, due in 90 days, discounted at 7 %.

4. The following blank shows the value of several notes and the periods for which they were discounted. Compute the bank discount at 6 %, and find the proceeds.

Note Due In		Value						
		$87.00	**$265.00**	**$962.50**	**$1000.00**	**$275.00**	**$765.00**	**$35.00**
30 days	Bank discount	.44						
	Proceeds	86.56						
60 days	Bank discount							
	Proceeds							
80 days	Bank discount							
	Proceeds							
90 days	Bank discount							
	Proceeds							
120 days	Bank discount							
	Proceeds							

245. Discounting the Paper of Other Persons. If you have a note signed by some person of good financial standing, you can borrow money on it by discounting it in the same way that you would discount a note made by yourself.

Suppose John Doe had given you the following note:

$ 500.00 CHICAGO, ILL., January 16, 1915.

Ninety days after date I promise to pay to the order of Your Name, Five Hundred Dollars.

JOHN DOE.

By the following indorsement,

Pay to the order of
The First National Bank
(Your Name)

you transfer the note to the bank. Moreover you promise to pay the bank $500 on April 16, in case Doe fails to do so. The bank will loan you $ 500 minus the discount.

When you discounted your own note (in the exercise just given), you did so on the same day that the note was made. The discount period and the time of the note were therefore the same. But you may keep the notes of other people some time before discounting them. The bank will charge you interest from the day you discount the note until it is due.

Thus, if you did not discount the above note until March 6, the bank would charge you interest for the number of days from March 6 to April 16, the date of maturity. How many days in this discount period?

What would be the discount period if you discounted the note January 23? February 16? March 20?

246. To find the discount period.

Find the maturity of the note.

Find the exact number of days between the date of discount and the date of maturity.

247. To find the maturity. Notes due a certain number of *months* after date fall due on the same day of the month as the day on which they were made, with the following exceptions:

Notes made on the 31st, maturing after a specified number of months, falling due in a month having only 30 days, mature on the 30th.

Notes made on the 28th, 29th, 30th, or 31st, maturing after a specified number of months, falling due in February, mature on the last day of February.

Examples.

Note Made	Due in	Maturity
Jan. 16, 1915	2 months	March 16, 1915
July 31, 1915	2 months	September 30, 1915
Dec. 31, 1915	2 months	February 29, 1916

Notes due a certain number of *days* after date, mature after the exact number of days has elapsed.

Examples.

	Note Made	Due in	Maturity
	1915		
a.	Feb. 16	30 days	March 18

Since February has only 28 days, the 30 days of the note will be divided as follows:

12 February days (after the 16th)
18 March days

b.	March 18	30 days	April 17

March has 31 days; 13 left after the 18th; the remaining 17 of the 30 days must be in April.

c.	April 17	30 days	May 17

April has 30 days; 13 left after the 17th; the remaining 17 of the 30 days will be in May.

From these examples, the following should be clear.

To find the maturity.

Change the days to months; (30 *days* = 1 *month*).

Assume that the note will mature on the same day of some following month.

Correct this result by subtracting 1 *day for each month of* 31 *days through which the note runs; and by adding* 2 *days if the note runs through February.* (*In case of leap year, add* 1 *day.*)

The following table provides a convenient method for indicating the months for which corrections are to be made.

MONTH	NUMBER OF MONTH		DAYS	CORRECTION
January	1	13	31	– 1
February	2	14	28	+ 2 (or + 1)
March	3	15	31	– 1
April	4	16	30	
May	5	17	31	– 1
June	6	18	30	
July	7	19	31	– 1
August	8	20	31	– 1
September	9	21	30	
October	10	22	31	– 1
November	11	23	30	
December	12	24	31	– 1

Examples. 1. Find the maturity of a note dated May 16, 1915, due in 4 months.

SOLUTION. May is the fifth month. Add 4 months. The note will be due in the ninth month, which by the table is September.

Maturity, September 16.

2. Find the maturity of a note of the same date, due in 120 days.

SOLUTION. Call 120 days 4 months.

May is the fifth month. The note is due in the ninth month, shown by the table to be September.

Call the maturity September 16. Correct as follows:

Note runs through May	Subtract	1 day
Note runs through July	Subtract	1 day
Note runs through August	Subtract	1 day
Total		3 days

Therefore the maturity is September 13.

3. Find the maturity of a note dated December 19, 1914, due in 150 days.

SOLUTION. Call 150 days 5 months.

December is the twelfth month. The note is due in the seventeenth month, which, by the table, is May.

Call the maturity May 19, 1915.

Correct as follows:

December	− 1
January	− 1
February	+ 2
March	− 1
April (has 30 da.)	
Note does not run through May.	
Total	− 1

Maturity, May 18, 1915.

4. Find the maturity of a note dated June 16, 1915, due in 70 days.

Solution. Call 70 days 2 months, 10 days.

June 16 plus two months is August 16; plus 10 days is August 26.

To correct:

For June	
For July	− 1
Total	− 1

Maturity, August 25, 1915.

Written Work

Enter the maturity of each note in the proper column:

Note Dated	Due in 1 Month	Due in 30 Days	Due in 2 Months	Due in 60 Days	Due in 150 Days	Due in 45 Days
Sept. 23, 1915						
Oct. 9, 1915						
May 1, 1915						
July 20, 1915						
Dec. 15, 1915						
Feb. 9, 1915						

248. To find the discount period. *Compute the actual number of days between the date of discount and the date of maturity.*

Illustration. A note, dated July 17, due in 4 months, was discounted on August 5. What was the discount period?

Solution. Four months from July 17 is November 17, the date of maturity. The discount period, therefore, extends from August 5, the date of discount, to November 17, the date of maturity.

To find the number of days between August 5 and November 17:

31	total number of days in August
5	number of August days expired before discounting
26	number of August days in the discount period
30	number of September days in the discount period
31	number of October days in the discount period
17	number of November days in the discount period
104	total number of days in the discount period

Oral Work

After studying the illustration, state a method for finding the number of days in the discount period.

Written Work

1. On a form ruled like the following, enter the date of maturity on the upper line, and the discount period on the lower line.

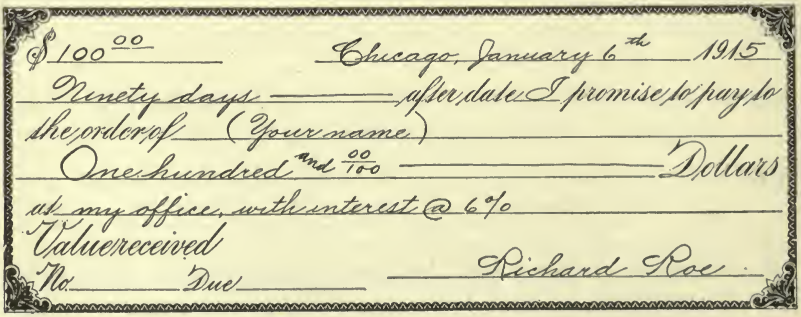
The notes dated February 12 were discounted April 8.
The notes dated February 17 were discounted March 2.
The notes dated March 23 were discounted April 29.
The notes dated March 17 were discounted April 15.
The notes dated March 30 were discounted May 20.

Dates Notes were Made	Time				
	60 Days	75 Days	100 Days	120 Days	6 Months
Feb. 12, 1915					
Feb. 17, 1915					
Mar. 23, 1915					
Mar. 17, 1915					
Mar. 30, 1915					

2. Complete a table ruled like the following model. Enter the date of maturity on the first line, the discount period on the second line, the discount on the third line, the proceeds on the fourth line, as illustrated by the first problem in the table.

Date of Paper and Face	Time, 90 Days	Time, 3 Months	Time, 120 Days	Time, 6 Months
	Discounted April 13	Discounted April 11	Discounted April 23	Discounted May 20
Jan. 16, 1915 $300	April 16			
	3 days			
	$.15			
	$299.85			
Jan. 30, 1915 $285				
Feb. 19, 1915 $126				

249. Discounting Interest-bearing Notes. If you had in your possession the following note and wished to discount it, you would first add 90 days' interest ($1.50) to the face of the note to determine its *value* at maturity. This note is really Roe's promise to pay you $101.50 at maturity, and the bank will give you as much for it as for a note for $101.50 without interest.



$100 00 Chicago, January 6th 1915

Ninety days after date I promise to pay to the order of (Your name)

One hundred and 00/100 Dollars

at my office, with interest @ 6%

Value received

No. Due Richard Roe

When discounting interest-bearing paper, compute the interest for the *full time* the note is to run, and add this interest to the face, to determine the *value* at maturity. Find the discount on this value. Subtract the discount from the *value* to determine the proceeds.

Written Work

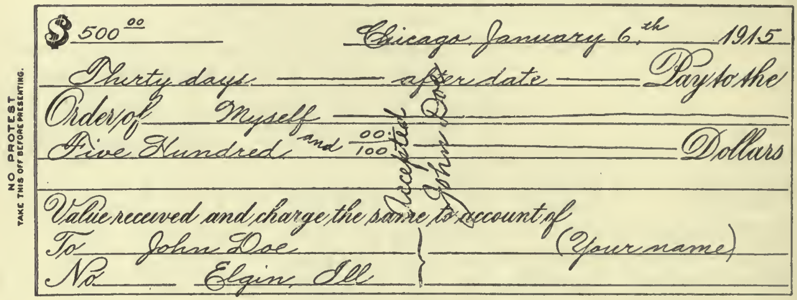
Complete a table ruled like the following model, entering the facts as illustrated in the first problem.

All notes discounted at 6 %.

		Time, 90 Days		Time, 4 Months		Time, 6 Months	
		Without Int.	With Int. 6%	Without Int.	With Int. 6 %	Without Int.	With Int. 5%
Principal,	Maturity	Sept. 1	Sept. 1				
$500.00	Interest		$ 7.50				
Date of Paper,	Value	$500.00	$507.50				
June 3, 1915	Dis. Period	11 days	11 days				
Date of Discount	Bank Dis.	$ .92	$ .93				
Aug. 21, 1915	Proceeds	$499.08	$506.57				
Principal,	Maturity						
$1250.00	Interest						
Date of Paper,	Value						
Aug. 6, 1915	Dis. Period						
Date of Discount	Bank Dis.						
Sept. 18, 1915	Proceeds						
Principal,	Maturity						
$47.96	Interest						
Date of Paper,	Value						
July 16, 1915	Dis. Period						
Date of Discount	Bank Dis.						
Aug. 27, 1915	Proceeds						
Principal,	Maturity						
$55.00	Interest						
Date of Paper,	Value						
Aug. 14, 1915	Dis. Period						
Date of Discount	Bank Dis.						
Oct. 5, 1915	Proceeds						

250. Discounting Acceptances. Time drafts were discussed in Chapter XIX. An accepted draft is a written promise of the acceptor to pay money at a definite date, and accepted drafts may be discounted, just as notes may be discounted. In case the person who has accepted the draft does not pay it when due, the person who discounted it will be required to pay it.

The following draft drawn by you on John Doe and accepted by him is just as truly his promise to pay $500.00 at a given date, as a promissory note signed by him would be.



NO PROTEST TAKE THIS OFF BEFORE PRESENTING.

\$500 ^{00} Chicago, January 6th 1915

Thirty days after date Pay to the

Order of Myself

Five Hundred and $\frac{00}{100}$ Dollars

Value received and charge the same to account of

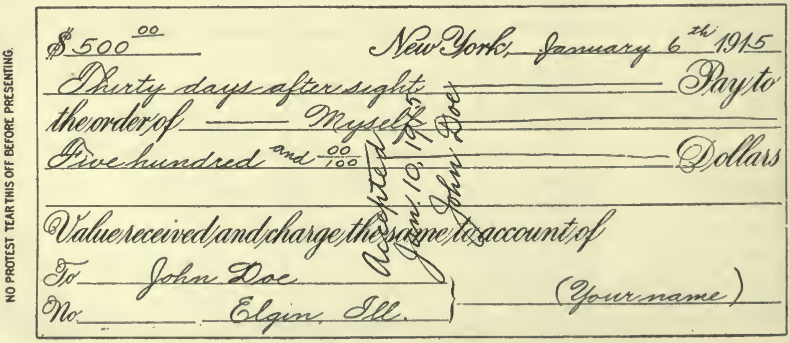
To John Doe (Your name)

No. Elgin, Ill.

Accepted John Doe

This draft could be discounted in exactly the same manner as a note dated January 6, payable 30 days after date.

If the draft were drawn payable 30 days after sight, instead of after date, the bank discount would be different because the date of maturity would be different.



NO PROTEST TEAR THIS OFF BEFORE PRESENTING.

\$500 ^{00} New York, January 6th 1915

Thirty days after sight Pay to

the order of Myself

Five hundred and $\frac{00}{100}$ Dollars

Value received and charge the same to account of

To John Doe

No. Elgin, Ill. (Your name)

Accepted Jan. 10, 1915 John Doe

Written Work

1. What is the date of maturity of the draft on page 300 payable 30 days after date?

2. What is the date of maturity of the draft on page 300 payable 30 days after sight?

3. What is the discount period of each draft if discounted on Jan. 14?

4. Find the bank discount and the proceeds of each draft.

Review

Write the notes and drafts called for in the following. Show all acceptances and indorsements.

On October 3, you received from B. A. Anderson, of Batavia, New York, a note dated October 1, due in two months, for $135.00, without interest.

On August 28, you sold to N. M. Davis, of Toledo, Ohio, an invoice of goods amounting to $84.00; terms 1/10; N/30. This invoice was due September 27, but Mr. Davis was unable to pay it. He agreed, however, to accept a thirty-day draft for the amount. You drew the draft on September 30, payable thirty days after date, and sent it to him for acceptance. He accepted it and returned it to you.

On October 5, you sold a bill of goods amounting to $68.00 to J. D. Robinson, of Burlington, Iowa. Terms, 30 day draft less 1 %. On October 5, you drew a draft on him, payable 30 days after sight; he accepted the draft on October 7.

On July 10, you received from J. F. Cook of Lime Springs, Iowa, a note made by him on July 8, in your favor, for $500.00, due six months after date, with interest at 6 %.

On June 4, you received a note for $600.00 from A. B. Hicks of Chicago. The note was drawn on April 16, by D. F. Fairchild of Omaha, Nebraska, in favor of Hicks, payable nine months after date, without interest, and was transferred to you by full indorsement.

On October 15, you took these notes and drafts to the Dairy State Bank to discount. The bank's discount rate was 6 %.

Finding that the proceeds would not be sufficient to meet your needs, you gave the bank your own note, dated October 15, due in 20 days, for $250.00, without interest.

1. Find the proceeds of each of the papers discounted.

Your balance at the bank before discounting this paper was $167.25. You deposited the proceeds of the discounted paper. What was your balance after discounting the paper?

2. On October 25, the bank wished to increase its deposit with its correspondent bank at Chicago. It therefore rediscounted the notes and drafts received from you. How much did the bank receive for the paper, when discounted on October 25 at 6%?

3. Marshall owed Daniels $485.90; on September 23 he gave Daniels the following note, properly indorsed.

$450.00 BRAINERD, IOWA, September 3, 1915.

Sixty days after date, I promise to pay to the order of J. F. Marshall, Four Hundred and Fifty Dollars.

Value received
Without interest. F. S. CROSBY.

How much did Marshall owe Daniels after transferring the note?

CHAPTER XXX

WAGES AND PAY ROLLS

THERE are several wage systems in use, each of which is designed to encourage employees to produce as much as possible. Several of the most common systems will be discussed in this chapter.

251. The Day or Hour Rate. This system is the one generally used, because it is the simplest. When the day or hour rate is used, the employee's wage depends upon the time he has labored. Work done overtime or on holidays is usually paid for at one and one half times the regular rate. In many factories time clocks are installed. As each employee begins work, he registers the time on his card; when he stops work, he again registers the time. At the end of the week, each employee's card shows his actual hours of labor. A pay roll is prepared showing the total number of hours each employee has worked, his hourly rate, and his total wage.

Written Work

1. Supply the missing facts in the following pay roll.

No.	Name	Hours worked per day						Time				Overtime			Total Wages Earned		Money Advanced		Wages Due	
		M	T	W	T	F	S	Hrs.	Rate	Amount		Hrs.	Amount							
1	Andrew White	9	8	9	9	9	8	52	30	15	60				15	60			15	60
2	August Piper	9	9	11	9	12	9	54	28	15	12	5	2	10	17	22			17	22
3	Edward Haglett	10	9	11	9	10	9	54	32	?		4	1	92	?		4	—	?	
4	James Davis	8	7	9	8	9	9	?	29	?					?				?	
5	Oscar Bennett	9	9	10	10	11	9	?	34	?		?	?		?		1	20	?	
6	James Belknap	8	8	9	9	9	8	?	32	?					?				?	
7	Fred Calkins	8	9	9	10	11	9	?	30	?		?	?				5	—	?	
8	Charles Davis	9	6	10	11	9	10	?	28	?		?	?		?				?	
9	Harry McGee	10	9	10½	9	10	11½	?	25	?		?	?		?		1	75	?	

In this factory a full day is 9 hours. Any over hours are paid for at 1½ times the regular rate. Thus, Calkins gets 1½ times regular pay for the overtime on Thursday and Friday, even though he did not work full time on Monday.

2. Rule a pay roll similar to the model, and enter the following data, finding the amount of wages due each employee.

The superintendent supplies the cashier with the following list of the employees, together with their numbers, and the hourly wage rate paid each. A full day in this factory is 9 hours, and overtime is paid for at 1½ times the regular rate.

No.	Employee's Name	Hourly Rate
1.	Fred Smith	$.35
2.	Frank Bailey	.32
3.	Charles Hanchett	.30
4.	Harry Davis	.36
5.	Carl Bell	.38
6.	Lewis Clark	.40
7.	Arthur Helms	.29
8.	Tony Martin	.31
9.	Henry Dickinson	.26
10.	Nathan Wright	.37

During the week ending March 15, time tickets were turned in by the foreman, showing the hours worked by each employee.

Monday: 1, 9 (Workman No. 1, 9 hours); 2, 11; 3, 8; 4, 11; 5, 8; 6, 10; 7, 9; 8, 12; 9, 9; 10, 8.

Tuesday: 1, 9; 2, 11; 3, 9; 4, 9; 5, 10; 6, 9; 7, 9; 8, 9; 10, 9.

Wednesday: 1, 10; 2, 12; 3, 11; 4, 12; 5, 9; 6, 11; 7, 8; 8, 11; 9, 10; 10, 9.

Thursday: 1, 9; 2, 9; 3, 11; 4, 10; 5, 8; 6, 10; 7, 11; 8, 11; 9, 12; 10, 10.

Friday: 1, 5; 2, 9; 3, 11; 4, 10; 5, 9; 6, 9; 7, 9; 8, 9; 9, 10; 10, 8.

Saturday: 1, 9; 2, 11; 3, 11; 4, 10; 5, 11; 6, 9; 7, 12; 8, 9; 9, 4; 10, 9.

The cashier has already advanced Charles Hanchett $3.00; Carl Bell, $2.25; Tony Martin, $3.75; and Nathan Wright, $5.00.

252. Piecework Wage System. The piecework wage system is based on the theory that if the employee is paid for the actual amount of work he accomplishes, he will produce more in a day than if he is paid a straight hour or day wage.

The pay roll prepared under a piecework system is similar to that prepared under the hourly rate system, with the exception that overtime is usually paid for at the same rate that regular time receives, and the basis of the wage is the number of pieces produced and not the number of hours employed.

Written Work

Find the missing facts in the following pay roll.

PIECEWORK PAY ROLL FOR WEEK ENDING JUNE 8, 19—

No.	Name	Operation Number	Number Produced						Total	Rate	Earned	Advanced	Due
			M.	T.	W.	T.	F.	S.					
1	John Sanford	264	16	14	17	13	15	18	93	.14	13 02	2	11 02
2	Irving Danner	178	23	26	22	27	24	25	?	.12	?		?
3	Byron Shepley	361	37	33	34	39	36	30	?	.08	?		?
4	Edward Magee	422	28	27	29	28	26	27	?	.05	?	3 25	?
5	Percival Conrad	267	42	45	41	39	43	20	?	.07	?		?
6	Ernest Anderson	207	24	26	20	23	30	27	?	.09	?	2 60	?

253. The Differential Rate. The differential rate is a modification of the piecework system. A standard day's work is determined, and each employee is expected to produce this standard amount of work. If he produces the standard, he receives a certain rate per piece. If he produces less than the standard, he receives a lower rate per piece. If he produces more than the standard, he receives a higher rate per piece.

This system is based on the theory that the expense of heat, light, rent, power, etc., remains about the same, whether the employees produce a small amount of product, or a large amount. By increasing the amount of output, these expenses are distributed over a larger quantity of manufactured goods, and the cost of making each article is thereby decreased. The saving effected by increasing the output is divided between the owner of the factory and the workmen who, by their skill or industry, increase the output. On the other hand, the employees who, by producing a small quantity, tend to increase the cost per article are paid at a lower rate.

Example. A manufacturer learned by experience that the average workman in his factory could produce ten articles per day, and that 25 cents per article could be paid for the work. He then adopted a differential rate, as follows:

NUMBER OF PIECES PRODUCED	RATE PER PIECE
7	$.20
8	.21
9	.23
10, Standard	.25
11	.26
12	.27

Jones produced the standard quantity on Wednesday, and received 10 × $.25 = $2.50.

Smith produced 8 pieces, and received 8 × $.21 = $1.68.

Brown produced 12 pieces, and received 12 × $.27 = $3.24.

Written Work

The following differential rate applies in a certain factory:

NUMBER OF PIECES	RATE
10	$.12
11	.13
12	.14
13	.15
14	.17
15	.19
16	.21
17	.22
18, Standard	.24
19	.25
20	.27
21	.29
22	.31
23	.33
24	.33½
25	.34

The production sheet for the week ending September 17, 19— was as follows :

No.	Name	Daily Production					
		M.	T.	W.	T.	F.	S.
1	Edward Dye	17	19	18	16	19	18
2	Frank Bartlett	14	15	14	16	15	14
3	Arthur Bassett	19	20	19	21	22	19
4	James Hamilton	22	21	21	23	20	21
5	Den Slocum	12	14	13	15	12	14
6	John Gaylord	13	16	15	16	15	17
7	Sam Bayliss	19	20	19	21	18	17
8	George Bates	19	22	23	20	19	18
9	Wm. Osgood	16	14	12	14	11	16

1. Devise a pay roll blank which will show all necessary facts, and enter each laborer's production and wages. See how good a pay roll form you can devise without asking for suggestions. It should be simple and convenient, and, while not being too elaborate, it should give all necessary information.

2. Find the average daily production of each workman. Prepare a graph, showing the standard production by a red line across the sheet, and the workmen's averages by a black curve.

COMMISSION

254. Definitions. An **agent** is a person who transacts business for another.

The **principal** is the one whose business is transacted by the agent.

Principals often pay their agents a commission for their services. The **commission** is usually a per cent of the money value involved in the transaction.

Example. A wholesale store engaged a salesman, agreeing to pay him 8 % of his gross sales as commission. How much did the salesman earn if he sold $1425 worth of goods during a month?

Solution.

$1425	sales
.08	rate of commission
$114.00	commission

Written Work

1. A real estate agent sold a farm belonging to Mr. Robinson for $20,450. The agent received 3 % of the selling price as payment for his services. How much did the agent receive? How much did Mr. Robinson receive after paying his agent?

2. A manufacturer engaged Mr. Seeley as a traveling salesman. Mr. Seeley's contract stipulated that he should receive 7 % of his gross sales as commission. During four weeks, his sales were as follows: $576.25; $498.49; $376.45; $723.50.

What was his commission each week?

3. Mr. Brooks, a merchant, engaged an attorney to collect a number of accounts. The attorney charged for his services 15 % of the amounts collected. What did the attorney receive for collecting $354.75? What was the net amount received by Mr. Brooks?

4. James Hunter bought eggs for a Chicago produce merchant. For his labor in purchasing and hauling them to the station, he received 1½ % of the cost of the eggs. On Tuesday he bought 27 cases, of 30 dozen each, paying 29 cents per dozen for them. How much did Hunter pay for the eggs? How much was his commission? How much did the eggs cost the Chicago merchant, not including the freight?

5. The salesmen of the Wickman Specialty Company receive the following commissions:

On goods sold in Department A, 9 %.
On goods sold in Department B, 13 %.
On goods sold in Department C, 12½ %.
On goods sold in Department D, 15 %.

The following table shows the sales made by five men.

SALESMAN	DEPT. A	DEPT. B	DEPT. C	DEPT. D
C.G. PINES	356.27	473.28	293.75	459.23
F.L.SNOW	523.74	625.90	483.37	824.84
E.D.VICKS	283.57	285.37	295.64	472.84
L.S.BASSETT	923.74	825.36	736.46	462.47
FRANK HOLTZMAN	735.84	375.47	375.47	462.85

Devise a form which will show the above data and enter the following:

Each salesman's commission on goods sold in each department.

The total commissions paid to each salesman.

The total commissions paid for each department.

The total commissions paid to all salesmen.

The total sales in each department.

The total sales of each salesman.

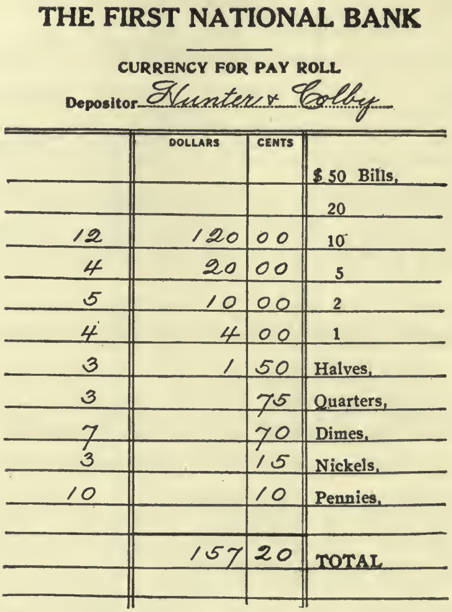
The total sales in all departments.

255. Pay Roll Slips. Laborers are sometimes paid by check, and sometimes by the envelope system. When the latter method is used, each employee receives an envelope containing the exact amount of his wages. A coin sheet is prepared from the pay roll, showing the change needed for each envelope. A pay roll

No.	Name	Wages		Bills				$.50	$.25	$.10	$.05	$.01
				$10.00	$5.00	$2.00	$1.00					
1	James Downs	22	67	2		1		1		1	1	2
2	Francis Canfield	31	69	3			1	1		1	1	4
3	Arthur Hubbard	26	38	2	1		1		1	1		3
4	Frank Munger	29	35	2	1	2			1	1		
5	George Seiley	28	96	2	1	1	1	1	1	2		1
6	Ezra Colfax	12	15	1	1	1	1			1	1	
		157	20	12	4	5	4	3	3	7	3	10

Coin Sheet

slip, designating the number and denomination of the bills and coins desired, is then filled out and sent to the bank. When the currency is delivered a check is made out to the bank for the amount of the pay roll.



THE FIRST NATIONAL BANK

CURRENCY FOR PAY ROLL

Depositor Hunter & Colby

	DOLLARS	CENTS	
			$ 50 Bills,
			20
12	120	00	10
4	20	00	5
5	10	00	2
4	4	00	1
3	1	50	Halves,
3		75	Quarters,
7		70	Dimes,
3		15	Nickels,
10		10	Pennies,
	157	20	TOTAL

Pay Roll Slip

Written Work

Rule a coin sheet and enter the number of coins of each denomination necessary to pay each of the employees in the pay roll prepared in the differential wage rate problem on page 307.

Make out a pay roll slip on the First National Bank of your city, to obtain the money to pay these employees.

CHAPTER XXXI

POSTAGE, FREIGHT, AND EXPRESS RATES

Postage

256. The Rates for Domestic Postage. Domestic mail matter includes all matter deposited in the mails for delivery to points in the United States or its possessions, including Porto Rico, Hawaii, the Philippine Islands, Guam, and the Canal Zone; with certain exceptions, it includes matter sent to Canada, Cuba, Mexico, and the Republic of Panama. The domestic rate applies also to letters, but not to other articles, addressed to Great Britain, Ireland, Newfoundland, and Germany.

CLASSES OF DOMESTIC MAIL

First Class: Written matter: Letters and all mail which is sealed.
Rate: 2 cents for each ounce or fraction thereof. Post cards and postal cards, 1 cent each.

Second Class: Unsealed. Newspapers and periodicals.
Rates: When mailed by the publisher, 1 cent per pound. When mailed by others than the publisher or a news agent, 1 cent for each four ounces or fraction thereof, on each separately addressed copy.

Third Class: Circulars, newspapers, and printed matter (except books) not admitted to the second class.
Rates: 1 cent for each two ounces or fraction thereof. The limit of weight of third-class matter is four pounds.

Fourth Class: Parcel post: All matter not embraced in the first, second, or third classes, and not likely to injure the employees of the postal service, or the mails

This includes merchandise, farm and factory products, seeds, books, etc.

Rates : Parcels weighing four ounces or less, except books, seeds, plants, etc., 1 cent for each ounce or fraction of an ounce, regardless of the distance.

Parcels weighing eight ounces or less containing books, seeds, cuttings, roots, etc., 1 cent for each two ounces or fraction thereof.

Parcels weighing more than eight ounces containing books, seeds, plants, etc., parcels of miscellaneous printed matter weighing more than four pounds, and all other parcels of fourth-class matter weighing more than four ounces are subject to pound rates, a fraction of a pound being considered a full pound. The pound rates vary with the distance the parcel is to be carried.

257. Units of Area and Zones. For the purpose of determining the various pound rates, the United States is divided into "units of area," each unit being thirty miles square. Each unit is the center of a series of zones:

First Zone : Territory within a radius of 50 miles.
Second Zone : Territory within a radius of 150 miles and beyond 1st zone.
Third Zone : Territory within a radius of 300 miles and beyond 2d zone.
Fourth Zone : Territory within a radius of 600 miles and beyond 3d zone.
Fifth Zone : Territory within a radius of 1000 miles and beyond 4th zone.
Sixth Zone : Territory within a radius of 1400 miles and beyond the 5th zone.
Seventh Zone : Territory within a radius of 1800 miles and beyond the 6th zone.
Eighth Zone : All territory beyond the 7th zone.

258. Size and Weight of Parcels. No package can be sent by parcel post, if its length plus its girth is more than seventy-two inches. For delivery in the first and second zones, parcels may weigh not more than fifty pounds; for delivery in other zones, they may weigh not more than twenty pounds.

259. Parcel Post Rates. The rates of postage on parcels exceeding 4 ounces in weight are as shown on the opposite page.

The local rate applies to parcels to be delivered from the same office at which they are mailed. The delivery may be made at the office, by city carrier, or by rural free delivery.

Books. Books weighing 4 ounces or less are mailed at the third-class rate of 1 cent for each 2 ounces or fraction thereof. Those weighing more than 4 ounces are mailed under the regular zone rates.

260. Special Regulations. When two classes of mail matter are mailed in the same package, the higher rate is charged on the entire package. For example, inclosing a letter in a parcel of merchandise subjects the whole parcel to the letter rate.

As a general rule the postage on all mail must be prepaid in stamps affixed to the package. Publishers, however, do not always affix stamps to their publications; and the postage on third-class matter, mailed in quantities of not less than 2000 identical pieces, may be paid in money.

Postage stamps are issued in the following denominations: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 15, 20, 30, and 50 cent, and 1, 2, and 5 dollars.

Special Delivery. By affixing a "special delivery" stamp (cost, 10 cents), or ten cents' worth of ordinary stamps, in addition to the regular postage, and writing the words "Special Delivery," prompt delivery by special messenger is obtained.

Registry. The registry system provides greater security for valuable mail matter. The registry fee is ten cents. If registered mail is lost, the sender is indemnified up to $ 50 for first-class, and up to $ 25 for third-class domestic mail.

Insurance of Parcels. Mail sent by parcel post may be insured against loss upon the payment of a fee of 5 cents for value not exceeding $ 25, or 10 cents for value not exceeding $ 50, in addition to the postage. It may not be registered.

TABLE OF POSTAGE RATES ON FOURTH-CLASS MAIL, EFFECTIVE JANUARY 1, 1914

WEIGHT	FIRST ZONE		SECOND ZONE RATE	THIRD ZONE RATE	FOURTH ZONE RATE	FIFTH ZONE RATE	SIXTH ZONE RATE	SEVENTH ZONE RATE	EIGHTH ZONE RATE
	Local Rate	Zone Rate							
1 pound	$0.05	$0.05	$0.05	$0.06	$0.07	$0.08	$0.09	$0.11	$0.12
2 pounds	.06	.06	.06	.08	.11	.14	.17	.21	.24
3 pounds	.06	.07	.07	.10	.15	.20	.25	.31	.36
4 pounds	.07	.08	.08	.12	.19	.26	.33	.41	.48
5 pounds	.07	.09	.09	.14	.23	.32	.41	.51	.60
6 pounds	.08	.10	.10	.16	.27	.38	.49	.61	.72
7 pounds	.08	.11	.11	.18	.31	.44	.57	.71	.84
8 pounds	.09	.12	.12	.20	.35	.50	.65	.81	.96
9 pounds	.09	.13	.13	.22	.39	.56	.73	.91	1.08
10 pounds	.10	.14	.14	.24	.43	.62	.81	1.01	1.20
11 pounds	.10	.15	.15	.26	.47	.68	.89	1.11	1.32
12 pounds	.11	.16	.16	.28	.51	.74	.97	1.21	1.44
13 pounds	.11	.17	.17	.30	.55	.80	1.05	1.31	1.56
14 pounds	.12	.18	.18	.32	.59	.86	1.13	1.41	1.68
15 pounds	.12	.19	.19	.34	.63	.92	1.21	1.51	1.80
16 pounds	.13	.20	.20	.36	.67	.98	1.29	1.61	1.92
17 pounds	.13	.21	.21	.38	.71	1.04	1.37	1.71	2.04
18 pounds	.14	.22	.22	.40	.75	1.10	1.45	1.81	2.16
19 pounds	.14	.23	.23	.42	.79	1.16	1.53	1.91	2.28
20 pounds	.15	.24	.24	.44	.83	1.22	1.61	2.01	2.40
25 pounds	.17	.29	.29						
30 pounds	.20	.34	.34						
35 pounds	.22	.39	.39						
40 pounds	.25	.44	.44						
45 pounds	.27	.49	.49						
50 pounds	.30	.54	.54						

Parcels weighing 4 ounces or less are mailable at the rate of 1 cent for each ounce or fraction of an ounce, regardless of distance. Parcels weighing more than 4 ounces are mailable at the pound rates, a fraction of a pound being considered a full pound.

261. Rates for Foreign Postage. The rates of postage applicable to articles for foreign countries are as follows:

	Cents
Letters for Germany, England, Ireland, Newfoundland, Scotland, and Wales, per ounce	2
Letters for all other countries:	
For the first ounce or fraction of an ounce	5
For each additional ounce or fraction of an ounce	3
Post cards, each	2
Printed matter of all kinds, for each two ounces or fraction of two ounces	1
Commercial papers (business documents without letters):	
For the first ten ounces or less	5
For each additional two ounces	1
Samples of merchandise:	
For the first four ounces	2
For each additional two ounces	1
Parcel post, per pound or fraction	12

Registry fee on all classes of mail, ten cents in addition to the postage.

Written Work

Compute the cost of postage on the following:

Domestic mail.

1. Letter weighing $1\frac{1}{2}$ oz.

2. Book weighing 5 oz.

3. Book weighing 19 oz., to a point in the fourth zone.

4. Newspapers weighing 26 pounds mailed by publisher.

5. Three separately wrapped newspapers mailed by a person not a publisher; weight, 3 oz., 5 oz., and 8 oz.

6. Ten separately addressed printed circulars, each weighing $1\frac{1}{2}$ oz.

7. Package of merchandise weighing 3 oz.

8. Package of seeds weighing 12 oz.

9. Merchandise weighing 16 lb. 7 oz., to a point in the sixth zone.

10. Merchandise weighing 35 pounds, to a point 130 miles distant.

11. Registered letter weighing 7 oz.

12. Books weighing 11 lb. 9 oz., to a point in the fifth zone; package insured. (Ins. fee 5¢.)

Foreign mail.

13. Letter to Berlin, weighing 1½ oz.

14. Letter to Paris, weighing 1½ oz.

15. Book to Madrid, weighing 9 oz.

16. Package of merchandise samples weighing 11 oz., to London.

FREIGHT RATES

A freight rate is the charge made by a railroad company for conveying a specified quantity of property between two stations.

The rate is usually stated in cents per hundred pounds; but it may be stated in dollars and cents per ton or other unit of weight.

262. Factors Determining Rates. Different rates are charged on different kinds of freight, the rate depending in general upon two things:

a. **The expense of handling.**

Suppose two boxes, each weighing 100 pounds, are sent by freight. One is a large box occupying 40 cubic feet of car space; the other occupies only 16 cubic feet. The rate per 100 pounds on the first box will probably be more than on the second because fewer such boxes can be placed in one car.

b. **The risk of loss or damage.**

Railroads are required by law to pay for goods lost or damaged. The rate will therefore depend upon:

1. The value. For example, the rate on a prize-winning cow, worth $ 300, will be higher than on a cow worth $ 60.

2. The risk of damage. The rate on glassware, for example, is higher than on wooden ware. Careful packing usually lowers the rate.

3. The risk of loss. The rate on small packages which may easily be misplaced or lost is higher than on those of larger size having the same value.

4. The risk of an article damaging other articles in the same car.

263. Classifications. For convenience in determining rates, commodities are grouped into ten different classifications. These classifications are made by the railroads after considering the elements of cost of transportation and risk. When goods are to be shipped from one state to another, the classification must be approved by the Interstate Commerce Commission.

A book called a "Classification" is published, containing an alphabetical list of commodities which can be shipped by freight, together with the classification in which each commodity is placed.

The following list is selected from a page of a Classification:

Notice that two classifications are given for some articles. The first applies when goods are shipped in less than car loads (L. C. L); the second applies to car load shipments.

		C.L.
Cutlery, blades or handles plated or sterling silver, boxed	1½	
Cutters, Sage Brush (knife attachments for grading machines):		5 Min. wt. 30,000 lbs.
In bundles	2	
In boxes or barrels	3	
Designs, Floral, Artificial or Natural, dried, in boxes	D 1	
Dextrine, in bags, bbls. or boxes, min. C. L. wt. 30,000 lbs.	2	4
Dish-Washing Machines, N. O. S., boxed or crated.	1	
Formaldehyde, in barrels or kegs or in glass bottles, boxed, min. C. L. wt. 30,000 lbs.	2	4

		C.L.
Display Cases, window shade, boxed or crated	1	
Dog Benches (for exhibition purposes only), K. D. flat, min. C. L. wt. 30,000 lbs.	4	B
Door Hangers and Rail:		
Barn Door Hangers (iron), boxed or crated	3	5 Min. wt. 36,000 lbs.
Barn Door Rails (iron), in bundles	3	
Barn Door Hanger Rails (wooden), in bundles	3	
Parlor Door Hangers, boxed	2	
Parlor Door Hanger Rail, wooden, in bundles	3	
Tubing, for Barn Door Hanger Rail, nested in bundles	4	

Terms Explained: N. O. S. means Not Otherwise Specified.
1½ means One and one half times first-class.
D 1 means Double the first-class rate.
C. L. means Car loads.
K. D. means Knocked down, taken apart.

264. Freight Tariff. The tariff is a list of the rates applying to different classifications of freight when shipped between the points named.

The following illustration is selected from a freight tariff.

BETWEEN	AND	CLASS RATES IN CENTS PER 100 POUNDS									
		1st Class	2d Class	3d Class	4th Class	5th Class	Class A	Class B	Class C	Class D	Class E
Stoddard, Wis. Calvert, Wis. La Crosse, Wis. Grand Crossing, Wis. Onalaska, Wis. Lytle, Wis. Trempealeau, Wis. East Winona, Wis. Winona, Minn.	Chicago, Ill. Group No. 1	50	42	33	23	18	23	18	15	12	11
	Springfield, Ill. Group No. 2	53	43	33	25	20	23	18	15	12^9	11
	St. Louis, Mo. Group No. 3	50	42	33	23	18	23	18	15	12	11
	Danville, Ill. Group No. 4	63	52^5	42	26	21	26	21	18	15	13^5
	Cairo, Ill. Group No. 4	80	65	52	33	26	33	26	22	19	17

NOTE. 52^5 means $52\frac{1}{2}$ cents.

265. To find the freight charge. The rates differ for each of the ten classifications. Therefore, when a shipment of goods is made, the following steps are necessary to determine the freight charge:

a. Weigh the commodity.

b. Find its classification.

c. Determine from the tariff the rate applying to that classification between the points of shipment.

d. Multiply the number of hundreds of pounds by the rate.

Example. A shipment of barn door hangers (crated) is sent from Chicago to La Crosse, Wis. The weight crated is 185 pounds. Find the freight charges.

SOLUTION. The classification for less than car load lots is 3.

The rate on third-class commodities between Chicago and La Crosse, as shown by the tariff, is 33 cents per hundred.

\$ .33	freight rate per hundred pounds
1.85	number of hundreds of pounds
\$ .61	freight charge

266. Minimum Charges. A minimum charge is fixed for shipment of any commodity between two points. When the charge is computed and is less than this minimum charge, the minimum charge is made. The minimum charge is sometimes the third-class rate per hundred, or it may be an arbitrarily fixed number of cents.

267. Car Loads. When goods are shipped in car load lots, they frequently go under a different classification and at a lower rate. Minimum weights for car loads are specified. For example, 36,000 pounds is the minimum weight for a car of door hangers. This does not mean that the car must contain 36,000 pounds, but the shipper will be charged freight on at least that weight, and more if the car contains a greater weight.

Example. Shipped from Chicago to La Crosse a car load 39,000 # of door hangers. Find the freight charges.

SOLUTION. Classification C. L., 5 (see classification, page 317).
Rate, $.18.
390.00 × $.18 = $70.20, the freight charge.

If the door hangers weighed only 32,380 #, a charge would be made for the minimum weight of 36,000 #.

SOLUTION. Classification, 5.
Rate, $.18 per 100 pounds.
360 × $.18 = $64.80, freight charge.

268. Commodity Rates. There are some articles which are not listed in the classification, but which have a rate of their own, called a "commodity rate." The most important of these commodities are lumber, grain, live stock, coal, coke, and hay.

Written Work

Find the freight on each of the following:

1. 360 # of formaldehyde in glass bottles, boxed, shipped from St. Louis, Mo., to Winona, Minn.

2. 500 # dextrine in bags, shipped from Winona, Minn., to Danville, Ill.

3. 260 # display cases (wood and wire, crated), shipped from Springfield, Ill., to Onalaska, Wis.

4. Car load of dextrine 28,000 #, shipped from Winona, Minn., to Cairo, Ill.

5. Box of silver-plated cutlery, 60 #, shipped from Chicago, to La Crosse, Wis.

6. Box of dish washing machines, shipped from Chicago to Stoddard, Wis. Weight 175 #.

7. Box of parlor door hangers, 80 #, shipped from Springfield, Ill., to Grand Crossing, Wis.

8. Flower design, 35 #, from Chicago to East Winona, Wis.

9. Car load of iron door hangers, 41,350 #, from St. Louis, Mo., to La Crosse, Wis.

EXPRESS RATES

269. The Classification. The express classification is simpler than the freight classification, most articles being unspecified and taking the first-class rate.

Rate Scale Numbers. The agent at each express station is supplied with a directory showing the scale number to be used in computing the cost of shipment to every express station in the United States. The scale numbers run from 1 to 294, depending on the distance between point of shipment and point of delivery.

The following table shows the rate scale numbers from the Chicago directory for shipment to certain Iowa points.

STATION	SCALE NO.
Burdette	27
Burlington	15
Burt	31
Buxton	21
C	
Calmar	17
Calumet	33
Cambridge	26
Campbell	26
Carlisle	26
Carnes	39
Carney	26
Carpenter	27
Carroll	29
Casey	30
Cedar Falls	23
Cedar Rapids	15

270. Schedule of Rates. The following table gives the rates per pound on shipments between points where scale numbers from 23 to 33 apply. The complete table states the rates on shipments weighing from 1 to 100 pounds.

SCALES 23 TO 33

SCHEDULE OF FIRST- AND SECOND-CLASS EXPRESS RATES IN CENTS

Pounds	Scale Numbers																						Pounds
	23		24		25		26		27		28		29		30		31		32		33		
	1st Class	2d Class	1st Class	2d Class	1st Class	2d Class	1st Class	2d Class	1st Class	2d Class	1st Class	2d Class	1st Class	2d Class	1st Class	2d Class	1st Class	2d Class	1st Class	2d Class	1st Class	2d Class	
25	56	42	57	43	59	45	60	45	61	46	62	47	64	48	65	49	66	50	67	51	69	52	**25**
26	58	44	59	45	60	45	62	47	63	48	64	48	65	49	67	51	68	51	69	52	71	54	**26**
27	59	45	60	45	62	47	63	48	65	49	66	50	67	51	69	52	70	53	71	54	73	55	**27**
28	61	46	62	47	63	48	65	49	66	50	68	51	69	52	70	53	72	54	73	55	75	57	**28**
29	62	47	63	48	65	49	66	50	68	51	69	52	71	54	72	54	74	56	75	57	77	58	**29**
30	63	48	65	49	66	50	68	51	69	52	71	54	72	54	74	56	75	57	77	58	78	59	**30**
31	65	49	66	50	68	51	70	53	71	54	73	55	74	56	76	57	77	58	79	60	80	60	**31**
32	66	50	68	51	70	53	71	54	73	55	74	56	76	57	78	59	79	60	81	61	82	62	**32**
33	68	51	69	52	71	54	73	55	74	56	76	57	78	59	79	60	81	61	83	63	84	63	**33**
34	69	52	71	54	73	55	74	56	76	57	78	59	79	60	81	61	83	63	85	64	86	65	**34**
35	71	54	72	54	74	56	76	57	78	59	79	60	81	61	83	63	85	64	86	65	88	66	**35**
36	72	54	74	56	76	57	78	59	79	60	81	61	83	63	85	64	87	66	88	66	90	68	**36**
37	74	56	75	57	77	58	79	60	81	61	83	63	85	64	87	66	88	66	90	68	92	69	**37**
38	75	57	77	58	79	60	81	61	83	63	85	64	86	65	88	66	90	68	92	69	94	71	**38**
39	77	58	78	59	80	60	82	62	84	63	86	65	88	66	90	68	92	69	94	71	96	72	**39**
40	78	59	80	60	82	62	84	63	86	65	88	66	90	68	92	69	94	71	96	72	98	74	**40**

271. To find the express charge for shipments of 100 pounds or under.

a. Find the weight of the commodity.

b. Determine the classification.

c. Find the rate scale number applicable between the points of shipment and delivery.

d. The table of rates will show the charge for first- and second-class commodities weighing 100 *pounds or less.*

Example. What is the express charge on a case of medicine shipped from Chicago to Cedar Falls, Iowa? Weight, 40 pounds.

SOLUTION. Classification, 1.

Rate scale number (see table) 23.

The schedule shows the charge under rate scale number 23, first class, 40 pounds, to be 78 cents.

272. To find the express charge for shipments over 100 pounds.

a. First-class Rates.

Find the charge for a shipment of 100 *pounds first class. Multiply this charge by the number of pounds in the shipment, and divide by* 100.

Example. Find the charge on 216 pounds of first-class matter when the rate scale number is 31.

Solution. Rate for 100 pounds, first-class matter, scale number 31, is found by the complete table to be \$2.05.

$$\frac{216 \times \$2.05}{100} = \$4.428.$$

Under the express company's rule that all fractions of a cent must be equalized as 1 cent, the charge would be \$4.43.

b. Second-class Rates.

Multiply the first-class rate on 100 *pounds by the number of pounds in the shipment, and divide by* 100.

Deduct 25% *of this result to find the second-class charge.*

Example. What is the charge on 216 pounds of second-class matter, when the rate scale number is 31?

Solution. Charge for 216 pounds, first class, scale number 31, is \$4.43 (from previous illustration).

75% of \$4.43 = \$3.3225, equalized as \$3.33.

Written Work

Find the charge for the following shipments by express from Chicago:

1. 26-pound box of cameras to Burdette, Iowa. Classification 1 (first class).

2. 25 pounds of candy to Calumet, Iowa. Classification 1.

3. 30 pounds of seeds to Carpenter, Iowa. Classification 2.

4. 28 pounds of telephone instruments, unboxed, to Casey, Iowa. Classification $1\frac{1}{2}$ times first class.

5. 163 pounds of statuary to Cedar Falls, Iowa. Classification 1. First-class rate for 100 pounds, scale number 23, \$1.65.

6. 191 pounds of shrubs to Burt, Iowa. Classification 2. First-class rate for 100 pounds, scale number 31, \$2.05.

CHAPTER XXXII

DEPRECIATION

273. Depreciation is the loss or expense incurred in business, due to the decline in the value of property. While repairs may be made to prolong the usefulness of a building or a machine, there is sure to come a time when the property is either entirely useless, or when it is good business economy to replace it.

Suppose a machine which cost $2400 can be used for ten years, at the end of which time it will be worth $400. At the end of the ten years, when the machine is replaced, there will have been a loss of $2000 due to depreciation. Unless a portion of the depreciation is charged as an annual expense, the entire $2000 loss will fall on the last year.

274. Methods of Computing Depreciation. There are a number of methods of computing depreciation. The following are those most frequently used:

First, a *fixed* rate, computed each year on the *original* value of the property.

Second, a *decreasing* rate, computed on the *original* value of the property.

Third, a *fixed* rate, computed on a *decreasing* value.

275. Fixed Rate Computed on the Original Value. This is the simplest method in use. It can best be explained by an illustration. A printing press is purchased at a cost of $8000 and it is expected that this press can be used for ten years, when it will have a value of $2000. Therefore, during the ten years of use, a depreciation of $6000 will occur. This is an annual depreciation of $600. $600 is $7\frac{1}{2}$ % of $8000.

Therefore $7\frac{1}{2}$ % of the original value is charged each year as an expense.

276. Decreasing Rate Computed on the Original Value of the Property. It is sometimes considered more reasonable to charge the largest amount of depreciation the first year, gradually reducing it each year thereafter. It is argued that a greater depreciation actually occurs during the first year than during any later year. For example, an automobile is "second hand" after only a few months' use, and the owner suffers a much greater loss from its use the first year than he does the second.

277. Fixed Rate Computed on a Decreasing Value. This method results in a similar decreasing annual charge for depreciation. For example, suppose the original value of the property is $1200 and the rate of depreciation is 10 % a year. The depreciation is computed as follows:

$1200.00	Original value
.10	
$120.00	Depreciation first year
$1200.00	
120.00	
$1080.00	Decreased or carrying value, beginning of second year
.10	
$108.00	Depreciation second year
$1080.00	
108.00	
$972.00	Decreased value, beginning of third year
.10	
$97.20	Depreciation third year.

Written Work

1. Find the annual depreciation on a building worth $15,560, if 4 % is charged off each year.

2. How much is charged off annually for depreciation by a manufacturer who owns property which depreciates at the following rates?

PROPERTY	VALUE	DEPRECIATION RATE
Factory building	$40,000	5 %
Machinery	4,800	7½ %
Tools	1,250	12½ %
Patents	5,000	6¼ %

3. The owner of a building estimates the annual depreciation as 3 % of its cost. The building cost $4000. What is the amount of the annual depreciation?

The building is rented at $40 per month. If the taxes, insurance, and other expenses amount to $80 per year, what net income does the owner of this property receive on his investment after allowing for depreciation?

4. It is estimated that a machine costing $2220 can be sold at the end of eight years for $500. What per cent should be charged annually for depreciation, and what will be the amount of the annual depreciation?

5. Machinery in a factory cost $24,000. Depreciation is computed as follows:

10 % of the original value the first year.
8 % of the original value the second year.
6 % of the original value the third year.
3 % of the original value the fourth year, and each year thereafter.

What was the amount of depreciation charged off each year for five years?

What was the inventory value of the machinery at the beginning of each year?

What was the inventory value of the machinery at the end of 12 years?

6. A flour mill was equipped with machinery costing $60,000. Depreciation was computed at 12½ % of its cost the first year, 8 % of its cost the second year, 5 % the third year, 2½ % the fourth year, and 2 % each year thereafter.

Find the amount of depreciation each year for seven years.

What was the inventory value of the machinery at the beginning of each year?

Find the annual depreciation and the decreased value each year, in the following problems:

7. Four years' depreciation on property costing $3200, depreciation computed at 8 % of the decreased value.

8. A manufacturer was engaged in business for 10 years. His machinery cost $ 14,500, and he charged 6 % depreciation annually on decreased values.

9. Complete the following table, showing the annual depreciation and decreased values per thousand dollars invested when depreciation is charged at various fixed rates on decreased values. (Correct to nearest mill.)

Year	2 %		3 %		5 %		7 %		12½ %	
	Depreciation	Inventory Value, End of Year	Depreciation	Inventory Value, End of Year	Depreciation	Inventory Value, End of Year	Depreciation	Inventory Value, End of Year	Depreciation	Inventory Value, End of Year
1st	$20	$980								
2d	19 60	960 40								
3d	19 208	941 192								
4th	18 824	922 368								
5th	18 447	903 921								
6th	18 078	885 843								
7th	17 717	868 126								

Use the table just prepared in solving the following problems.

10. Machinery in a factory cost $ 7460.00, and depreciation was computed at 7 % on decreased annual values. What was the depreciation during the fourth year, and the inventory value at the end of the fourth year?

11. Cost of equipment, $ 5600.00; depreciation computed at 3 % on decreased annual values. What was the depreciation during the third year, and the inventory value at the end of the third year?

12. Cost of machinery $ 23,746.00; depreciation computed at 12½ % on decreasing annual values. What was the amount of the depreciation during the sixth year, and the reduced value at the end of that year? The depreciation during the sixth year was what per cent of the original cost of the machinery?

CHAPTER XXXIII

ADVERTISING

During the last fifty years the annual cost of advertising in this country has increased to more than seven hundred million dollars. The advertising methods employed are numerous. The principal mediums are newspapers, magazines, street car signs, posters, electric signs, circulars, booklets, and novelties.

278. Newspapers. Newspaper space is sold by the page, inch, and agate line. The word "agate" refers to the size of the type.

This is a line of agate type.

An inch contains nine lines of agate type when a lead is placed between each line; or 12 lines when set solid. The price varies with the circulation of the newspaper; 3 cents per inch per thousand subscribers is considered a fair basis on which a newspaper may determine its rates. On this basis, a newspaper with a circulation of 5000 would charge 15 cents per inch for its space. The larger newspapers generally compute their rates on the basis of 7 cents per line per thousand subscribers, but there are many exceptions to this.

Rate Card of a Daily Newspaper

Display Rates per Agate Line		
	Daily	Sunday
Run of paper, 5 lines or more	$.40	$.50
Next to reading matter	.50	.60
Specified page	.45	.55
Discounts for Yearly Contracts		
Lines	Per Cent	Insertions
2,500 or more lines	10	26
5,000 or more lines	25	52
10,000 or more lines	28	312

279. Magazines. The rate charged by magazines for advertising space also depends upon the circulation, a fair basis being considered $\frac{1}{2}$ cent per line, or $1.00 per page, per thousand subscribers. Higher rates are charged for the desirable positions, such as cover page, and for printing in colors.

Specimen Rate Card of the —— Magazine

Rates per Issue	
One page	$100.00
Half page	50.00
Quarter page	27.00
Eighth page	15.00
Less than $\frac{1}{8}$ page, per agate line	.75
Inside cover page	150.00
Back cover, 3 colors	350.00

Three per cent discount for cash.

Forms close on the 10th day of the month preceding date of issue.

One well-known magazine with a circulation of one million charges $4000 per page, and there are several magazines which charge $250 per page.

280. Street Car Signs. Space is provided above the windows of most street cars for printed signs. These signs differ in size, but the standard size is 11 in. × 21 in. Some cards of double length are used; these are 11 in. × 42 in. The ordinary rate for street car advertising space is 50 cents per month per car for a half run, which means six months in all the cars, or one year in one half the cars. It is 45 cents a car per month when all cars are used for a full year. Exceptions to this rate apply in New York, Chicago, and other large cities. The cards are furnished at the expense of the advertiser.

281. Posters. Posters are printed in various sizes, but the standard unit is a sheet 28 in. × 42 in. The billboards are usually about 10 feet high, in order to accommodate 4-sheet posters. The charge made for rent of boards and posting signs depends upon the size of the city and the desirability of the location. The rates range from 4 cents to 30 cents a sheet per month. A discount of 5 % is usually given on a contract for three months' service, and 10 % on a contract for six months' service. The cost of printing the posters ranges from $1\frac{1}{2}$ cents to 4 cents per sheet depending on the quality and quantity.

282. The Advertising Agency. In many of the larger cities there are advertising agencies which assist advertisers in preparing copy, in directing advertising campaigns, and in selecting the best medium for advertising a particular article. On the theory that the agency creates new business, publishers usually allow agencies a commission of from 10 % to 15 % of the gross cost of all advertising placed with them for publication.

283. Checking Results. Large advertisers make special efforts to determine the mediums which bring the greatest number of inquiries, and which result in the most sales and the largest amount of profit. For this purpose, the advertisements in the various newspapers and magazines are "keyed"; for example, the address may be differently stated in each medium, or different booklets may be offered, so that when the replies are received in the office, it will be possible to determine the medium which attracted the reader. Each periodical is credited with the replies received from its readers. Further records are kept to determine the value of the goods sold to the readers of the periodical, and the profit resulting therefrom.

Written Work

Refer to the newspaper rate card on page 327 to obtain facts for Problems 1, 2, and 3.

1. On the basis of nine agate lines to the inch, what is the cost of a four-inch, single-column advertisement? The advertisement appears next to reading matter.

2. What is the cost of a double-column, 40-line display advertisement, printed by request on the financial page? How much would the same advertisement cost if printed on a specified page of the Sunday paper? Can you account for the increased cost?

3. A clothing store contracted for 10,000 lines to be printed in the course of a year. 8000 lines were printed in the daily editions, "run of paper." 1200 lines were printed in the daily editions on specified pages, and the balance appeared in the Sunday editions next to the reading matter. What was the total cost?

4. A breakfast food manufacturer ran the following amount of advertising in the magazine whose rate card is printed on page 328.

January	$\frac{1}{8}$ page	July	Inside cover page
February	1 page	August	Inside cover page
March	$\frac{1}{2}$ page	September	Back cover
April	12 agate lines	October	1 page
May	1 page	November	1 page
June	Inside cover page	December	$\frac{1}{2}$ page

What was the cost for the year if he paid cash?

5. The advertiser in Problem 4 placed his copy through an agency which received from the publishers $12\frac{1}{2}\%$ of the gross cost as its commission. How much commission did the agency receive? How much did the publishers receive?

6. An insurance company advertised in the street cars of a city in which one of its offices was located. There were 850 cars, and the insurance company contracted for a "full run" at the rate of 45 cents per month per car. What was the cost (not including the cost of printing)?

If the cards were changed every two months, and if 400 extras were printed to allow for "torns" and "dirties," how many were required for the year? What was the cost of the cards at $15 per thousand?

What was the total cost of this company's street car advertising?

7. A manufacturer advertised his product with posters. Each poster was 112″ × 126″. How many sheets 28″ × 42″ to each poster?

He signed a six months' contract for 250 posters at 16 cents per sheet per month, posters to be changed only when worn out, and received the customary 10% discount. What was the cost of the service?

He purchased 15% more posters than were actually required for the boards, estimating that this number would be necessary to allow for damaged sheets. What was his printing bill at $2\frac{1}{4}$ cents per sheet?

What was the total cost of the six months' advertising?

8. Complete the following table :

RECORD OF ADVERTISING FOR JUNE, 1917

PUBLICATION	SPACE	COST OF ADVERTISING	NUMBER OF REPLIES	COST PER REPLY	NUMBER OF SALES	PER CENT OF REPLIES SOLD	COST PER SALE	VALUE OF GOODS SOLD	ADVERTISING COST WHAT PER CENT OF SALES	PROFIT FROM SALES	NET PROFIT AFTER DEDUCTING ADVERTISING COST	PER CENT OF NET PROFIT
Literary Guide . .	page	$400	500	$ 80	400	80	$1	$2000	20	$600	$200	50
Readers' Monthly .	½ page	250	385		156			863		315		
Inland Monitor .	page	180	600		88			1275		420		
Firelight	⅛ page	60	93		68			1640		395		
Romance and Story	¼ page	75	370		148			1290		396		
Youth and Age . .	80 ag. lines	30	147		95			670		115		
Farming Age . .	¼ page	60	320		90			280		38		

NOTE. Cost per reply is found by dividing the total cost by the number of replies.
Per cent of replies sold is found by dividing the number of sales by the number of replies.
Cost per sale is found by dividing the total cost of advertising by the number of sales.
To find what per cent the advertising cost is of the sales, divide the advertising cost by the value of the goods sold.
Net profit is found by subtracting the advertising cost from the gross profit.
To find what per cent the net profit is of the cost of advertising, divide the net profit by the cost of the advertising.
If a net loss should occur, show the net loss and the per cent of net loss in red ink.

Which columns do you think give the best evidence as to the value of each magazine as an advertising medium for this product?

Which is the better evidence of the value of an advertising medium, the cost per reply or the cost per sale?

List these magazines in the order of their value as advertising mediums for this particular advertiser (on the basis of per cent of net profit).

CHAPTER XXXIV

PROPERTY INSURANCE

PROPERTY is in danger of destruction from many causes; insurance companies have been organized to assume the risks of destruction, thus relieving the owners from anxiety and loss. Any person having a financial interest in a property may insure himself against its loss or destruction by paying an insurance company a certain per cent of the value of the property insured.

The person who purchases insurance is called the **Insured.**

The company which assumes the risk is called the **Underwriter.**

The contract between the insured and the underwriter is called a **Policy.**

284. The Value of Insurance.

a. In case of loss the insured receives a certain sum, called an indemnity, which wholly or partially repays his loss.

b. Even though the property is not destroyed, the insured has the security which the insurance gives.

c. Some wholesale houses are unwilling to sell goods on credit to persons whose property is inadequately insured.

285. Risks for which Insurance is Written. Insurance companies are organized to furnish protection against numerous kinds of risks. Some of the most common classes of insurance are:

Fire.
Lightning and Tornado.
Marine, on vessels and cargoes at sea.
Live stock.
Transportation.
Burglary.
Plate Glass.
Boiler Explosion.
Automobile.
Crop.

The principles underlying all types of property insurance are the same, and for the purposes of this text fire insurance only will be explained. It will be understood that other forms of insurance may be written, settled, or canceled in a similar manner.

286. Process of Insuring. When a person wishes to secure a policy insuring his property, he should first ascertain the value of the property. To do so, it may be necessary to take an inventory, applying to the goods a price at which they would sell in the market at the time of taking the inventory.

After the valuation has been determined, the next thing to decide is the amount of insurance it is desirable to carry. If the property is insured at its total valuation, the owner will bear no loss in case of fire. But since, in most fires, a portion of the property is saved, it is customary to insure only a fraction of the total valuation. Thus, if a stock of goods worth $10,000.00 is insured at $\frac{3}{4}$ of its value, in case of fire the insurance company will pay all loss up to $7500.00. If goods to the value of $2500.00 or more are carried from the building and saved, the merchant will not suffer any loss. If only $1000.00 worth of goods are saved, the merchant will lose $1500.00.

It is a matter of choice how much risk the insured wishes to take. If he wishes to be free from all risk, he should insure his property at its full valuation. Insurance companies, however, are rarely willing to insure property at its full value. Business men usually insure from $\frac{1}{2}$ to $\frac{4}{5}$ of the valuation of their property.

In some rural communities where the fire protection is poor, insurance companies sometimes refuse to insure more than $\frac{3}{4}$ of the valuation of property. On the other hand, in cities where the fire protection is good and the probability of total loss is comparatively slight, people are not inclined to insure a large fraction of the valuation. To counteract this tendency, the insurance companies frequently offer a lower rate, or price per hundred dollars of insurance carried, to those who insure 80% or more of the valuation than they offer to persons insuring less than 80%.

287. The Cost of Insurance. Insurance companies charge a certain number of cents or dollars for insuring each $100.00 worth of property. Thus, if the insurance rate were 60 cents per hundred dollars of insurance carried, and you insured a building worth $5000.00 at its full value, the insurance cost would be computed as follows:

$.60 the rate per $100 insurance
50 the number of hundred dollars of insurance purchased
$30.00 the premium, or cost of the insurance per year

288. To find the premium:

Multiply the valuation of the property by the fraction of the value insured.

Point off two places in this product, to determine the number of hundred dollars of insurance purchased.

Multiply by the rate per $100.

Example. What is the premium for insuring, at 80% of its value, a building worth $10,000.00, if the rate is 90 cents per year?

SOLUTION. $10,000.00 value of the property
.80 fraction of property to be insured
$8,000.00 insured value

$.90 rate per hundred
80 number of hundred dollars of insurance carried
$72.00 premium

289. Insurance Rates. The insurance rate per hundred dollars per year varies from $.30 on brick dwellings in cities, where the fire protection is good, to $7 or more on very dangerous risks, such as garages storing gasoline, and buildings located in districts where the fire protection is poor.

Thus it is seen that the rate depends upon the hazard, or the danger of the property burning. This hazard is determined by four things, which are always taken into account in fixing the rate on different pieces of property. They are:

a. The **Construction** of the building. If it is built of wood, a higher rate is charged than if it is built of brick, or other slow-burning material. If it has many stories, so that a fire starting near the roof will be difficult to extinguish, a higher rate will be charged.

b. Its **Occupancy.** If the building is occupied by some hazardous business, such as a woodworking mill in which wood shavings accumulate, or a clothes-cleaning establishment containing dangerous chemicals, a higher rate will be charged than if the business caused no particular hazard.

c. Its **Exposure.** By "exposure" is meant: first, the distance from other buildings from which it might take fire; and second, any hazard which might arise from its being in the neighborhood of a dangerous building.

d. Its **Protection.** The rate will be lower if the building is close to a fire hydrant, in a city with an efficient fire department, or if the building is equipped with automatic sprinklers which are opened by the heat from a fire. If a night watchman is engaged, this also may reduce the rate.

The rate depends also upon the length of time for which the insurance is purchased.

290. Typical Rates. The following table will illustrate the rates which apply on different classes of property. On business houses and factories the rate is fixed after an inspection of the property made by a representative of the insurance company. The insurance companies of a state frequently join in organizing an "inspection bureau" which fixes the rates.

The rates on city property will be lower than those which apply generally throughout the country, because of the more adequate fire protection of a large city.

A CITY INSURANCE TARIFF

DWELLINGS

	ONE YEAR, PER $100
Brick, stone, tile, cement-block, or concrete dwellings and contents	$0.30
Single frame dwellings and contents, detached not less than 50 feet in all directions	.50
Single frame dwellings and contents, detached not less than 25 feet in all directions	.60
Single frame dwellings and contents, detached 50 feet on one side	.60

	One Year, per $100
All other frame dwellings and contents, not less than	$0.75
Brick-veneered or tile-veneered dwellings and contents	.40
Dwellings plastered outside, with tile or other noncombustible roof, and contents	.40

Dwellings plastered outside, with shingle or other combustible roof, rate same as frame dwellings.

Dwellings in part brick and frame rate as frame, subject to survey and rating by the board.

Streets, without reference to their width, to be considered as cutting off charge for exposure.

BRICK BUILDINGS AND CONTENTS, OCCUPIED FOR APARTMENT HOUSES OR FLATS

	One Year, per $100
Three stories in height	$0.40
Four stories in height and 50 feet or less wide	.50
Five stories in height and 50 feet or less wide	.60
Six stories or more in height and 50 feet or less wide	.75

BRICK BUILDINGS (OTHER THAN APARTMENT HOUSES, CHURCHES, SCHOOLS, FLATS AND WOODWORKING RISKS) IN PROCESS OF CONSTRUCTION

	One Year, per $100
Two stories or less in height	$0.40
Three stories in height	.50
Four stories in height	.60
Five stories in height	.75

Over five stories in height, add 25 cents for each story.

SCHOOLHOUSES

	One Year, per $100
Brick or stone, with metal, slate, or composition roof, and contents	$0.60
Brick or stone, with shingle roof, and contents	.75
Frame, and contents	1.00

CHURCHES

	One Year per $100
Brick or stone, and contents	$0.75
Frame, and contents	1.00

TERM RATES

When policies are written for several years, the following will apply:

2 years, 1½ annual rates.
3 years, 2 annual rates.
4 years, 2½ annual rates.
5 years, 3 annual rates.

FARM PROPERTY

	One Year, per $100	Three Years, per $100	Five Years, per $100
Dwellings, barns, outbuildings, and contents, when written under same policy	$0.50	$1.00	$1.50
When farm barns and contents are written without the dwellings	.75	1.50	2.25

Written Work

Compute the premiums on the following fire insurance policies. The proper rates will be found by a study of the preceding tariffs.

ONE-YEAR POLICIES — CITY PROPERTY

1. Single frame building, value $4000. No buildings nearer than 60 feet. Insured at 80 % of its valuation.

2. Single frame dwelling, value $5400. House on north, 70 feet away; store on south, 25 feet away. Insured at full value.

3. Brick dwelling, value $6800. Insured at 75 % of its value. Nearest building, 50 feet.

4. Dwelling, plastered outside; shingle roof. Value, $4200. Insured for $3600. No building nearer than 35 feet.

5. Five-story apartment house, 48 feet wide. Value, $36,000. Insured at ¾ of value.

6. Four-story brick store in process of construction. Value, $3900. Insured at 80 % of its valuation.

TERM POLICIES — CITY PROPERTY

7. Three-year policy on brick schoolhouse, with tile roof. Value $40,000. Insured at full value.

8. Five-year policy on frame church. Value, $7500. Insured at full value.

POLICIES ON HOUSEHOLD GOODS — CITY

9. Three-year policy of $1000 on household goods in frame dwelling. Building on north, 10 feet; building on south, 60 feet.

10. One-year policy of $500 on household goods in brick-veneered building.

FARM POLICIES

11. One-year policy of $8500, covering dwelling and barns.

12. Five-year policy of $3000 on barn and silo.

291. Insurance Agent's Commission. Local agents of the fire insurance companies are located in almost every city and town. They act as the representatives of the companies, soliciting the business, and collecting the premiums. For this service they receive a certain per cent of the premiums, usually varying from 15% on mercantile risks to 25% on dwellings.

To find the agent's commission.

Example. Suppose that a store building valued at $6000 were insured for $\frac{3}{4}$ of its value, the rate being 70 cents per hundred. What would be the agent's commission if he received 15% of the premium?

SOLUTION. $\frac{3}{4}$ of $6000 = $4500, the insured value.
45 × $.70 = $31.50, the premium.
15% of $31.50 = $4.73, the agent's commission.

Written Work

In each of the following problems find
The premium;
The agent's commission;
The amount of the premium received by the insurance company.

1. A one-year policy of $3000 on a store building. Rate, 90 cents. Agent's commission, 20%.

2. A three-year policy on a dwelling valued at $6500, insured at 80% of its valuation. Rate, 65 cents. Three-year policy for 2 annual rates. Agent's commission, 18%.

3. A one-year policy of $800 on household goods. Rate, 50 cents. Agent's commission, 15%.

292. Settlement of Losses. When a fire occurs, causing a small loss, settlement may be made by the local agent. When the loss is large, a special **adjuster** is sent by the company to make the settlement.

The adjuster first determines with the owner the actual cash value of the property immediately preceding the fire. This may be done by finding the cost of the property when new, and subtracting a certain amount for **depreciation**; that is, the decrease in value due to use, age, change of style, or any other cause. This cash value is called the **sound value.**

The adjuster next determines the value of goods which were saved from the fire. In determining this value he deducts any loss due to damage by the fire, or by water used in extinguishing the fire. The property saved is called **salvage.**

It is evident that:

Sound Value − Salvage = Loss.

The companies pay this actual loss up to the value of the insurance carried.

Examples.

1. Valuation of property by insured when taking the policy	$10,000.00
Insured for 80 % of its value; Insured value	8,000.00
Sound value agreed upon by adjuster and insured after allowing $500 for depreciation	9,500.00
Salvage	3,000.00
Actual loss	$6,500.00

Since the insured carried $8000 insurance, the loss would be paid in full.

2. Valuation of property by insured when taking policy	$8,000.00
Insured for $\frac{3}{4}$ of its value; Insured value	6,000.00
Sound value, after deducting 10 % for depreciation	7,200.00
Salvage	500.00
Actual loss	$6,700.00

Since the insured carried only $6000 insurance, the company is liable for only $6000 of the loss; the insured bearing the remaining $700 loss.

In adjusting losses, the insurance companies reserve the right to:

a. Settle the loss as shown above; or

b. Pay the sound value of the property, taking possession of all the salvage; or

c. Replace the property by other property of like kind and quality.

Written Work

1. An apartment building worth $45,000 was insured at 80 % of its valuation. A fire occurred resulting in a total loss. If the sound value was considered to be the same as the original valuation, how much insurance did the owner of the property receive, and what was his loss?

2. A dwelling valued at $3500.00 was insured at its full valuation. A fire resulted in a partial loss, the salvage being estimated at $1250.00. If the sound value of the property was $3325.00, how much insurance did the owner of the property receive?

3. If the property in Problem 2 had been insured for $1500.00, how much insurance would have been paid?

4. A policy of $1500.00 was written on a man's household goods. When adjustment of the loss was made, the following facts were determined: Value of household goods immediately preceding the fire, $1855.00; salvage, $575.00. How much money did the insured receive as a settlement?

5. If the salvage in Problem 4 had been $215.00, how much would the insured have received?

6. A building worth $5000.00 was insured under a five-year policy at its full value. Four years after taking the policy, the house was completely destroyed by fire. In making adjustment of the loss, it was agreed that the sound value was 5 % less than the insured value. How much did the owner of the house receive in settlement?

7. If the salvage in Problem 6 had been $1350.00, how much insurance would the insured have received?

8. Cost of building, $6000; insured for 80% of its value under five-year policy secured by the payment of three annual rates of 75 cents. Burned at end of third year. Sound value determined by deducting 4% annual depreciation. Salvage, $1200. Find premium and loss, if any, to owner.

293. Canceling Policies. Both the insurance company and the insured have the right to cancel an insurance policy at any time.

When the policy is canceled by the *insurance company*, the portion of the premium to be repaid to the insured is determined *pro rata*.

The exact fraction of the time of the policy which has not expired is determined and this fraction of the premium is returned to the insured.

Example. On July 7, 1917, the owner of a building insured his property for one year. The premium was $24.00. On September 17, the policy was canceled by the insurance company. What rebate did the insured receive for the unexpired term?

Solution. Time from September 17, 1917, to July 7, 1918, 293 days, unexpired term of policy.

$\frac{293}{365}$ of $24.00 = $19.27, amount of premium returned.

The table on page 267 will be of assistance in finding the unexpired term of a policy.

Example. An insurance policy for one year was purchased on July 9, and was canceled by the company on the 17th of the following December. How many days in the unexpired term of this policy? What fraction of the premium will be refunded?

Solution. The number of days between December 17 and July 17 is shown by the table to be 212. Subtracting 8, we have 204, the number of days from December 17 to July 9.

$\frac{204}{365}$ of the premium will be refunded.

When the policy is canceled by the *insured*, the amount of premium returned is determined by the "short rate." The short rate is an arbitrary per cent fixed by the insurance companies, and is shown by a table, a portion of which is shown on page 343.

Per Cents of the Annual Premiums to be Charged or Retained for Periods less than One Year. Arranged by Days from One to Three Hundred and Sixty Days

Days	Per Cent		Days	Per Cent		Days	Per Cent		Days	Per Cent	
66	34	32	71	36	30	76	37	50	81	38	85
67	34	97	72	36	58	77	37	90	82	39	13
68	35	46	73	36	79	78	38	20	83	39	34
69	35	79	74	36	93	79	38	40	84	39	48
70	35	95	75	37	00	80	38	50	85	39	55

Table for Cancellation of Term Risks

Three-Year Policies	
For 3 mo. or less	20 per cent of term premium
Over 3 and not exceeding 6 mo.	30 per cent of term premium
Over 6 and not exceeding 9 mo.	40 per cent of term premium
Over 9 and not exceeding 12 mo.	50 per cent of term premium
Over 12 and not exceeding 15 mo.	60 per cent of term premium
Over 15 and not exceeding 18 mo.	70 per cent of term premium
Over 18 and not exceeding 21 mo.	75 per cent of term premium
Over 21 and not exceeding 24 mo.	80 per cent of term premium
Over 24 and not exceeding 27 mo.	85 per cent of term premium
Over 27 and not exceeding 30 mo.	90 per cent of term premium
Over 30 and not exceeding 33 mo.	95 per cent of term premium
Over 33 mo.	100 per cent of term premium

Example. 1. On October 28, 1917, a one-year policy for $2500.00 was written on a dwelling. Premium, $12.50. On January 15, 1918, the policy was canceled at the request of the insured. What rebate did the insured receive?

Solution. Time from October 28, 1917, to January 15, 1918, 79 days.

The table shows that 38.40% of the premium is to be *retained* when the policy has been in force 79 days.

38.40% of $12.50 = $4.80.

$12.50 – $4.80 = $7.70, amount of premium returned.

2. On August 12, 1914, a three-year policy was written at a premium cost of $78.00. On February 11, 1915, it was canceled at the request of the policyholder. What was the amount of the premium rebate?

SOLUTION. Time from August 12, 1914, to February 11, 1915, 5 months 29 days.

The table states that when a three-year policy has been in force 3 months and not exceeding 6 months, 30 per cent of the premium shall be retained.

30 % of $78.00 = $23.40, amount of premium retained.

$78.00 – $23.40 = $54.60, premium returned.

Written Work

Find the premium rebates on the following canceled policies:

1. One-year policy of $2400.00. Premium, $16.00. Policy written May 19, 1917; canceled by the company on July 29, 1917.

2. Same facts as Problem 1, except that the policy is canceled at the request of the policyholder.

3. Three-year policy, dated July 23, 1917. Face of policy, $3600.00. Rate, 65 cents, three-year policy being written for term rates. Policy canceled at the request of the policyholder on October 7, 1918.

CHAPTER XXXV

TAXATION

294. Purpose of Taxation. The national government requires money to support the army and navy, to pay the salaries of government employees, to pay pensions, and to finance other activities carried on by the nation. During a recent year, Congress appropriated $1,098,678,788 for the annual budget.

The state governments require money for the expense of their officers, and to support their various institutions, schools, universities, asylums, and penitentiaries.

The counties require money for the building of bridges, the trial of criminal cases, the salaries of officers, the relief of the poor, etc.

Cities must pay for police and fire protection, care of streets, etc.

School districts contribute to the support of the public schools.

The money required for all these expenses is raised by taxes, licenses, fees, assessments, and fines.

295. A **tax** is a sum of money levied by the proper officers of a government to defray government expenses.

The funds of the national government are raised largely in three ways:

Customs Duties on Imports ;
Internal Revenue ;
Income Tax.

The funds of the state and local governments are raised largely by direct taxes on real estate and personal property.

STATE AND LOCAL TAXES

296. Taxable Property. The amount of property tax paid by any individual to state and local governments depends upon the value of the property which he owns and the tax rate.

Real estate consists of land and buildings.

Personal property consists of movable property, such as merchandise, furniture, machinery, live stock, cash, notes, stocks, bonds, and mortgages.

297. Determining the Assessed Value of Property. The value of each person's taxable property is determined by the assessor. The **assessed value** is usually a fractional part of the real value.

For example, in a certain state, property is assessed at $\frac{1}{3}$ of its real value. Jones owns a farm which the assessor considers to have a real value of $30,000. He therefore assesses it at $10,000.

Although the law gives the assessor the power to determine the taxable value of property, property owners usually have the privilege of appearing before a Board of Equalization to prove a claim that their property has been assessed at too high a valuation. The Board of Equalization judges between the values fixed on property by the assessor and by the owner.

298. The Tax Rate. The tax rate may be expressed in several ways, the most common of which are:

A per cent;

A certain number of mills on each dollar of assessed valuation;

A certain number of dollars or cents on each hundred dollars of assessed valuation.

The following tax rates are equivalent:

1.6 %;
16 mills (on the dollar);
$1.60 (on each hundred dollars).

299. To find the amount of tax. To find the amount of tax to be paid by any property owner:

Multiply the assessed value of the property by the tax rate.

(*a*) When the rate is stated as a per cent.

Example. Bennet's property is assessed at $3000; the tax rate is 1.6 %.

Solution.

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 $3000 assessed valuation
  .016 tax rate
$48.00 tax
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(*b*) When the rate is stated as a certain number of mills on the dollar.

Example. Taylor's property is assessed at $3800. The rate is 24 mills.

SOLUTION.

$$\begin{array}{rl} \$3800 & \text{assessed valuation} \\ .024 & \text{tax rate in mills} \\ \hline \$91.20 & \text{tax} \end{array}$$

(*c*) When the rate is stated as a certain number of dollars on each hundred dollars of assessed value.

Example. Finch's property is assessed at $5470. The tax rate is $1.95.

SOLUTION.

$$\begin{array}{rl} \$1.95 & \text{the rate per hundred dollars} \\ 54.70 & \text{the number of hundreds of dollars assessed value} \\ \hline \$106.67 & \text{the tax} \end{array}$$

Oral Work

1. State each of the following tax rates in two other ways:

3.6 %   27 mills $2.83

2. A certain state assesses property at $\frac{2}{5}$ of its real value. What is the assessed value of a house worth $5000? Of a factory worth $15,000? Of a building worth $4000?

3. Dickinson has a house assessed at $2000. The rate is 2.5 %. What is his tax?

4. Barnes has a store building assessed at $3000 and a house assessed at $2000. The tax rate is 30 mills. What is his tax?

5. If property is assessed at $\frac{1}{3}$ of its real value, what is the assessed value of a city lot worth $1200? How much tax will the owner pay if the tax rate is 22 mills?

6. Property valued at $6000 is assessed at $\frac{1}{5}$ of its value and taxed at a rate of $2.50. What tax does the owner pay?

7. Three men living in different cities were comparing their tax rates. A's rate was $3.56; B's rate was 28 mills; C's rate was 3.14 %. Which had the largest rate? How much tax should each pay on an assessed value of $2000?

8. Bailey owns property worth $15,000 and is taxed $4.50 per hundred dollars on $\frac{1}{5}$ of the real value. Osborne owns property worth $15,000 and is taxed 32 mills on $\frac{1}{3}$ the real value. Which pays the larger tax? How much does each pay?

9. Powell pays 3 % on $\frac{1}{3}$ of the real value of his property. What per cent of the value of his property does he pay annually as taxes?

10. A loaned money at 6 % and took a mortgage. Since a mortgage is personal property, it is taxable. The mortgage was taxed at a rate of $4 on $\frac{1}{5}$ the real value. What per cent of the value of the mortgage did A pay annually as taxes? What was his per cent of net income on the loan?

Written Work

1. Complete the following table:

REAL VALUE OF PROPERTY	FRACTION OF VALUE ASSESSED	ASSESSED VALUE	RATE	TAX
$16,250	$\frac{1}{5}$		$5.42	
7,920	$\frac{1}{3}$		3.18 %	
22,000	$\frac{2}{5}$		$2.89	
960	$\frac{1}{5}$		64 mills	
4,000	$\frac{1}{3}$		2.92 %	

2. Hill rents his house for $30.00 per month. The house cost him $3850.00. What is the per cent of net profit on his investment, after paying the following annual expense?

Insurance, $15.40.

Repairs, $60.00.

Taxes: Property assessed at $\frac{1}{5}$ its value and taxed at $6.23 per hundred dollars.

300. To determine the tax rate. Each of the taxing governments (state, county, township, city, school district, etc.) prepares a budget or estimate of the money needed for the following year.

The assessed value of the property available for taxes is found from the assessors' lists.

The amount of funds needed, divided by the assessed value of the property, gives the rate which must be levied in order to obtain the required amount.

The rates for the state, county, city, school district, etc., are added to form a single rate.

The following illustration shows the method of determining a tax rate.

STATE TAX

State budget	$3,600,000
Assessed value of the property in the state	$1,000,000,000

$36,000 ÷ $1,000,000,000 = .36 % state rate

COUNTY TAX

County budget	$30,000
Assessed value of the property in the county	$4,000,000

$30,000 ÷ $4,000,000 = .75 % county rate

CITY TAX

City budget	$12,000
Assessed value of property in the city .	$1,000,000

$12,000 ÷ $1,000,000 = 1.20 % city rate

SCHOOL TAX

School district budget	$24,000
Assessed value of the property in school district	$1,000,000

$24,000 ÷ $1,000,000 = 2.40 % school tax rate

4.71 % total tax rate

Example. Mr. Owen has a building worth $15,000.00, which is assessed at $\frac{1}{3}$ of its real value. What is the tax on the building at the above rate?

SOLUTION. Assessed value, $5,000.00.
Rate, 4.71 %.
.0471 × $5,000.00 = $235.50, tax.

Oral Work

Explain why the tax rate in your city may be different from the rate in a neighboring city.

Written Work

1. Find the tax rate (approximate).

State budget, $2,850,000.
Assessed value of property in the state, $1,450,000,000.
State rate?

County budget, $33,000.
Assessed value of property in county, $5,600,000.
County rate?

City budget, $14,800.
Assessed value of property in city, $1,345,000.
City rate?

School district budget, $32,500.
Assessed value of property in school district, $1,345,000.
School tax rate?

Total rate?

2. The following is the tax rate in the town of M.

State tax	$ .60
County tax	1.25
Township tax	.26
City tax	2.18
School district tax	2.86
	$7.15

In this town, property is assessed at $\frac{1}{5}$ of its real value.

Snyder owns the following property:

	REAL VALUES
Residence	$6,500.00
Store	4,950.00
Office building	13,000.00

What is his total tax?

3. How much of this tax goes to the state, and how much to each of the other taxing units?

4. What per cent of his tax goes to each of the taxing units?

5. The assessed value of Miller's real estate is $13,395.00. What is his total tax?

6. How much school tax does Miller pay?

CHAPTER XXXVI

THE INCOME TAX

301. Basis of the Tax. The income tax bill, which became a law on October 3, 1913, and which was amended Sept. 8, 1916, places a tax on all net incomes in excess of $3000.00 a year in the case of single persons, whether adults or minors, and in excess of $4000.00 per year in the case of the head of a family. The normal tax is based on the *excess* only. The tax is levied on American citizens living in this country and abroad, on aliens living in the United States, on corporations, joint-stock companies or associations, and insurance companies. Exemptions are not allowed except in the case of individual citizens.

302. Gross Income. Gross income consists of salaries, wages, interest, rent, and all profits or income arising or accruing from whatsoever source, with the exception of interest on the bonds of the United States government and its political subdivisions. Property acquired by gift or bequest, and the proceeds of life insurance policies paid on the death of the insured, are not considered as income.

303. Net Income. Only the net income is taxed. The net income for the purpose of the tax is found by deducting from the gross income all necessary expense actually paid in carrying on the business (but not personal, living, or household expenses); interest paid on indebtedness; taxes; fire losses not covered by insurance or otherwise, loss from bad debts, if actually charged off; a reasonable depreciation on the value of property.

304. The Normal Tax. The rate of the normal tax is 2% of the net income above the amount exempted.

Example. A married man living with his wife has a net income of $7000. How much income tax is he required to pay?

Solution.

$7000	Net income
4000	Exemption
$3000	Taxable income
.02	Normal rate
$ 60	Income tax

305. The Additional or Super Tax. In addition to the normal rate of 2% levied on incomes in excess of the exemption, an additional tax is assessed on large incomes.

The following table gives the additional rates charged on the portions of net income which exceed specified amounts.

On the amount by which the total net income (not deducting the $3000 or $4000 exemption applying to the normal tax).

exceeds	$ 20,000	but does not exceed	$ 40,000	1%
exceeds	40,000	but does not exceed	60,000	2%
exceeds	60,000	but does not exceed	80,000	3%
exceeds	80,000	but does not exceed	100,000	4%
exceeds	100,000	but does not exceed	150,000	5%
exceeds	150,000	but does not exceed	200,000	6%
exceeds	200,000	but does not exceed	250,000	7%
exceeds	250,000	but does not exceed	300,000	8%
exceeds	300,000	but does not exceed	500,000	9%
exceeds	500,000	but does not exceed	1,000,000	10%
exceeds	1,000,000	but does not exceed	1,500,000	11%
exceeds	1,500,000	but does not exceed	2,000,000	12%
exceeds	2,000,000			13%

Example. Dudley, an unmarried man, has a net income of $125,000 per year. What is his income tax?

Solution.

$125,000	Total net income
3,000	Exemption
$122,000	Basis of normal tax

2% of $122,000	$2440	Normal tax
1% of $ 20,000 (excess between $20,000 and $40,000)	200	Additional tax
2% of $ 20,000 (excess between $40,000 and $60,000)	400	Additional tax
3% of $ 20,000 (excess between $60,000 and $80,000)	600	Additional tax
4% of $ 20,000 (excess between $80,000 and $100,000)	800	Additional tax
5% of $ 25,000 (excess of total net income over $100,000)	1250	Additional tax
	$5690	Total tax

306. Collection at the Source. The law provides that the tax on incomes derived from interest, rent, salaries, wages, and other fixed annual gains payable to an individual, shall be deducted by the debtor and paid by him to the government. This is called collection at the source.

No deduction is made unless the total interest, rent, or salary, etc., paid to one individual during the year exceeds $3000.00. If the annual payment exceeds $3000.00, 2% of the total is deducted, unless the creditor files with the debtor a certificate claiming the exemption of $3000.00 or $4000.00 provided by law. When exemption is claimed, 2% is deducted from the amount in excess of the exemption.

Examples. 1. Jones loans $5000 to Smith at 6% interest. The annual interest is $300. Since the annual payment is less than $3000, Smith pays Jones the full amount of the interest without deduction.

2. Jones loans Smith $60,000 at 6% interest. The annual interest is $3600.

a. Jones has already claimed his exemption on some other income, and is therefore not permitted to file a certificate of exemption with Smith. Smith therefore deducts 2% of $3600, or $72, and pays Jones the balance, $3528. Forms are provided for notifying the collector of internal revenue, who later sends Smith a bill showing the amount to be paid.

b. Jones, being a single man, and not having filed certificate of exemption elsewhere, files a certificate with Smith, claiming an exemption of $3000. Smith deducts 2% of $600 (the excess of the interest over $3000) and pays Jones the balance, $3588. He notifies the collector of internal revenue that he has deducted the $12 tax at the source, and on receiving a bill from the collector he forwards all money so retained.

c. If Jones were a married man, he could file a certificate of exemption for $4000. Smith would send this certificate to the government collector, and pay Jones the entire amount of the interest, $3600.

In illustration *c*, Jones has not exhausted his exemption. He can claim a $400 exemption from some other debtor.

The certificates are forwarded by the debtors to the collector of internal revenue so that the government can check up the exemptions and prevent any person from claiming more than his legal right. If a person files certificates at the source, claiming greater exemption than he is entitled to, he becomes liable to a fine of $300.

Collecting at the Source from Corporations. If the debtor is a corporation, 2% of *all* interest must be deducted, whether the annual amount exceeds $3000 or not, unless a certificate of exemption is filed.

Example. Osborne owns a bond of $1000, payable by the Franklin Company, a corporation, and bearing 6% interest, payable semi-annually. July 1, he clips the interest coupon for $30, and presents it to his bank for collection. The coupon must be accompanied by a certificate which states either that Osborne claims exemption, or that he does not claim exemption.

a. Osborne is married: he attaches a certificate to the coupon claiming exemption of $4000. He will receive the full $30.

b. Although Osborne is entitled to an exemption of $4000, he presented other coupons yesterday for $4000 and claimed his full exemption on them. He, therefore, attaches a certificate to this coupon, stating that he does not claim further exemption, and 2% of the $30 is deducted.

307. The Corporation Tax. Corporations are required to pay a tax on their entire net income without any exemption. They are subject to the normal tax of 2% only.

Example. The Hayes Corporation had a net income during the year 1916 of $15,250. What was its tax?

Solution. 2% of $15,250 = $305.00, the corporation tax.

Since the corporation has paid the tax on its net earnings, the individual stockholder is permitted to deduct the dividends received from stock from his gross income, when determining his net income, on which the normal tax is computed. But dividends must be added to the income to determine the basis of the additional tax. This provision is made so that taxes will not be paid twice on the same income.

Written Work

Find the income tax on the following:

1. Net income, $5000; single man.
2. Net income, $5000; married man.
3. Net income, $63,000; single man.
4. Net income, $63,000; married man.
5. Net income, $126,000; married man.
6. Net income, $580,000; single man.

7. Find the net income and the income tax (married man). Salary, $3500; rent on buildings owned by him, $1250; interest on money loaned by him, $600.

He pays $1400 interest on money which he has borrowed; repairs, insurance, and depreciation on rented property, $165; state and local taxes, $255.

8. Find the net income and the tax (single man). Salary, $5000; profit from sale of land, $2400; royalties on a patent, $2875.

During the year, a fire caused a loss of $14,000 partially covered by a $10,000 insurance policy. State and local taxes, $420.

9. Find the net income and the tax (married man).

Cost of goods sold during the year .	$146,000
Gross sales	197,000

Rent, insurance, wages of employees, and other business expenses, $14,000. Loss from bad debts, actually incurred, determined, and charged off, $985. He had borrowed $5000, on which he paid a year's interest at 5%. His store building was worth $8000 and he estimated the annual depreciation at 2% of the value of the building.

10. Carter, a married man, owned a factory which made a net annual profit of $15,000. Rent on houses owned by him, $3400; repairs, taxes, and depreciation on rented property, $815. He received $1800 dividends from a corporation which had already paid the corporation tax. He had loaned $25,000 to Mitchell for a year at 6%.

What was his gross income, net income, and total income tax?

11. Potter, a single man, had $80,000 invested in bonds paying 5% interest. What was the annual interest on these bonds? How much would his annual income tax amount to? If he filed certificates for his full legal exemption, how much of the tax would be paid at the source?

12. Walker, a married man, had the following gross income, expenses, and losses during the year:

Gross Income

Salary, $4000; interest on money loaned on notes, $750; dividends from stocks of a corporation, $1200; gain on sale of farm, $3200; interest on bonds, $5200.

Expenses and Losses

Interest on money borrowed, $60; taxes, $945.

What was his gross income?

What was his net income?

What was his total income tax?

If he filed certificates when collecting the bond coupons, claiming his full exemption, how much of his tax was paid at the source?

13. How much income tax is paid by corporations making net profits of $1800? $2500? $74,000? $825,000?

CHAPTER XXXVII

CUSTOMS DUTIES

308. Tax on Imports. The national government collects a tax from persons who import certain kinds of merchandise into the United States from foreign countries. The taxes thus imposed and collected are called **Customs Duties.**

The process of importing goods is, very briefly, as follows:

The foreign shipper prepares an invoice of the goods, showing their value in the coinage of the exporting country.

Invoices of goods with a value of more than $100.00 are certified by the United States consul in the foreign city. The consul retains one copy of the invoice and sends another copy to the collector of the United States port to which the goods are shipped.

When the goods are delivered to the steamship company, the shipper receives a bill of lading, which is a receipt for the merchandise. This bill of lading and an invoice are sent to the American importer.

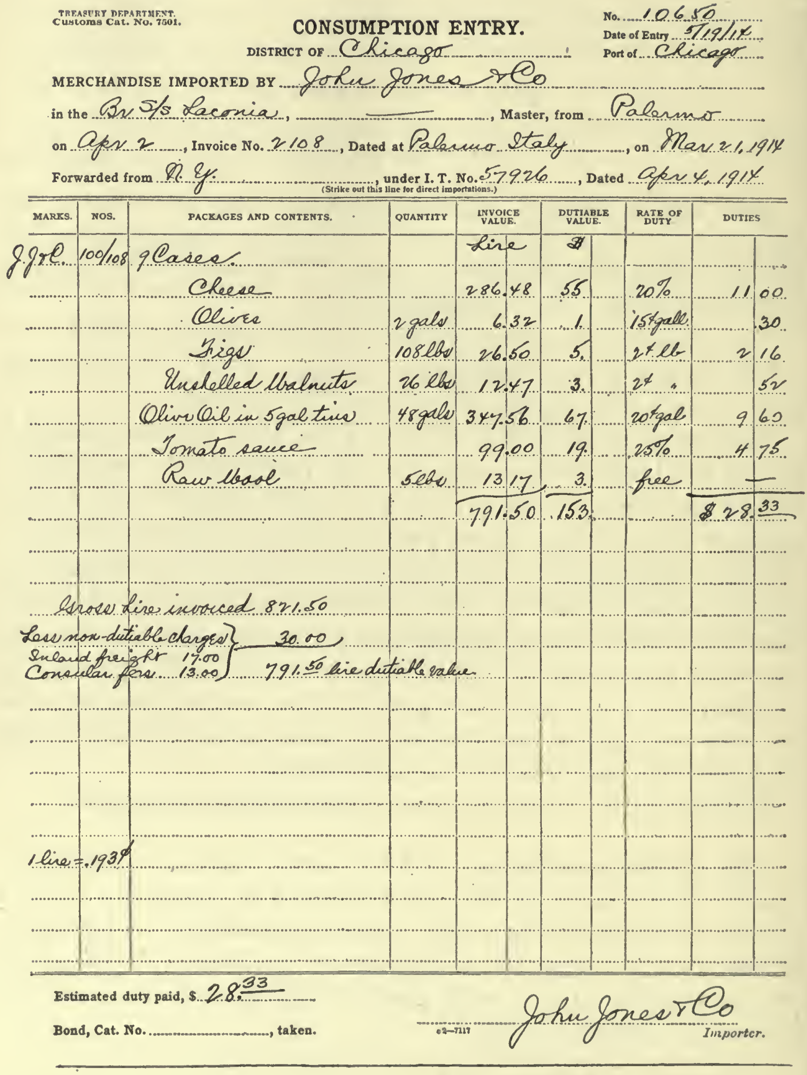
When the goods arrive at port in this country, the importer presents the invoice and the bill of lading at the custom house, pays the duty, and receives his merchandise.

Customs duties are of two kinds: **ad valorem** and **specific.**

309. Ad Valorem Duties. "Ad valorem" means: *according to value.* When an ad valorem duty is placed on goods, the tax is a certain per cent of the value of the merchandise. Ad valorem duties are not computed on fractions of a dollar. If the cents are less than fifty, they are dropped; if fifty or more, they are considered an additional dollar.

To compute ad valorem duty:

Reduce the value of the goods stated in foreign money to United States money. Multiply the value by the tariff rate.



TREASURY DEPARTMENT.
Customs Cat. No. 7501.

CONSUMPTION ENTRY.

No. 10650
Date of Entry 5/19/14
Port of Chicago

DISTRICT OF Chicago

MERCHANDISE IMPORTED BY John Jones & Co

in the Br S/s Laconia, ——, Master, from Palermo

on Apr 2, Invoice No. 2108, Dated at Palermo Italy, on Mar 21, 1914

Forwarded from N. Y., under I. T. No. 57926, Dated Apr 4, 1914
(Strike out this line for direct importations.)

MARKS.	NOS.	PACKAGES AND CONTENTS.	QUANTITY	INVOICE VALUE.	DUTIABLE VALUE.	RATE OF DUTY	DUTIES
J. J. & C.	100/108	9 Cases.		Lire	$		
		Cheese		286.48	55	20%	11 00
		Olives	2 gals	6.32	1.	15¢ gall.	.30
		Figs	108 lbs	26.50	5.	2¢ lb	2 16
		Unshelled Walnuts	26 lbs	12.47	3.	2¢ "	52
		Olive Oil in 5 gal tins	48 gals	347.56	67.	20¢ gal	9 60
		Tomato sauce		99.00	19.	25%	4 75
		Raw Wool	5 lbs	13.17	3.	free	—
				791.50	153.		$ 28.33

Gross lire invoiced 821.50
Less non-dutiable charges } 30.00
Inland freight 17.00
Consular fees 13.00 } 791.50 lire dutiable value

1 lire = .193¢

Estimated duty paid, $ 28.33

Bond, Cat. No. ...., taken.

e2—7117

John Jones & Co
Importer.

The Secretary of the Treasury proclaims the value of foreign coinage in terms of United States money, and the most recent proclamation is used as the basis of money reduction. A copy of such a proclamation will be found on page 234.

Example. What is the duty on an importation of woolen dress goods valued at £30, the tariff rate being 35 % ad valorem?

Solution. Value of shipment in foreign coinage, £30.
Value in United States coinage, $146.00.
35 % of $146.00 = $51.10, amount of duty.

310. Specific Duties. Specific duties are levied according to the quantity of the goods imported, without respect to their value. The rate is usually stated as a certain number of cents per pound, per hundred pounds, per ton (2240 lb.), per yard, or other standard of measure. Allowance is sometimes made for tare and breakage.

To compute specific duty. When the metric system of measurement is used in the foreign invoice, the quantity must be reduced to the American standards. Duties are not computed on fractions of a unit of measure. Fractions less than $\frac{1}{2}$ are dropped; $\frac{1}{2}$ or larger fractions are considered as an additional unit.

Example. The tariff rate on macaroni is 1 cent per pound. What is the amount of the duty on a shipment of 200 kilos of macaroni?

Solution. 1 kilo = 2.2046 lb.
200 kilos = 440.92 lb., or 441 lb.
Duty on 441 lb. at 1 cent per pound is $4.41.

311. Ad Valorem and Specific Duties. In some cases both an ad valorem and a specific duty are levied on the same importation of merchandise.

Example. The duty on perfumery containing alcohol is 40 cents a pound and 60 % ad valorem. What is the amount of the tariff tax on an importation of 10 kilos of perfumery valued at 970 francs?

Solution. 10 kilos = 22.046 lb., or 22 lb.

$$\begin{array}{rl} \$\ .40 & \text{duty per pound} \\ \underline{22} & \text{number of pounds} \\ \$8.80 & \text{specific duty} \end{array}$$

1 franc = $.193
$.193 × 970 = $187.21 value of perfumery
60 % of $187 = $112.20 ad valorem duty
$ 8.80 specific duty
112.20 ad valorem duty
$121.00 total duty

312. The Tariff. Congress revises the tariff law at various intervals. The law prescribes the tariffs, or rates, to be charged on various imports. The *free list* comprises those articles on which no duty is placed.

The following are some of the customs duties from a recent tariff.

Artificial flowers, 60 % ad valorem
Cabbage seed, 6 cents per pound
Castile soap, 10 % ad valorem
Cheese, 20 % ad valorem
Cigars, $4.50 per pound, and 25 % ad valorem
Cocoa, unsweetened, 8 % ad valorem
Cotton batting, 25 % ad valorem
Lithographed booklets, 7 cents per pound
Oriental rugs, 50 % ad valorem
Poppyseed, 15 cents per bushel
Silk ribbons, 45 % ad valorem
Toys, 35 % ad valorem
Tulip bulbs, $1.00 per thousand
Woolen dress goods, 35 % ad valorem

Written Work

Refer to the tariff rates above, and compute the duty on the following importations of merchandise.

1. 50,000 tulip bulbs from Holland.

2. Woolen dress goods from England, valued at 125 pounds sterling.

3. Castile soap from Italy, valued at 40 liras.

4. An invoice of cotton batting valued at £20 15*s.* 6*d.*

5. Lithographed booklets, weight 247 lb. 9 oz.

6. A shipment of cheese valued at 820 marks.

7. Silk ribbons from Paris, invoiced at 5400 francs.

8. Toys from Germany, valued at 1000 marks.

9. An invoice of 3000 cigars from Germany. The cigars weigh $12\frac{3}{8}$ ounces per hundred, and are invoiced at 52 marks per thousand.

10. The tariff rate on Cuban cigars is 20 % less than the rate applying to cigars shipped from other countries. What is the duty on a shipment of 7 cases, each containing 3000 cigars, weight $14\frac{3}{8}$ oz. per hundred; value, $50.00 per thousand?

BUSINESS ORGANIZATION

CHAPTER XXXVIII

INDIVIDUAL PROPRIETORSHIP

BUSINESS enterprises are usually organized in one of the following ways:

Individual Proprietorship. **Partnership.** **Corporation.**

313. Terms Employed. The property owned by, and the debts owed to, a business are its **Resources,** or **Assets.**

The debts owed by a business are its **Liabilities.**

When the resources exceed the liabilities, the difference is called the **Net Worth, Net Capital,** or **Net Resources.**

When the liabilities exceed the resources, the difference is called the **Net Insolvency.**

314. Statement of Resources and Liabilities. The net worth, or the net insolvency, of a business is shown by a **Statement of Resources and Liabilities.** The following is a statement of the financial condition of the business owned by J. W. Waterbury.

J. W. WATERBURY
STATEMENT OF RESOURCES AND LIABILITIES
June 30, 1917

		Resources					
		Cash	Balance on hand	3500	00		
		Real Estate	Store and lot	7500	00		
		Merchandise	Inventory	6325	00		
		James Fitch	Owes the business	110	00		
		F. G. Beach	Owes the business	65	35		
		Total Resources				xxxx	xx
		Liabilities					
		T. E. Madison	Business owes him	123	60		
		Henry Marsden	Business owes him	962	85		
		Total Liabilities				xxxx	xx
		J. W. Waterbury's net worth				xxxx	xx

What are the total resources?
What are the total liabilities?
What is the net worth of the business?

315. Notes Receivable and Notes Payable. Notes Receivable include such written obligations as promissory notes, and acceptances, payable to the business. Their number and value are shown by the record in the Notes Receivable Book. Notes Payable include such written obligations as promissory notes, and acceptances, payable by the business. Their number and value are shown by the record in the Notes Payable Book.

Computing the Interest or Discount on Notes Receivable and Notes Payable. Interest-bearing Notes Receivable and Notes Payable are worth their face *plus* accrued interest. The accrued interest should therefore be included in the statement of resources and liabilities. The accrued interest is the interest on the principal for the expired time.

Example. J. C. Danner is making a statement of resources and liabilities on December 31, 1917.

Among his resources is a note for $600.00, made by I. M. Henderson, on December 1, 1917, due in 90 days, with interest at 6%. Thirty days have expired since the note was made, and thirty days' interest has therefore accrued. The amount of this interest, $3.00, being an obligation payable to the business, should be entered among the resources.

Among his liabilities is a note for $500.00, made by him on December 16, payable in two months, with interest at 6%. Fifteen days' interest has accrued on this note. The amount of the accrued interest, $1.25, being an obligation payable *by* the business, should be entered among the liabilities.

Non-interest-bearing Notes Receivable and Notes Payable are worth, on any day, their face value, *minus* the discount from that day until the day of maturity. Therefore:

Non-interest-bearing Notes Receivable should be entered at their face value among the resources, and the amount of the discount should be entered among the liabilities.

Non-interest-bearing Notes Payable should be entered at their face value among the liabilities, and the discount should be entered among the resources.

Written Work

1. Complete the following:

HENRY PETERMAN
STATEMENT OF RESOURCES AND LIABILITIES
June 30, 1917

	Resources		
Cash	On hand	2365.90	
Merchandise	Inventory	8926.95	
Notes Receivable	A. B. Hines's note	400.00	
Notes Receivable	H. E. Benton's note	975.00	
Interest	24 days' accrued on Benton's note	?	
Discount	15 days' discount on note favor of Row	?	
Smith & Edwards	Owe the business	825.19	
Total Resources			xxxx.xx
	Liabilities		
Notes Payable	My note favor E. A Clark	360.00	
Notes Payable	My note favor A. B. Row	420.00	
Interest	12 days' accrued on note favor of Clark	?	
Discount	25 days' discount on A. B. Hines's note	?	
William Davis	Business owes him	38.26	
Total Liabilities			xxx.xx
Henry Peterman's Net Capital			xxxx.xx

Explanation of Notes Receivable and Notes Payable. A. B. Hines's note was dated June 25, due in 30 days, without interest. It was, therefore, due on July 25, and was worth its face minus 25 days' discount.

H. E. Benton's note was dated June 6, payable in 3 months, with interest at 6%. 24 days' interest accrued since June 6.

Peterman's note in favor of Clark was made on June 18, with interest at 6%. 12 days' interest accrued on this note.

Peterman's note in favor of Row was drawn on May 15, payable in 2 months, without interest. It was due on July 15, or 15 days from the day the statement was made. Fifteen days' discount must therefore be computed on this note.

Written Work

1. Prepare Statements of Resources and Liabilities from the following facts:

Facts for statement of the financial condition of O. E. Benton, December 31, 1917.

Cash, $4624.27; merchandise inventoried at $12,417.95; value of store and lot, $5000.00; furniture and fixtures valued at $420.00; delivery equipment, estimated value $270.00.

Personal Accounts

F. E. Fairchild owes the business $215.20.
George Finch owes the business $28.45.
Edgar Knight owes the business $103.25.
Business owes Justin & Co. $856.75.
Business owes F. R. Rickert $49.00.

Notes and Acceptances

On October 27, 1917, Benton gave a note to Eldridge & Peabody for $300.00. The note is due in 4 months, and bears interest at 6%.

Compute interest for exact number of days.

On December 26, 1917, Benton accepted a draft payable 30 days after sight, for $85.00, without interest, in favor of S. B. Gridley.

316. The Profit and Loss Statement.

Gains – Losses = Net Gain.
Losses – Gains = Net Loss.

The statement showing Net Gain or Net Loss is called the Profit and Loss Statement.

D. L. Bailey

Profit and Loss Statement, Month Ending October 31, 1917

Gains		
Net Sales during October		6210.72
Merchandise inventory, Oct. 1, 1917	1233.60	
Purchases, October	4394.80	
Total	5628.40	
Less inventory, October 31, 1917	975.62	
Cost of goods sold during October		4652.78
Gross trading profit		1557.94
Interest On money loaned	35.60	
On money borrowed	6.92	28.68
Total Gains		1586.62
Losses and Expenses		
Rent	175.00	
Salaries	460.00	
Advertising	56.20	
Heat and light	32.00	
Delivery charges	56.35	
Insurance and taxes	73.85	
General expense	92.74	
Depreciation	70.00	
Loss from bad debts	15.75	
Total losses and expenses		1031.89
Net Gain		554.73

Written Work

1. Percentage Analysis of Profit and Loss Statement.

Answer the following questions:

The net profit shown by the above statement is what per cent of the net sales?

Each of the losses or expenses is what per cent of the net sales?

2. Prepare a Profit and Loss Statement and a Percentage Analysis from the following facts:

Statement for the year 1917, made December 31, 1917. Business owned by Oscar Clark.

Net Sales, 1917, $ 26,746.27.
Purchases, 1917, $ 22,860.39.
Inventory, December 31, 1916, $ 4,219.26.
Inventory, December 31, 1917, $ 5,126.99.

Interest:

6 months' accrued interest at 6 % on Clark's note of $ 345.00 in favor of Jenkins.

30 days' accrued interest at 5 % on Clark's note of $ 280.00 in favor of Michelson.

2 months' accrued interest on Bailey's note in favor of Clark. Face of note, $ 375.00; rate, 6 %.

28 days' interest has accrued on a note given by Snyder to Clark on December 3, 1917. Face of note, $ 240.00; rate, 6 %.

An extra salesroom was rented for the year at a monthly rental of $ 20.00.

Salaries:

Clerk, 12 months at $ 35.00 per month.
Delivery boy, 3 months at $ 15.00 per month;
9 months at $ 16.00 per month.

Heat and light, $ 119.25.

Feed and care of delivery horse, $ 64.00.

Value of store building, $ 3000.00; value of lot, $ 800.00.

Store and lot assessed at $\frac{1}{3}$ of its value and taxed at a rate of $2.16.

Stock valued at $4000.00; assessed at $\frac{1}{3}$ of its value and taxed at a rate of $ 2.16.

An insurance policy was carried on the store and one on the stock.

Store insured at 80 % of its value; rate, $ 1.15.

Stock insured under a policy of $ 3000.00; rate, $ 1.15.

Annual depreciation on store building charged at 3 %.

Loss from bad debts estimated at $\frac{1}{4}$ % of gross sales.

General expense, $ 114.17.

CHAPTER XXXIX

PARTNERSHIP

"A **partnership** or **firm** is an association composed of two or more persons, formed for the purpose of carrying on, as co-owners, a business with a view to profit." * Partners are liable individually for the debts of the partnership which cannot be paid from the resources of the partnership.

317. Sharing Profits and Losses Equally. When partners make equal investments and contribute equal values of skill and service, it is customary to divide gains and losses equally. If the partnership contract does not specify the ratio in which profits and losses are to be shared, the law provides that they shall be shared equally.

Written Work

1. Oldham, Barnes & Snowden each invests $4000 in a partnership, the gains and losses of which are to be shared equally. The following statements show the distribution of the net gain. Complete the statements.

OLDHAM, BARNES & SNOWDEN

Profit and Loss Statement for the Year Ending December 31, 1917

		Gains				
	Mdse. Sales			18,226.55		
	Mdse. Purchases		17,847.90			
	Mdse. Inventory Dec. 31, '17		3,472.00			
	Cost of goods sold			xx,xxx.xx		
	Mdse. Gain (gross trading profit)				xxxx.xx	
	Purchase Discounts			135.85		
	Sales Discounts			57.80	xx.xx	
	Gain					xxxx.xx
	Total Gains					
		Losses				
	Expense	For the year			1245.25	
	Interest	On borrowed funds		28.35		
	Interest	On money loaned		3.50	xx.xx	
	Total Losses					xxxx.xx
	Net Gain	For the year				xxxx.xx

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* Sec. 8 of Draft of an Act to make uniform the Law of Partnership.

OLDHAM, BARNES & SNOWDEN

STATEMENT OF DISTRIBUTION OF PROFIT

Year Ending December 31, 1917

G. F. Oldham	Investment	4000.00		
	$\frac{1}{3}$ net gain	xxx.xx		
	Total	xxxx.xx		
	Withdrawals	150.00		
	Net worth		xxxx.xx	
D. M. Barnes	Investment	4000.00		
	$\frac{1}{3}$ net gain	xxx.xx		
	Total	xxxx.xx		
	Withdrawals	125.00		
	Net worth		xxxx.xx	
Wm. A. Snowden,	Investment	4000.00		
	$\frac{1}{3}$ net gain	xxx.xx		
	Total	xxxx.xx		
	Withdrawals	160.00		
	Net worth		xxxx.xx	
Oldham, Barnes & Snowden, Net worth				xxxx.xx

OLDHAM, BARNES & SNOWDEN

STATEMENT OF RESOURCES & LIABILITIES

December 31, 1917

	Resources		
Cash	On hand	3240.90	
Merchandise	Inventory	9872.35	
Accounts Receivable	Per schedule	3132.35	
Notes Receivable	Per notebook	275.00	
Interest	Accrued on Notes Receivable	3.50	
Total Resources			xxxx.xx
	Liabilities		
Accounts Payable	Per schedule	1800.50	
Notes Payable	Per notebook	500.00	
Total Liabilities			xxxx.xx
Net Capital	Of the partnership		xxxx.xx
Divided as follows:			
G. F. Oldham		xxxx.xx	
D. M. Barnes		xxxx.xx	
Wm. A. Snowden		xxxx.xx	
Total (as above)			xxxx.xx

2. Prepare completed copies of the following statements:

WADHAMS & LEE

FINANCIAL STATEMENT

December 31, 1917

	Resources		
Cash	On hand	246.90	
Merchandise	Inventory	5724.60	
Real Estate	Store and lot	4195.00	
Notes Receivable	Per notebook	1600.00	
Interest	Accrued on notes receivable	8.00	
Accounts Receivable	Per schedule	2640.35	
Total Resources			xx,xxx.xx
	Liabilities		
Notes Payable	Per notebook	2000.00	
Interest	Accrued on notes payable	12.50	
Accounts Payable	Per schedule	1347.23	
Total Liabilities			x,xxx.xx
Net Capital			xx,xxx.xx

WADHAMS & LEE

STATEMENT OF PROFITS AND LOSSES

Year Ending December 31, 1917

	Losses			
Taxes	On store and lot	32.00		
	On stock	24.00	xx.	
Insurance	On store	48.00		
	On stock	32.00	xx.	
Interest	Accrued on notes payable	12.50		
	Accrued on notes receivable	8.00	x.xx	
Sales Discounts		86.40		
Purchase Discounts		69.25	xx.xx	
General Expense			1378.90	
Total Losses				x,xxx.xx
	Gains			
Merchandise Sales			11,147.80	
Merchandise Inv. 1/1/'17		3,000.00		
Merchandise Purchase		12,416.20		
Cost of Merchandise		xx,xxx.xx		
Inventory 12/31/' 17		x,xxx.xx		
Cost of Goods Sold			xxxx.xx	
Gross Earnings				x,xxx.xx
Net Loss				xxx.xx

WADHAMS & LEE
CAPITAL ACCOUNTS

James Wadhams	Investment	6000.00		
	Withdrawals	426.93		
	Net Investment	xxxx.xx		
	$\frac{1}{2}$ Net Loss	xx.xx		
Net Capital			xxxx.xx	
Edwin C. Lee	Investment	6000.00		
	Withdrawals	340.70		
	Net Investment	xxxx.xx		
	$\frac{1}{2}$ Net Loss	xx.xx		
Net Capital			xxxx.xx	
Wadhams & Lee, Net Capital				xx,xxx.xx

318. Sharing Profits and Losses According to Arbitrary Proportion. When partners contribute unequal amounts of time or skill, they frequently divide the profits in some arbitrary proportion.

Written Work

1. Davis, a lawyer, admits two younger lawyers, Fritz and Healy, into partnership. Since Davis has an established practice, it is agreed that he shall receive $\frac{2}{5}$ of the net earnings, and that Fritz and Healy shall divide the remainder equally.

The net earnings the first year are $8000 ; the second year they are $9260. Compute the division of the profits each year.

319. Sharing Profits and Losses According to Investments. When partners contribute unequal amounts of capital, gains or losses are often shared in proportion to the capital invested.

Example. Nelson invested $2000, Baker $3000, and Anderson $5000 in a business which gained $1256.70. What was each partner's share of the gain?

SOLUTION.

PARTNERS	INVESTMENT	PROPORTION OF INVESTMENT	SHARE OF GAINS OR LOSSES
Nelson	$2000	$\frac{2000}{10000}$	$\frac{1}{5}$
Baker	$3000	$\frac{3000}{10000}$	$\frac{3}{10}$
Anderson	$5000	$\frac{5000}{10000}$	$\frac{1}{2}$

The gain for the year was $1256.70.

$\frac{1}{5}$ of $1256.70 = $251.34, Nelson's share.

$\frac{3}{10}$ of $1256.70 = $377.01, Baker's share.

$\frac{1}{2}$ of $1256.70 = $628.35, Anderson's share.

Written Work

1. A invests $4500. B invests $6000. C invests $7500. Profits and losses are to be shared in proportion to investment. The first year there was a gain of $2461.25. What was the amount of profit received by each partner?

2. On July 1, 1918, D. M. Paine, F. R. Adamson, and B. O. Cone entered into partnership. Gains and losses were to be shared in proportion to investment.

Paine, who had been conducting a grocery business, turned into the partnership his entire resources and liabilities, as shown by the following statement:

D. M. PAINE

STATEMENT OF RESOURCES AND LIABILITIES

July 1, 1918

	Resources		
Cash	On hand	316.75	
Merchandise	Per inventory	4135.60	
Real Estate	Store and lot	3000.00	
Furniture and Fixtures		350.00	
Accounts Receivable	Per ledger	1237.80	
Total			xxxx.xx
	Liabilities		
Accounts Payable	Per ledger	215.60	
Notes Payable	Per bill book	1245.00	
Interest	Accrued on notes payable	3.11	
Total			xxxx.xx
Net Capital	D. M. Paine		xxxx.xx

He gave his note with interest at 6%, dated July 1, 1918, payable to Paine, Adamson & Cone, for an amount sufficient to make his total investment $8000.

Adamson invested the following:

Merchandise, $1200.

A note in his favor, signed by O. G. Dunlap, for $2500, dated April 1, 1918, due in 1 year, bearing 6% interest. (Is this note worth its face, its face plus accrued interest, or its face minus the discount until maturity?)

Cash to make his total investment $ 4000.

Cone invested a draft for $ 740, drawn by himself on Graf & Knight, on June 18, 1918, payable 30 days after sight, without interest. Graf & Knight accepted the draft on June 20, 1918. Cone also invested enough cash to make a total investment of $ 3000.

Prepare a statement of resources and liabilities, showing the net capital of the firm of Paine, Adamson & Cone, at the beginning of business on July 1, 1918. The following illustration will show the form of the statement:

PAINE, ADAMSON & CONE

Statement of Resources and Liabilities, July 1, 1918

	Resources			
Cash	Invested by Paine	xxx.xx		
	Invested by Adamson	xxx.xx		
	Invested by Cone	xxxx.xx	xxxx.xx	
Merchandise	Invested by Paine	xxxx.xx		
	Invested by Adamson	xxxx.	xxxx.xx	
Real Estate			xxxx.	
Furniture and Fixtures			xxx.	
Notes Receivable	Paine's note favor P. A. & C.	xxx.xx		
	Dunlap's note favor Adamson	xxxx.		
	Cone's draft on G. & K.	xxx.	xxxx.xx	
Interest	Accrued on Dunlap's note		xx.xx	
Accounts Receivable from Paine			xxxx.xx	
Total				xx,xxx.xx
	Liabilities			
Notes Payable	Received from Paine		xxxx.	
Interest	Accrued on note rec'd from Paine	x.xx		
	Discount on Cone's draft	x.xx	x.xx	
Accounts Payable	Received from Paine		xxx.xx	
Total				x,xxx.xx
Net Capital	Paine, Adamson & Cone			xx,xxx.

What fraction of the net gains should each partner receive if gain is shared in proportion to capital invested?

On December 31, 1918, a statement of resources and liabilities of the firm showed the following facts:

STATEMENT OF RESOURCES AND LIABILITIES
PAINE, ADAMSON & CONE
December 31, 1918

		Resources		
		Cash On hand	2,215.60	
		Merchandise Inventory	15,465.70	
		Real Estate	2,910.00	
		Furniture and Fixtures	339.50	
		Accounts Receivable	4,285.37	
		Total		xx,xxx.xx
		Liabilities		
		Accounts Payable	1941.20	
		Total		x,xxx.xx
		Net Capital Paine, Adamson & Cone		xx,xxx.xx

What was the increase in the capital of the business during the 6 months' period?

This increase represents the profits remaining in the business. During the 6 months' period, the partners withdrew profits as follows:

PARTNER	AMOUNT
Paine	$1413.32
Adamson	1206.66
Cone	1654.99
	$4274.97

What was the total gain for the 6 months' period?

What share of the gain was each partner entitled to receive?

Each partner has already drawn out a portion of his profits. How much is he still entitled to receive?

The partners decide to add the remainder of their profits to their investments. What was the investment of each partner on January 1, 1919?

Prepare a Statement of Distribution of Profit similar to the model on page 369, showing the original investment, gains, withdrawals, and net worth of each partner as of June 30, 1919.

On the basis of this investment what fractional part of the profits of 1919 should each partner receive?

On June 30, 1919, a statement of resources and liabilities showed the following facts:

STATEMENT OF RESOURCES AND LIABILITIES
PAINE, ADAMSON & CONE
June 30, 1919

Resources			
Cash	On hand	1846.03	
Merchandise	Inventory	8420.50	
Real Estate		2820.00	
Furniture and Fixtures		329.00	
Accounts Receivable	Per ledger	5627.38	
Total			xx,xxx.xx
Liabilities			
Accounts Payable	Per ledger	1240.65	
Notes Payable	Per bill book	1000.00	
Interest	Accrued on notes payable	13.33	
Total			x,xxx.xx
Net Capital Paine, Adamson & Cone			xx,xxx.xx

What was the net gain or net loss for the 6 months' period?

Prepare a statement showing the investment of each partner on January 1, 1919, the distribution of gain or loss, and the investment of each as of June 30, 1919.

320. Computing Interest on Investments and Withdrawals. It is sometimes agreed that each partner shall receive interest on his investment, and that after this interest has been paid, or credited, to the partner, the remaining profits shall be divided according to agreement.

Example. At the beginning of the year Briggs invests $ 8000.00 and Duncan invests $ 7500.00. It is agreed that each partner shall receive 6 % interest on his investment, payable out of the profits, and that the remaining profits shall be divided equally. The net gain for the year is $2800.00. What interest and what share of the remaining gain does each partner receive?

Solution. 6% interest for 1 year on $8000 = $480, Briggs's interest.
6% interest for 1 year on $7500 = $450, Duncan's interest.
$2800 Net gain
930 Total interest
$1870 Remaining profits
$1870 ÷ 2 = $935. Share of profit for each partner
Briggs's total interest and gain, $480 + $935 = $1415.
Duncan's total interest and gain, $450 + $935 = $1385.

Written Work

1. Jones and Morgan began a partnership July 1, 1917; Jones invested $9500.00 and Morgan invested $11,500.00. Each partner was to receive interest on his investment at the rate of 5% per year, the remaining net profit or loss to be divided equally. On December 31, 1917, a statement of the losses and gains of the firm showed a net gain of $815.00. What amount of interest and remaining profit should each partner have received?

2. Wallace and Montgomery entered into partnership on January 1, 1917. Each of these men had formerly conducted a business of his own, and each turned into the partnership his entire resources and liabilities, as shown by the following statements:

L. F. Montgomery

Statement of Resources and Liabilities, December 31, 1916

		Resources			
		Cash	On hand	1215.75	
		Merchandise	Per inventory	3565.80	
		Real Estate	Store and lot	3000.00	
		Fixtures	Per inventory	419.75	
		Accounts Receivable	Per schedule	625.90	
		Interest	Discount on notes	4.18	
		Total			xxxx.xx
		Liabilities			
		Accounts Payable	Per schedule	1275.90	
		Notes Payable	Per bill book	1418.00	
		Total			xxxx.xx
		Net Investment, L. F. Montgomery			xxxx.xx

IRVING WALLACE

Statement of Resources and Liabilities, December 31, 1916

Resources			
Cash	On hand	865.00	
Merchandise	Per inventory	3462.90	
Fixtures		319.50	
Accounts Receivable	Per ledger	2419.17	
Total Resources			x,xxx.xx
Liabilities			
Accounts Payable	Per ledger	1425.00	
Total Liabilities			x,xxx.xx
Net Investment	Irving Wallace		x,xxx.xx

It is agreed that each of the partners shall receive interest on his investment at the rate of 6% per annum, remaining profit to be divided equally.

On December 31, 1917, the following statement showed the condition of the firm:

WALLACE AND MONTGOMERY
STATEMENT OF RESOURCES AND LIABILITIES

December 31, 1917

Resources			
Cash	On hand	1462.80	
Merchandise	Per inventory	7292.90	
Real Estate	Store and lot	3000.00	
Fixtures	Per schedule	632.86	
Accounts Receivable	Per schedule	1853.27	
Interest	Discount on notes	2.09	
Total			xx,xxx.xx
Liabilities			
Accounts Payable	Per schedule	615.20	
Notes Payable	Per bill book	1418.00	
Total			x,xxx.xx
Net Capital, Wallace and Montgomery			xx,xxx.xx

What amount of interest should each of the partners receive?

After paying interest out of profits, what amount of profit remains, and what share of this profit should each partner receive? No withdrawals were made by either partner.

CHAPTER XL

INSOLVENCY AND BANKRUPTCY

INDIVIDUALS, firms, or corporations which are not able to pay their debts when due, or whose liabilities exceed their resources, are said to be insolvent. Under the Federal law, any person in a condition of insolvency may be declared a bankrupt if he so desires. His resources, frequently called assets, with the exception of certain exempt property, are turned over to a Trustee representing the creditors, and divided proportionately among them, all creditors except those whose claims are preferred or secured receiving the same fractional part or per cent of the amount due them.

321. Bankruptcy Proceedings. The creditors of any insolvent person except a "wage-earner" earning less than $1500 per year, or a "person engaged chiefly in farming or tillage of the soil," may institute bankruptcy proceedings against him, if he has done one of the following acts:

a. Transferred or concealed any of his property with intent to delay payment or to defraud his creditors.

b. Transferred property to one creditor for the purpose of giving him an advantage over the other creditors.

c. Allowed one creditor to obtain a preference through legal proceedings.

d. Made an assignment of all his property to a trustee for creditors.

e. Admitted in writing his inability to pay his debts, and his willingness to be adjudged a bankrupt.

After a debtor has "gone into bankruptcy" and made a settlement by turning over his assets, he is legally discharged from his indebtedness. Any resources which he may possess at a later time cannot be taken in further payment of debts settled in bankruptcy.

Illustration of a Settlement in Bankruptcy. On October 20, 1917, J. D. Murphy was declared a bankrupt, and a trustee was elected by the creditors to take over his assets, convert them into cash, and pay the creditors. Mr. Murphy filed the following schedule of assets and liabilities at book values:

SCHEDULE OF ASSETS & LIABILITIES

ASSETS		
Cash	$ 465.00	
Merchandise	3225.00	
Real estate	8500.00	
Accounts receivable	3688.18	$15,878.18
LIABILITIES		
Taxes	$ 165.00	
Wages	233.00	
Mortgage on real estate	2000.00	
Hartman, Oelberg & Co.	2146.80	
D. G. Barnhard	364.00	
Franklin & Gertz	7685.25	
D. O. Mitchell	8146.30	$20,740.35

The trustee sold the real estate to the holder of the mortgage for $5800 in excess of the mortgage.

Cash receipts were as follows:

From Murphy	$ 465.00	
From the sale of the real estate after paying the mortgage	5800.00	
From the sale of merchandise	2900.00	
From accounts receivable collected	3500.00	$12,665.00

He made payments as follows:

Wages (which are a preferred claim and must be paid before the other liabilities)	$ 233.00	
Taxes (which are also a preferred claim)	165.00	
His fee as receiver, allowed by court	250.00	$ 648.00
Balance of cash on hand		$12,017.00

The unpaid liabilities are as follows:

Hartman, Oelberg & Co.	$2146.80	
D. G. Barnhard	364.00	
Franklin & Gertz	7685.25	
D. O. Mitchell	8146.30	$18,342.35

The receiver will be able to pay $\frac{12017.00}{18342.35}$ or 65.5% of the liabilities. Each creditor will receive 65.5% of his claim. The common expression of this settlement is, "Mr. Murphy paid 65 cents on the dollar."

Written Work

1. On August 25, 1917, Fred Helms made an assignment to his creditors, turning over the following assets to a receiver:

Cash on hand and in bank	$ 85.00
Merchandise inventoried at	2300.00
Store property, book value	3500.00
Accounts receivable	1632.50

His liabilities were as follows:

Wages, preferred claim	$ 95.00
Accounts payable:	
Weston and Burton	1036.49
George Seeley & Co.	378.57
J. G. O'Shea	1275.85
Holden & Plehn	5300.00
Wm. Meyers	3000.00

The receiver is able to realize $2000 on the merchandise. The store property is old and no depreciation has been deducted on Mr. Helms' books. The receiver is able to sell it for only $2400. He collects all of the accounts receivable with the exception of $212.50.

The receiver pays the wages, $95; the expenses of the receivership, $75, and his own fee of $100, which is allowed by the court. The remaining cash is distributed among the creditors.

How much does each creditor receive?

CHAPTER XLI

CORPORATIONS, STOCKS, AND BONDS

THE method and the advantages of organizing a business as a corporation may be shown by an illustration.

William Austin has $75,000.00; $25,000.00 of this amount is invested in business. He has $50,000.00 remaining, which he wishes to invest in a factory for manufacturing carriages.

George Fox has $30,000.00, which represents his entire capital, and he is willing to join Austin in the organization of a carriage manufacturing business.

Austin and Fox believe that a capital of $100,000.00 is necessary for the successful operation of a business such as they wish to organize, but together they are able to invest only $80,000.00. In order to obtain the remaining $20,000.00, they may take one or more persons into partnership, or they may organize a corporation.

322. Partnership and Corporation Contrasted. The disadvantages of a partnership are:

a. Each partner is personally liable for the debts of the partnership.

b. Partners frequently have difficulty in withdrawing from a partnership.

Unless at the time of organizing a partnership a definite date is set when the partnership may be dissolved, no partner can withdraw from the business without the consent of the other partners. The law does not permit a partner to sell his interest in the partnership without the consent of the other partners.

c. The death, bankruptcy, or insanity of a partner dissolves the partnership.

The advantages of a corporation are:

a. Each member (stockholder) of the corporation is liable for only the amount he contributes to the capital of the corporation.

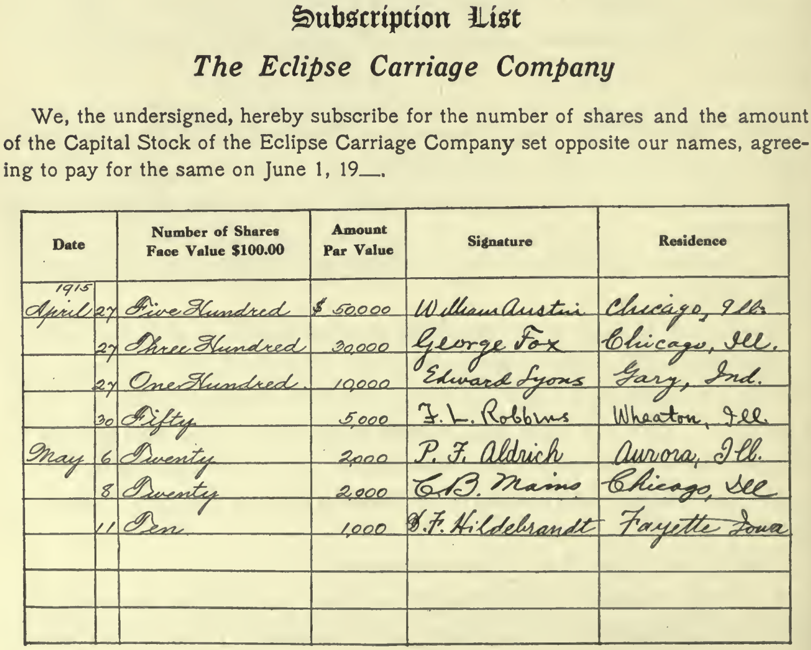
His private resources cannot be taken for the debts of the corporation.

b. Any stockholder may sell his interest in the corporation without the consent of the other stockholders, and without dissolving the corporation.

c. The death or bankruptcy of a stockholder does not dissolve a corporation.

d. By the method of selling shares of stock (to be explained later) an individual may invest a small amount of money in a large corporation. The capital of the railroads and other large corporations is usually furnished by thousands of stockholders. Partnerships of such a size would be unwieldy and impracticable.

Considering these advantages, Austin and Fox decide to incorporate, and after complying with the legal requirements they prepare a subscription list on which to receive subscriptions to the capital stock.



Subscription List

The Eclipse Carriage Company

We, the undersigned, hereby subscribe for the number of shares and the amount of the Capital Stock of the Eclipse Carriage Company set opposite our names, agreeing to pay for the same on June 1, 19__.

Date		Number of Shares Face Value $100.00	Amount Par Value	Signature	Residence
1915 April	27	Five Hundred	$ 50,000	William Austin	Chicago, Ill.
	27	Three Hundred	30,000	George Fox	Chicago, Ill.
	27	One Hundred	10,000	Edward Lyons	Gary, Ind.
	30	Fifty	5,000	F. L. Robbins	Wheaton, Ill.
May	6	Twenty	2,000	P. F. Aldrich	Aurora, Ill.
	8	Twenty	2,000	C. B. Mains	Chicago, Ill
	11	Ten	1,000	J. F. Hildebrandt	Fayette Iowa

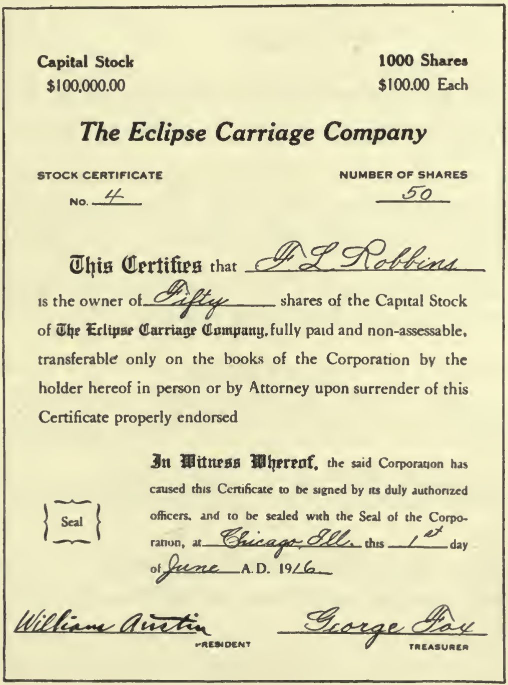
323. Terms Used. The **Capital Stock** of a corporation is the amount of capital authorized by the charter.

Shares. The capital stock is divided into shares of equal size, having a nominal or face value, usually of $100.00. Shares may be of any denomination within the limits fixed by law.

When the stock has been subscribed (and, in some states, paid in) a meeting of the stockholders is held at which meeting a board of directors is elected to manage the business.

At this meeting each share owned by a stockholder entitles him to one vote. (How many votes should Austin have?) By this method of voting it is possible for a few stockholders owning a majority of the shares to control the business.

324. Stock Certificates. Each stockholder receives a certificate, signed by the officers of the corporation, stating the number of shares for which he has subscribed and paid.



Capital Stock
$100,000.00

1000 Shares
$100.00 Each

The Eclipse Carriage Company

STOCK CERTIFICATE
NO. 4

NUMBER OF SHARES
50

This Certifies that F. L. Robbins is the owner of Fifty shares of the Capital Stock of The Eclipse Carriage Company, fully paid and non-assessable, transferable only on the books of the Corporation by the holder hereof in person or by Attorney upon surrender of this Certificate properly endorsed

Seal

In Witness Whereof, the said Corporation has caused this Certificate to be signed by its duly authorized officers, and to be sealed with the Seal of the Corporation, at Chicago, Ill. this 1st day of June A.D. 1926

William Austin
PRESIDENT

George Fay
TREASURER

Oral Work

Refer to the stock certificate, and answer the following questions:

1. What is the total capital stock of the corporation?
2. Into how many shares is the capital stock divided?
3. What is the par (face) value of each share?
4. How many shares does Robbins own? To how many votes is he entitled?
5. What is the par value of his stock?

325. Dividends. The profits of the corporation which are divided among the stockholders are called Dividends. The board of directors has the authority to determine the amount of dividends which shall be declared out of the profits.

326. Surplus. The undivided profits of a corporation are held as *surplus.*

Examples. 1. Let us suppose that the Eclipse Carriage Company's profits at the end of one year amounted to $6800. What per cent of dividends could the directors declare?

SOLUTION. $6800 (Profits) ÷ $100,000 (Capital Stock) = 6.8 %.

Therefore, the largest dividend which the directors could declare is 6.8 %. If such a dividend were declared, each stockholder would receive 6.8 % of the par value of his stock.

Austin, owning 500 shares, with a par value of $50,000, would receive a dividend of 6.8 % of the par value of $50,000, or $3400.

2. Let us suppose that the directors vote to declare a 5 % dividend. How much of the profits would be divided among the stockholders, and how much would remain as surplus?

SOLUTION. 5 % of $100,000 = $5000, Total Dividend.
$6800 (Profits) – $5000 (Dividends) = $1800, Surplus.

327. Par Value and Market Value of Stock. The par value or nominal value of stock is its face value, shown by the stock certificate.

The market value of a share is the amount for which it can be sold. The market value depends upon several factors, the following being the most important:

a. The value of the property owned by the corporation.

When the Eclipse Carriage Company was organized, each stockholder paid for his stock in cash, and for each share of stock there was $100 in the treasury. Each share, therefore, was worth $100. But let us assume that a surplus of $1600 has been created. Since there are one thousand shares, this surplus adds $1.60 to the property represented by each share.

b. The dividends paid by the corporation.

Let us suppose that 9% dividends are declared each year. $9 is 6% income on $150, and Jones might be willing to pay $150 for a share of this stock.

If only 4% dividends were paid, Jones would probably not be willing to pay $100 for a share because he would not receive 6% on his investment.

When stock sells for more than its par value, it is said to be above par, or **at a premium**. Stock selling for $105 per share is quoted at 105.

When stock sells for less than its par value, it is said to be below par, or **at a discount**. Stock selling for $95 is quoted at 95.

328. Assessments. When money is required to cover losses, or to extend the business, in some companies the stockholders may vote to levy an assessment. The assessment is a per cent of the par value of the stock, and is paid by the stockholders to the treasurer of the corporation. Shares of stock may be non-assessable.

Written Work

1. The stock of a corporation has all been subscribed and is owned as follows:

A, 150 shares; B, 220 shares; C, 80 shares; D, 50 shares.

Par value per share, $100.

What is the capital stock of the corporation?

2. How much will a dividend of 8% yield each stockholder?

3. If the annual profits are $4500, what per cent dividend can be declared?

4. If a 5 % dividend is declared from the profits of $ 4500, how much will remain as surplus?

5. If a $2\frac{1}{2}$ % assessment is levied, how much will each stockholder pay to the corporation?

329. Kinds of Stock. There are two principal classes of stock, **common** and **preferred.**

Common Stock entitles its owner to a proportionate share of the net profits of the corporation when dividends are declared by the directors. The rate of dividend is not fixed, but depends upon the success of the business.

Preferred Stock entitles its owner to a fixed per cent of dividend provided the profits are sufficient to pay this dividend. Thus, 5 % preferred stock entitles the owner to a 5 % dividend before any dividends are declared on the common stock.

Cumulative Preferred Stock. Preferred stock may be cumulative or non-cumulative. If the stock is cumulative, a dividend which is unpaid one year becomes a claim against the corporation, to be paid out of future profit.

Participating Preferred Stock. In some states unless the preferred stock is expressly declared to be non-participating, the preferred stock participates equally with the common stock in any dividends after both common and preferred stock have received an equal dividend.

That is, after 6 % preferred stock has received its 6 % dividend and the common stock has also received 6 %, remaining dividends must be shared equally.

Example. The Eclipse Carriage Company desires to build and equip a new factory at a cost of $25,000. A meeting of the stockholders is held, and it is decided to issue 5 % cumulative non-participating preferred stock, in order to obtain the required funds. Two hundred and fifty shares, with a par value of $ 100 each, are issued.

At the end of the year, a profit of $ 9750 is shown. How may this profit be divided among the holders of the common and preferred stock?

SOLUTION. 5 % of $25,000 (Preferred Stock) = $1250, dividend on preferred stock.

$9750 − $1250 = $8500, remaining for dividends on common stock.

An 8½ % dividend may be declared on the common stock.

If an 8½ % dividend is declared, no surplus will remain.

Written Work

If an 8½ % dividend is declared on the common stock:

1. How many dollars are paid as dividend on each share?

2. How many dollars are paid as dividend on each share of 5 % preferred stock?

The profits of the second year are only $500, and the directors are able to pay only a 2 % dividend on the preferred stock. No dividends are declared on the common stock.

3. How much does Clark, who owns one share of preferred stock, receive?

4. Comstock owns five shares of preferred stock. How much does he receive?

5. What is the total unpaid dividend on the preferred stock?

The third year a profit of $12,000 is made. Settlement among the shareholders is made as follows:

$12,000	Profits
750	Last year's unpaid dividends on preferred stock
$11,250	Balance
1,250	Dividends on preferred stock for current year
$10,000	Available for dividends on common stock, and surplus

Clark, the owner of one share of preferred stock, now receives:

3 % unpaid dividends of previous year,	$3.00
5 % dividends of current year,	5.00
	$8.00

Dividends up to 10 % may be declared on the common stock.

From this illustration you should understand:

a. The claim of the owners of cumulative preferred stock for unpaid dividends in case sufficient profits are made in future years.

b. The fact that preferred stock is usually an investment, but common stock is more of a speculation.

c. In a business of proved success, where the preferred stock is non-participating, the common stock may be a better investment than the preferred stock.

330. Buying Stocks. Stocks are bought and sold at market value which may be the same as par value, but which is usually either more or less than par. The quotation is the statement of market value.

Example. Mr. Ross purchased 10 shares of the Eclipse Carriage Company's stock from F. L. Robbins at 115. What was the cost of each share? What was the total cost of the stock purchased?

SOLUTION. $ 115 market value per share
10 number of shares purchased
$1150 cost of ten shares

When this sale of stock is made, Robbins surrenders his stock certificate of fifty shares to the secretary of the corporation, who issues a new certificate to Robbins for forty shares, and a certificate to Ross for ten shares.

331. Dealing in Stock through a Broker. Stock is usually bought and sold through a stock broker. Brokers charge a certain per cent of the par value of the stock which they buy and sell for customers. $\frac{1}{8}$ of one per cent of the par value is a common charge for such services when stock is sold in large blocks.

Examples. 1. What is the cost of 100 shares of stock quoted at $112\frac{1}{2}$, purchased through a broker? Brokerage $\frac{1}{8}$%.

SOLUTION. $112\frac{1}{2}$ or $112.50 market price per share
$\frac{1}{8}$ or $.12\frac{1}{2}$ brokerage added to cost
$112\frac{5}{8}$ or $\$112.62\frac{1}{2}$ cost including brokerage

$100 \times \$112.62\frac{1}{2} = \$11{,}262.50$, cost to purchaser.

2. What is received by the seller of 100 shares of stock quoted at $112\frac{1}{2}$, sold through a broker? Brokerage $\frac{1}{8}$%.

SOLUTION. $112\frac{1}{2}$ or $112.50 price per share
$\frac{1}{8}$ or $.12\frac{1}{2}$ brokerage deducted
$112\frac{3}{8}$ or $\$112.37\frac{1}{2}$ returns per share after deducting brokerage

$100 \times \$112.37\frac{1}{2} = \$11{,}237.50$, returns from sale of 100 shares.

332. Profits from Stocks. Stocks are usually purchased with the hope of making a profit in the following ways:

a. Holding the stock and receiving dividends; or

b. Selling the stock when the market price advances.

Income from Dividends. The per cent of income from dividends depends upon two things:

a. The price paid for the stock.

b. The dividends, which are always expressed in terms of a per cent of the *par value.*

Example. The par value of the common stock of the Butler Dairy Company is $100.00. A dividend of 8% is declared. What per cent of income does the holder of stock receive if the stock cost him $119\frac{7}{8}$, plus brokerage?

SOLUTION.

$\$100.00$	par value of stock
$.08$	rate of dividend
$\$8.00$	dividend per share
$\$119\frac{7}{8}$	market value when purchased
$\frac{1}{8}$	brokerage
$\$120$	cost per share

$\$8.00 \text{ (dividend)} \div \$120 = 6\frac{2}{3}\%$ income on investment.

Profit or Loss from Buying and Selling Stock. Example. Perry buys 200 shares of stock at $107\frac{1}{8}$, and sells at $108\frac{5}{8}$, buying and selling through a broker. What profit does he make?

SOLUTION.

$\$107\frac{1}{8}$	market value when purchased
$\frac{1}{8}$	brokerage added
$\$107\frac{1}{4}$	total cost per share
$\$108\frac{5}{8}$	market value when sold
$\frac{1}{8}$	brokerage deducted
$\$108\frac{1}{2}$	received for each share
$\$108\frac{1}{2}$	received per share
$\$107\frac{1}{4}$	cost per share
$\$1\frac{1}{4}$	profit per share
$\$1.25$	profit per share
200	number of shares
$\$250.00$	profit on 200 shares

Written Work

1. What would have been the total loss if Perry had sold at $106\frac{7}{8}$?

2. What would have been the total loss if he had sold at $107\frac{1}{8}$?

3. 7% preferred stock is purchased at 110, without brokerage. What per cent income does the purchaser receive on his investment?

4. What is the cost of 7 shares of stock, par value 100, purchased through a broker at $109\frac{3}{8}$? Brokerage $\frac{1}{8}$%.

5. $6\frac{1}{2}$% dividends are declared on this stock. What per cent income does the purchaser receive on his investment?

The following is a partial list of the quotations of a day's sales by brokers on the New York Stock Exchange. Current quotations can be found in the daily newspapers.

SALES	DESCRIPTION	OPEN	HIGH	LOW	CLOSE
1,200	Am. Smelting pfd.	$104\frac{1}{2}$	105	102	$103\frac{1}{4}$
2,300	Am. Sugar common	$108\frac{7}{8}$	$108\frac{7}{8}$	107	$108\frac{1}{2}$
400	Am. Sugar pfd.	$113\frac{3}{4}$	$113\frac{3}{8}$	$112\frac{1}{8}$	$112\frac{1}{4}$
1,600	Central Leather pfd.	$96\frac{3}{4}$	$97\frac{1}{2}$	$96\frac{3}{4}$	$97\frac{1}{2}$
22,900	C. M. & St. P. common	$105\frac{1}{4}$	$106\frac{7}{8}$	104	$106\frac{5}{8}$
400	C. M. & St. P. pfd.	$141\frac{1}{8}$	$141\frac{1}{4}$	140	141

6. What is received for 400 shares of Am. Smelting pfd. sold through a broker at the opening price in the table? Brokerage $\frac{1}{8}$%.

7. What is the cost of 400 shares of this stock if purchased through a broker? Brokerage $\frac{1}{8}$%.

NOTE. Other problems may be given by the teacher, either from the above table or from the quotations of sales of stock printed in the daily newspapers.

Written Review

Martin, Owen, and Rathbun wish to organize a corporation. They circulate a subscription list, on which they receive subscriptions for the entire authorized capital stock, as follows:

Subscription List

The Roadbed Grader Company

We, the undersigned, hereby subscribe for the number of shares and the amount of the Capital Stock of the Roadbed Grader Company, set opposite our names, agreeing to pay calls upon the said stock as they shall be made by the directors of company.

Date		**Number of Shares**	**Amount**	**Signatures**
1915 August	4	One Hundred Fifty Shares	$ 15,000	George R. Martin
	4	One Hundred Twenty Shares	12,000	F. G. Owen
	4	One Hundred Ten Shares	11000	D. L. Rathbun
	5	Ninety Shares	9000	D. R. Wilbur
	6	Thirty Shares	3,000	R. V. Sanders

1. What is the capital stock of the company?

2. Into how many shares is the capital stock divided?

3. What is the par value of each share of stock?

4. How many votes does each stockholder have in the meetings of the stockholders?

5. Which two of the stockholders have votes enough (combined) to control the management of the corporation?

6. The Roadbed Grader Company is duly organized, and a call is made on each of the subscribers for one half of his subscription, payable on September 1. Martin, Owen, and Sanders pay the call on September 1. How much is received by the treasurer of the company on that date?

7. Rathbun pays his "call" on September 6, and is charged 6% interest on the $5500 for the 5 days during which the call remained unpaid. How much does Rathbun pay, including call and interest?

8. Wilbur pays his call on September 10, including interest on the delinquent payment. Mr. Wilbur sends a check to the treasurer for $ ——.

9. Call is made for one quarter of the subscriptions, payable on September 20. The subscribers pay as follows:

Martin, September 20;
Owen, September 20;
Sanders, September 22;
Wilbur, September 25;
Rathbun, September 27.

How much does each subscriber pay, including interest?

10. Final call for subscriptions is made payable on October 1, at which time each of the subscribers pays the balance of his subscription. How much does each subscriber pay?

11. The net profits the first year were $4075. How much is the income tax?

12. How much profit remains for dividends and surplus?

13. A 6% dividend is declared on the stock. The total dividend amounted to $——.

14. What was the amount of the surplus?

15. What was the amount of dividend received by each stockholder?

16. Considering (*a*) the dividends, and (*b*) the surplus, if you owned a share of the stock of the Roadbed Grader Co., would you be willing to sell it at par?

17. The net profits for the second year were $4690. After paying the income tax, how much remains?

18. What is the largest per cent of dividend which the directors can declare out of the second year's profits and accumulated surplus?

19. What amount of dividend would each stockholder receive?

20. After the payment of the dividend indicated in question 18, how much surplus remains?

21. Rathbun sells 7 shares of his stock to S. F. Cromer at 114$\frac{3}{8}$, without brokerage. How much does Rathbun receive?

22. The Roadbed Grader Company issues $15,000.00 of 6% non-participating stock, which it sells at 103. How much does the company receive for the stock?

23. The stock is issued in $100 shares and is sold to the following persons:

F. R. Gridley, 45 shares,
G. C. Moore, 105 shares.

How much does each pay for his stock?

24. The net earnings for the third year were $6230.00, which were distributed as follows:

2% income tax on net earnings.
Dividends on preferred stock, $——.

The directors declare $4000.00 dividends on the common stock. What is the per cent of dividend on the common stock?

25. What is the amount of surplus remaining from this year's profits? What is the total surplus accumulated to date?

26. What per cent of income do the holders of the preferred stock, who purchased it at 103, without brokerage, receive on their investment?

27. F. R. Taylor and J. E. Price engaged in partnership. Taylor invested $6000.00 and Price invested $9000.00. The business was conducted as a partnership for several years, when it was decided to reorganize as a corporation. A statement of resources and liabilities showed that the net capital of the partnership was $23,619.50. The corporation was organized with a capital stock of $25,000.00. The capitalization was how much larger than the actual net resources of the firm?

28. The stock was divided between Taylor and Price in proportion to their original investment. If the stock was issued in $100 shares, how many shares did each partner receive?

29. Burns, Randall, and Anderson were partners. Burns invested $8000, Randall $9000, and Anderson $13,000. They decided to organize as a corporation with a capital stock of $50,000. $45,000 of the stock was distributed among the partners in proportion to their original investments. How many $100 shares did each receive?

30. The remaining stock is sold to Chapman at 98½. How much did Chapman pay for his stock?

31. The profits for the first year, after paying the income tax, were \$4500. What dividend could be declared on the stock?

32. A corporation is organized with \$100,000 of common stock and \$50,000 of participating 5% preferred stock. Dividends of \$13,500 are declared. What is the total amount of dividends paid on the common stock? on the preferred stock? What rate of dividend does the holder of a share of preferred stock receive? What rate of dividend does the holder of a share of common stock receive?

333. Bonds. When an individual borrows money, he usually gives a promissory note.

When a corporation borrows money in large amounts, it usually issues bonds.

Bonds are the promissory notes of a corporation. They are usually issued in denominations of \$1000.00, although there are bonds of smaller amounts. There is an increasing tendency to issue bonds in denominations of \$500.00 or \$100.00, so that they may be sold to small investors.

Bonds are issued by:

Business Corporations, called Industrial Bonds
U. S. Government called Government Bonds
State Governments called State Bonds
Cities called Municipal Bonds
Counties called Municipal Bonds
School Districts called Municipal Bonds

Security of Bonds. A first-mortgage bond is secured by a mortgage on the property of the corporation. If the bond is not paid, the property of the corporation may be sold to pay the bondholders.

Second-mortgage and third-mortgage bonds are also secured by mortgage on the property, but the bondholders are paid, if the mortgages are foreclosed, in the numerical order of the mortgages.

Maturity of Bonds. A bond states the time at which the principal is payable. Some bonds state that the corporation reserves the right to redeem the bond at any time after a specified date, either at par or at some specified rate above par.

Interest. Bonds bear interest payable annually, semi-annually, or quarterly, as specified. The method of collecting the interest depends upon whether the bond is

a. A Registered Bond, or

b. A Coupon Bond.

When a person owns a registered bond, his name is recorded on the books of the corporation. When the interest is due, a check is sent to the bondholder in payment.

A coupon bond is one to which small coupons are attached, there being one coupon for each interest payment. For example: a bond maturing in ten years with interest payable quarterly would have forty coupons. On the date when each interest payment is due, the coupon bearing that date is clipped from the bond and delivered to a bank to collect, or it may be collected directly from the treasurer of the corporation.

Value of Bonds. Bonds, unlike common stock, pay a fixed income. The market value of a bond may be above or below par, depending in general upon the security and the rate of interest.

Bonds are usually known by the name of the issuing corporation or government, the nature of the security, the rate of interest, and the date of maturity. Thus first-mortgage bonds issued by the Eclipse Carriage Company, bearing 5% interest and payable in 1925, would be called Eclipse Carriage Company, 1st Mortgage 5% Bonds, 1925.

334. Buying and Selling Bonds. When a bond is sold, the buyer usually pays not only the market value of the bond, but also the interest which has accrued since the last interest day.

Example. Murdock owned an Eclipse Carriage Company 5% bond which he wished to sell. The par value of the bond was $1000.00, and it was quoted at 95. Henderson bought the bond on April 1, at its market value plus accrued interest. Interest is payable semi-annually, June 1 and December 1. How much did Murdock receive?

Solution. Since the bond was sold at 95, the owner received 95 % of its par value.

95 % of $1000.00 = $950.00.

Interest at 5 % has accrued on the bond from December 1 to April 1, four months.

Interest on $1000 for 4 months at 5 % = $16.67, accrued interest.

$950.00	market value of bond
16.67	accrued interest
$966.67	selling price

Many banks and trust companies buy an entire series of bonds from corporations and sell them to investors. Prices are quoted by these bond houses in the following ways:

Price, Par and Accrued Interest;
Price, at the Market and Accrued Interest.

335. Dealing in Bonds through a Broker. When bonds are purchased or sold through a broker, brokerage is charged in the same manner as when stocks are transferred. $\frac{1}{8}$ % of the par value of the bonds is the customary brokerage for large transfers.

Written Work

1. S. & J. 6 % coupon bonds are quoted at 104 and accrued interest. Interest payable June 1 and December 1. What is the cost of a $500 S. & J. bond purchased on August 15 at the market and accrued interest? No brokerage.

2. In delivering the coupon for collection on December 1, the owner files a certificate claiming exemption under the income tax law. What is the amount of interest which he receives?

3. What would the bond have cost if purchased through a broker? Brokerage $\frac{1}{8}$ %.

4. A bond house quotes Murdock Apartment $5\frac{1}{2}$ % bonds of $1000.00 at par and accrued interest. Interest due March 26 and September 26. What is the cost of a bond purchased May 12? October 19?

5. Laclede Gas Light Company's $1000 5 % bonds were quoted by the Harris Trust and Savings Bank at $101\frac{1}{2}$ and interest. Semi-annual interest payable April 1 and October 1. What is the cost of a bond purchased June 15?

6. Wilmington Power Company's 5% bonds are quoted at 97½ and interest. Interest payable semi-annually January 1 and July 1. The Wilmington Power Company reserves the right to redeem the bonds on January 1, 1919, or on any interest day thereafter at 105 and interest.

What is the cost of a $1000 bond purchased October 15, 1917, at the quoted price?

If the bond is redeemed on July 1, 1920, what gain is made by the increase in the redemption price over the purchase price?

The following is a partial list of a day's transfers of bonds through Chicago Brokers. Brokerage ⅛%.

No. Sold	Par Value	Description	Open	High	Low	Close
1	$1000.00	Armour 4½'s	92	92	92	92
67	1000.00	Chicago City Ry. 5's	100⅜	101½	100¼	100½
16	1000.00	Com. Edison 5's	101¾	101¾	101⅝	101⅝
214	1000.00	Peo. Gas 5's	99½	99⅞	99½	99⅞

7. What was the cost of 5 Chicago City Ry. $1000 bonds, purchased at the opening price in the table?

8. What did the seller receive for these bonds? How much was the brokerage on the sale?

9. What was the gain or loss from purchasing 60 Peo. Gas $1000 bonds at the opening price and selling them at the closing price? Brokerage ⅛% on purchase and ⅛% on sale.

TABULATIONS TO PROMOTE EFFICIENT MANAGEMENT

CHAPTER XLII

BUYING EXPENSES; SELLING EXPENSES; NET PROFIT

336. Expenses and Profits. In Chapter XXIII the principles of gross earnings were discussed. You have now studied many of the expenses incurred in conducting a business, and are ready to consider the problems which arise in computing net profit.

Net Profit is the difference between the gross earnings of an enterprise and the total expense of conducting it.

Expenses are divided into two classes:

Buying Expenses.

Selling Expenses.

337. Buying Expenses include all the costs of buying goods, such as freight and drayage, customs duties on goods imported, commissions of purchasing agents, insurance, and the cost of keeping goods in warehouses and placing them on the shelves of the salesroom. Buying expenses are added to the original purchase price of goods to determine the total cost. Thus:

Prime Cost of Merchandise is the original or purchase price;

Total Cost of Merchandise is the prime cost plus the buying expenses.

Example. A retail furniture store purchases goods during the year, amounting to $36,450.00. This is the prime cost. The freight, drayage, rent of warehouse, labor of warehouse employees, and other costs of placing the goods in the salesroom are $2187.00.

What is the total cost of the merchandise purchased?

SOLUTION.		
	$36,450.00	Prime cost
	2,187.00	Buying expenses
	$38,637.00	Total cost

338. Marking Cost of Goods. When the cost price is marked on goods, some merchants add a certain per cent to the prime cost to cover the buying expenses. The records of previous years are used as a basis to determine the per cent to be added.

In the illustration on page 398, the prime cost is $36,450.00, and the buying expenses $2187.00, or 6% of the prime cost. The furniture dealer might say, "Next year's buying expenses will probably be about the same as they were this year, and I will therefore add 6% to the prime cost to cover them."

A safer method, however, is to base the computation on the average expenses of several years.

YEAR	PRIME COST	BUYING EXPENSES
1916	$35,275.00	$1840.13
1917	41,391.00	2069.55
1918	62,387.00	3618.45
	$139,053.00	$7528.13

$$\$7528.13 \div \$139,053.00 = 5.41^{+}\%.$$

For convenience, 5.4% probably would be adopted as the per cent of buying expenses.

Illustration. An article cost $8. At what price is it marked to cover buying expenses?

SOLUTION.		
	$8	Prime cost
	.055	Per cent of buying expenses
	$ .44	Buying expenses
	$8.00	Prime cost
	.44	Buying expenses
	$8.44	Total marked cost

Written Work

The prime cost of a merchant's purchases and the buying expenses for several years are given below. Find the per cent of buying expenses for each year and the average for the entire interval.

	YEAR	PURCHASES	BUYING EXPENSES
1.	1914	$ 9,426.90	$ 643.92
	1915	12,318.26	812.29
	1916	14,126.29	830.12
	1917	11,341.72	690.19
	1918	14,985.29	752.29
2.	1916	39,286.26	2326.80
	1917	47,562.83	3146.29
	1918	59,286.28	4210.08

3. The prime cost of merchandise in a certain store is increased 8% to include buying expenses; the selling price is determined by adding 9% to the total cost. Mark the total cost (using "blacksmith" as the key and "g" as a repeater) and the selling price (in figures) of articles, the prime cost of which was as follows:

Stock No.	Prime Cost
A 326	$2.19
A 394	6.25
F 246	.90
A 385	2.78

4. The manager of one of the departments in a large store adds 6% to the prime cost to cover the buying expenses. He receives quotations of prices from manufacturers who offer different discounts. He prepares a table showing the total cost after deducting the discount offered and adding the 6% buying expenses. The following is a similar table which you will complete:

$	.00			.25			.50			.75		
	Less 2% Plus 6%	Less 5% Plus 6%	−7% +6%	−2% +6%	−5% +6%	−7% +6%	−2% +6%	−5% +6%	−7% +6%	−2% +6%	−5% +6%	−7% +6%
0				.2597	.25175	.24645						
1	1.0388	1.007	.9858									
2												
3												
4												
5												
6												
7												
8												
9												
10												

5. By referring to the table, find the total cost, including buying expenses, of an article quoted at $2.25 less 5%; $3 less 2%.

Refer to the table, and find

6. Which is the better price, and how much better: $1.75 each, less 2%, or $2 each, less 7%? How much will be saved by purchasing one gross at the cheaper quotation?

339. Selling Expenses. After the goods have been placed on sale, all further costs, such as rent of store, salaries of clerks, advertising, heat and light, delivery, store supplies, insurance and taxes, depreciation and shrinkage, bad debts, and general expenses, are considered as selling expenses.

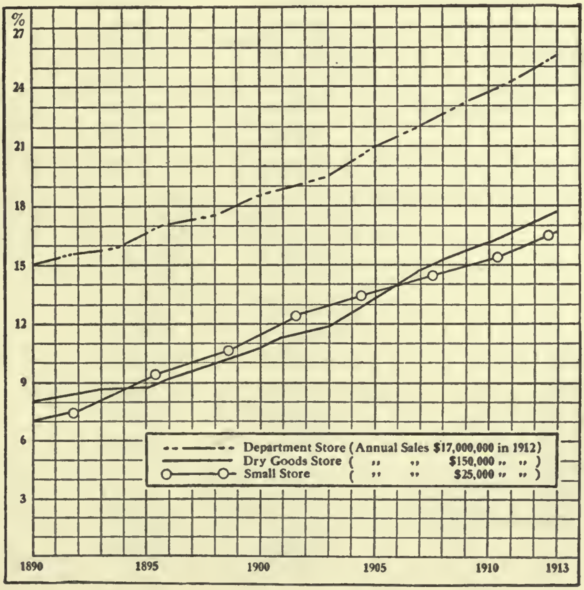
340. Finding the per cent of selling expenses. The per cent of selling expenses is a matter of valuable information to a merchant. It is determined by the formula:

Selling Expenses ÷ Gross Sales = % of Selling Expenses.

Example. A merchant's gross sales during one year are $124,265; the selling expenses for the same year are $29,078.01. The selling expenses are what per cent of the gross sales?

Solution. $29,078.01 ÷ $124,265.00 = 23.4 %.

341. Increase of Selling Expenses. During recent years, as shown by the graph reprinted from *System, a Magazine of Business*, selling expenses, such as advertising, delivery, etc., have greatly increased.



Oral Work

Trace the rise in the per cent of selling expenses in each business, stating what per cent the selling expenses in each business are of the gross sales in each year.

Written Work

The following statistics are reprinted from *System*, and show the result of an investigation made by that magazine to determine *average* or *standard* per cents of selling expenses in various businesses.

Find what per cent each item of expense is of the total sales.

Find what per cent the total selling expenses are of the sales.

	Dry Goods Store	General Store	Shoe Store
	Gross Sales, $50,000.00	Gross Sales, $50,000.00	Gross Sales, $25,000.00
Rent	$1,550.00	$1,154.85	$ 778.13
Salaries	4,800.00	4,067.09	2,786.21
Advertising	750.00	351.48	376.51
Heat and light	200.00	251.04	225.91
Delivery	450.00	954.01	75.30
Supplies	200.00	150.63	100.40
Insurance and taxes	550.00	200.84	301.21
General expense	2,200.00	150.63	1,029.14
Depreciation and shrinkage .	700.00	301.27	150.61
Bad debts	150.00	150.63	25.10

	Vehicle Store	Hardware Store	Furniture Store
	Gross Sales, $57,000.00	Gross Sales, $45,000.00	Gross Sales, $100,000.00
Rent	$1,094.63	$1,305.61	$3,507.36
Salaries	5,818.81	5,042.35	8,317.60
Advertising	633.73	405.20	2,906.15
Heat and light	345.67	225.10	801.70
Delivery	518.51	270.13	1,102.33
Supplies	230.45	180.08	501.06
Insurance and taxes	460.90	495.23	1,402.97
General expense	230.45	360.17	1,803.82
Depreciation and shrinkage .	403.28	225.10	1,904.03
Bad debts	115.22	180.08	1,202.54

	CLOTHING STORE	DRUG STORE	JEWELRY STORE
	Gross Sales, $60,000.00	Gross Sales, $20,000.00	Gross Sales, $30,000.00
Rent	$1,322.24	$ 924.65	$1,080.40
Salaries	5,469.29	2,191.01	3,361.23
Advertising	2,043.47	522.64	1,050.38
Heat and light	180.31	160.81	180.07
Delivery	360.61	80.40	30.01
Supplies	120.20	60.30	270.10
Insurance and taxes	661.12	281.40	540.20
General expense	1,681.84	482.42	630.23
Depreciation and shrinkage	721.22	100.50	360.13
Bad debts	240.41	40.20	90.03

Save your results for use in the problems which follow.

342. Efficient Management to Determine Economies and "Leaks." When a merchant has a reliable standard with which to compare the various per cents of selling expenses in his business, he is able to determine the items of expense which are too large.

Written Work

A dry goods store has gross sales during one year amounting to $40,000.00. The expenses are as follows:

Rent	$1200.00	Supplies	$165.00
Salaries	3240.00	Insurance and taxes	180.00
Advertising	250.00	General expense	155.00
Heat and light	225.00	Depreciation and shrinkage	120.00
Delivery	1135.00	Bad debts	42.00

1. Each item of expense is what per cent of the gross sales?

2. The total selling expenses are what per cent of the sales?

3. Accepting the per cents found in the dry goods store problem on page 402 as a standard, determine which expenses are larger than the standard, and which are smaller.

4. Complete the following table:

PROFITS IN 12 DEPARTMENTS

DEPT.	STOCK, JAN. 1, 1915	PURCHASES DURING 1915	TOTAL COST FOR 1915	STOCK, DEC. 31, 1915	COST OF GOODS SOLD	SALES, 1915	GROSS PROFITS	%	EXPENSES	NET PROFITS	%	NET LOSS	%
A	$15,223 40	$118,427 56		$17,327 72		$162,341 72			$26,514 95				
B	21,714 32	168,427 92		25,314 71		184,231 76			25,601 51				
C	26,361 71	176,204 91		25,747 39		197,432 65			21,430 13				
D	13,247 91	112,347 29		15,261 03		120,261 32			29,675 55				
E	20,403 71	150,265 97		20,374 76		174,231 61			14,512 53				
F	27,349 61	160,231 39		28,417 19		174,628 32			16,669 17				
G	25,732 92	237,511 86		27,961 31		284,162 39			13,635 20				
H	21,113 65	107,221 13		22,213 47		135,261 34			15,509 01				
I	23,416 47	121,376 41		24,316 19		142,221 32			13,459 85				
J	30,265 45	164,213 96		29,262 47		174,227 42			11,581 19				
K	18,462 31	174,216 91		19,417 13		202,346 21			9,420 57				
L	15,271 34	121,216 09		16,237 41		161,213 47			14,748 47				
Total													

Per cent of Gross Profit is found by dividing the Gross Profit by the Sales.
Per cent of Net Profit is found by dividing the Net Profit by the Sales
Per cent of Net Loss is found by dividing the Net Loss by the Sales.

5. Plot a curve in black ink to show the per cent of gross earnings in each department. Plot a curve on the same paper in red ink to show the per cent of net profit or net loss in each department.

SUGGESTION. Draw a line horizontally across the paper representing 0 %; scale the paper to show per cents of gain above this line, and per cents of loss below the line.

343. Common Omissions in Computing Selling Expenses. Merchants sometimes underestimate their selling expenses by omitting the following:

(*a*) Rent on store owned by the proprietor.

A man who owns his own store should include among his expenses the sum which he would be able to obtain if he rented the store to some other person, because he loses this rent by occupying the store himself.

(*b*) Salaries of the proprietor and members of his family employed in the store.

Such salaries, even though they may not be paid, should be included in the expenses, since the proprietor and the members of his family would probably be able to obtain the salaries elsewhere.

(*c*) Depreciation.

Store buildings and fixtures depreciate in value, and while such depreciation may not be noticeable, it should be computed annually. Goods also become shopworn or out of style and the depreciation in value should be charged off in the year in which it occurs.

(*d*) Bad debts.

Careful merchants make every effort to collect their accounts, but unless a strictly cash business is conducted, some accounts are almost certain to prove uncollectable. In estimating the probable loss from bad debts, some merchants compute a certain per cent of the sales, while others compute a certain per cent of past due accounts.

Written Work

A merchant invests $5000.00 in a general store. His gross sales during a certain year are $17,350.00. He owns his store building valued at $3000.00. He lists his selling expenses as follows:

Salaries (one clerk)	$600.00
Advertising	85.00
Heat and light	125.00
Delivery	140.00
Insurance and taxes	95.00
General expenses	110.00

1. What are the total expenses as shown by the list?

2. These expenses are what per cent of the gross sales?

The following items have been omitted:

The store building could be rented for $360 per year.

The owner and one member of his family work in the store. If employed elsewhere, the two could earn $950.

7% would be a fair rate for depreciation on the building.

Shop wear and change of style have decreased the value of goods in stock, $100.

An examination of this merchant's books shows that his annual loss from bad debts is about 1% of his gross sales.

3. Make a list of the selling expenses of this business, including those which the merchant omitted.

4. What are the total expenses?

5. The total expenses are what per cent of the gross sales?

6. The cost of the goods sold is $14,520.00.
What is the gross profit?

7. What does the merchant consider to be his net profit after deducting the expenses as stated in his list?

8. After deducting his real expenses as shown by your corrected list, did he have a net gain or a net loss? How much?

9. Find what per cent each item of selling expenses is of the gross sales.

10. By comparing these per cents with the average or standard per cents, we may find the "leaks" of this business.

Which of these per cents are smaller than the standard?

Which of the per cents are larger than the standard?

Although the general expenses are large, they include supplies for which no separate charges are made.

11. What is the worst "leak" in this business?

344. Reducing the Per Cent of Selling Expenses by Increasing the Gross Sales. It is evident that as the per cent of selling expenses increases, other factors remaining the same, the per cent of profit on the sales decreases. It is therefore desirable to decrease the per cent of selling expenses. The per cent of selling expenses may be reduced by

Reducing the selling expenses,
or
Increasing the gross sales.

Written Work

1. If the gross sales were $50,000.00 and the selling expenses $10,000.00, what was the per cent of selling expenses?

2. If the selling expenses could be reduced to $8000.00, without reducing the gross sales, what would be the per cent of selling expenses?

3. If the selling expenses remained $10,000.00, and the gross sales were increased to $60,000.00, what would be the per cent of selling expenses?

4. If an increase of the gross sales to $80,000.00 resulted in an increase of selling expenses to $12,000.00, what would be the per cent of selling expenses?

Increasing the Gross Sales. The gross sales may be increased in three ways:

By increasing the selling price; the turnover remaining the same.

By increasing the turnover; the selling price remaining the same.

By increasing the selling price and the turnover.

Increasing the selling price of a commodity may result, however, in decreasing the gross sales by causing a decreased demand. For this reason the more popular method of increasing gross sales is to sell at a small profit and to make frequent "turnovers."

The "turnover" means the number of times by which the gross sales exceed the average inventory.

Thus, if the average value of the stock kept is $5000.00 (selling price) and the gross sales for a year are $5000.00, there is one turnover. If the gross sales are $15,000.00, there are three turnovers.

345. Small Profits and Frequent Turnovers. The following illustration shows how a larger profit may be made from several turnovers at a small markup,* than by a single turnover at a large markup.

Let us suppose that a merchant's stock inventories at cost prices $10,000.00 (which includes buying expenses). If the selling expenses are $5000.00, the merchant must obtain $15,000.00 before he begins to make any profit. This means a markup of 50 %. In order to make a 5 % profit on the cost, he must mark his goods at 55 % above cost. An article costing $1.00 must sell for $1.55. Thus a single turnover at 55 % markup will net a profit of $500.00.

Now let us suppose that a 30 % markup (selling an article costing $1.00 for $1.30) will result in two turnovers without increasing the selling expenses.

Each turnover of a stock costing $10,000 means gross sales of $13,000.00; two turnovers make the gross sales $26,000.00, and the gross profits $6000. After deducting the selling expenses of $5000 a profit of $1000 remains.

If a markup of 20 % results in four turnovers of stock costing $10,000 with selling expenses of $6000, what is the net profit?

---

* By "markup" is meant the per cent added to prime cost and buying expenses to cover selling expenses and profit.

Written Work

1. A certain merchant takes an inventory once a month. The following are the inventories for the year at cost prices.

January	$ 27,365.80	July	$ 28,346.70
February	25,398.25	August	28,134.62
March	26,285.00	September	26,347.80
April	26,947.30	October	27,228.35
May	27,235.80	November	30,562.95
June	29,165.20	December	31,728.73

What is the average stock carried?

2. Plot a curve on graphically ruled paper to show the monthly inventories.

3. The selling expenses for the year are $ 15,342.55, or —— % of the average inventory.

4. If one turnover is made, what must be the per cent of markup in order to make a net profit of 4 % on the average inventory?

The gross sales would be $——, and the profit would be $——.

5. If four turnovers are made, what is the cost of the goods sold?

The selling expenses increased to $18,420; what may be the cause of the increase in the selling expenses?

6. The selling expenses in Problem 5 are what per cent of the cost of the goods sold?

7. In order to make a total profit of 4 % of the average inventory during the year, what per cent of profit must be made on each turnover?

8. What is the necessary per cent of markup on each turnover, selling expenses being 16.5 %?

9. What would be the selling price of an article which cost $ 2?

What is the profit or loss in each of the following?

	Average Inventory	Selling Expenses	Turnovers	Markup
10.	$ 2,000.00	$ 432.00	1	20 %
11.	8,340.25	1,365.00	3	15 %
12.	12,375.70	3,174.25	4½	13 %
13.	27,320.00	12,278.60	9	8 %

14. With $12,000 invested in stock, and with selling expenses of $4000, how much profit is made by

3 turnovers with a markup of 12 %?
5 turnovers with a markup of 10 %?
8 turnovers with a markup of 6 %?

CHAPTER XLIII

FINDING THE PROFITABLE DEPARTMENTS

In a business which is divided into several departments, one or more departments may be operating at a loss, while the business as a whole shows a profit. Unless these departments where losses occur are discovered and expenses curtailed, or gross sales or gross profits increased, such departments remain a burden to the business.

In order to find the profit by departments, the following facts must be known for each department:

Cost of Goods Sold;
Gross Sales;
Expenses.

The first two of these facts may be easily determined, since each department may be considered as a separate store, with separate records of purchases, sales, and inventories. But the difficulty comes in determining the expenses — a difficulty due to the fact that the expenses are of two kinds.

346. Local and Overhead Expenses. Local Department Expenses consist of items of expense paid for each department separately, such as salaries of clerks who work in only one department. There is no difficulty in determining the local expenses of each department.

Overhead Expenses consist of all items of expense paid by the store as a whole for the general benefit of all departments. Among these expenses are Rent, Taxes and Insurance, Heat and Light, Janitor and Elevator Service, Salaries of Executive and Office Employees, Delivery Service, Advertising.

347. Prorating Overhead Expenses. All overhead expenses must be divided among the various departments according to some reasonable basis. The proper basis of division depends on the nature of the expense. The method of distribution is called prorating.

Bases of Prorating Overhead Expenses. The following illustrations show some of the bases of distributing overhead expenses.

a. Rent: Prorated by floor space.

Suppose a store with four departments occupies a building with 4800 square feet of floor space. If each department occupies 1200 square feet, each should pay $\frac{1}{4}$ of the rent. But if one department occupies only 400 square feet, it should be charged with only $\frac{4}{48}$ or $\frac{1}{12}$ of the rent. Each department should be charged rent in proportion to the floor space which it occupies. In some concerns the relative values of different floors and locations is taken into consideration.

b. Insurance and Taxes on Stock: Prorated according to the average value of the stock carried in each department.

c. Insurance and Taxes on Building: Prorated by floor space.

d. Heat and Light: Determined by separate meters or prorated by floor space.

e. Expenses of Alterations Department: Charged according to the actual expense of alterations made for each department.

f. Expenses of Returned Goods Department: Prorated among the various departments on the basis of the number of articles returned to each department by customers.

g. Salaries and Expenses of Traveling Salesmen: Prorated according to the sales made by traveling salesmen for each department.

h. Advertising: Prorated according to the space occupied in the advertising mediums.

Oral Work

Discuss the reasons for the basis of prorating adopted in each instance above.

What would be a reasonable basis for prorating the following expenses?

Executive Salaries;
Salaries of Office Force;
Cost of Delivery Service;
Depreciation on Building.

Method of Prorating.

a. Determine the most reasonable basis;

b. Divide the total expenses by the total number of units of the basis used, to determine the prorate;

c. Multiply the prorate by the number of units of the basis in each department.

Example. The rent of a store building is $925, to be prorated on the basis of floor space. The space occupied by each department is as follows:

DEPT.	DIMENSIONS	AREA
I.	15′ × 20′	300 sq. ft.
II.	80′ × 40′	3200 sq. ft.
III.	25′ × 52′	1300 sq. ft.
Total		4800 sq. ft.

What share of the rent should be charged to each department?

SOLUTION. $925.00 ÷ 4800 = $.19270, the prorate, or the rent per square foot.

Dept. I.	$.19270	prorate
	300	sq. ft., Dept. I
	$57.81	share of rent
Dept. II.	$.19270	prorate
	3200	sq. ft., Dept. II
	$616.64	share of rent
Dept. III.	$.19270	prorate
	1300	sq. ft., Dept. III
	$250.51	share of rent

To check the above computations:

$57.81	rent, Dept. I
616.64	rent, Dept. II
250.51	rent, Dept III
$924.96	total rent

The total rent thus computed is four cents less than the actual rent. A discrepancy so small is entirely justifiable in such computations. In order to avoid a larger error, the prorate should include as many significant figures as there are dollars and cents in the expenses. A prorate of .0932 has three significant figures. If the expenses were $36.45, the prorate should have four significant figures; if the expenses were $3125, the prorate should have six significant figures.

Written Work

A blank form similar to the following illustration should be prepared for each of the following problems of prorating overhead expenses.

1. Prorating rent by floor space:

Dept.	Floor Space	Dept.	Floor Space
1.	260 sq. ft.	6.	470 sq. ft.
2.	380 sq. ft.	7.	630 sq. ft.
3.	694 sq. ft.	8.	763 sq. ft.
4.	876 sq. ft.	9.	856 sq. ft.
5.	340 sq. ft.	10.	940 sq. ft.

Total annual rent, $36,000.

PRORATING RENT BY FLOOR SPACE

Total Rent______________

Prorate________________

Department	Floor Space	Prorated Rent	
Totals			

2. Prorating insurance on stock according to the average value of the stock carried in each department:

Dept.	Average Stock	Dept.	Average Stock
1.	$3,600.00	6.	$6,890.00
2.	5,400.00	7.	3,750.00
3.	4,990.00	8.	8,460.00
4.	6,030.00	9.	6,560.00
5.	3,740.00	10.	4,700.00

Cost of insurance on stock, $540.00.

3. Prorating general expenses, heat, light, janitor service, etc., according to floor space.

The total general expenses for the year are $6346.80. The floor space of the different departments is given in Problem 1.

4. Prorating executive salaries and interest on investment, according to capital invested in each department, as shown by the average stock carried. The total executive salaries and interest on the investment for the year are $12,350.00. The average stock carried in each department is shown in Problem 2.

5. Prorating expenses of returned goods department according to the number of sales returned.

Dept.	No. of Sales Ret'd	Dept.	No. of Sales Ret'd
1.	1980	5.	1746
2.	3764	6.	1329
3.	Goods purchased from this department cannot be returned.	7.	1426
		8.	1276
		9.	1362
4.	189	10.	1147

The total expenses of the returned goods department for the year are $375.94.

348. Comparing Profits or Losses by Departments. After the overhead expenses have been prorated among the various departments, and after the sales, purchases, local expenses, and other facts of each department have been determined, the profit or loss and the per cent of profit or loss can be computed, and the departments compared on a tabulated form similar to the one used in the written exercise on page 404.

CHAPTER XLIV

FINDING THE PROFIT OR LOSS ON EACH SALE

349. Factors which Determine Cost of Each Sale. Large business houses keep records to show not only the profit or loss of the entire store and of each department, but also the profit or loss on every sale. In order to do this, a cost book is prepared, showing the cost of every article for sale in the store. The cost shown in the cost book is the total cost; it includes

a. the wholesale cost of the article;
b. a share of the buying expenses;
c. a share of the selling expenses.

Preparing the Cost Book. The three items of cost required in preparing a cost book are found as follows:

a. The wholesale cost of each article is taken from the invoice when the goods are purchased.

b. The share of buying expenses to be added is determined by the following formulas:

* Direct Buying Expenses + Prorated Overhead Buying Expenses = Total Buying Expenses.

Total Buying Expenses ÷ Total Cost of Goods Purchased = % of Buying Expenses to be added to the wholesale cost of each article.

Example. What is the per cent of buying expenses in a department reporting the following:

Salaries of buyers, freight, and other direct buying expenses, $3500.

Prorated share of overhead buying expenses (warehouse rent, labor, etc.), $2400.

Cost of purchases for the year, $84,285.50.

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* Direct buying expenses are the expenses which may be charged directly to the department purchasing the goods.

SOLUTION.	$3500.00	Direct buying expenses
	2400.00	Share of overhead buying expenses
	$5900.00	Total buying expenses

$5900 ÷ $84,285.50 = 7%.

7% is therefore added to the cost of each article to cover the buying expenses.

c. The share of selling expenses to be added, is computed by the following formulas:

Direct Selling Expenses + Prorated Share of Overhead Selling Expenses = Total Selling Expenses.

Total Selling Expenses ÷ Cost of Goods Sold = % of Selling Expenses included in Cost.

Example. The department furnishing the facts for the preceding illustration reported the following:

Salaries of salesmen, and other direct selling expenses, $5160;

Prorated share of overhead selling expenses, $7340;

Cost of goods sold, $96,153.85.

What per cent should be added to the cost of goods to cover the selling expenses?

SOLUTION.	$5160.00	Direct selling expenses
	7340.00	Overhead selling expenses
	$12,500.00	Total selling expenses

$12,500 ÷ $96,153.85 = 13%.

13% would, therefore, be added to the cost of each article in this department to cover the selling expenses.

350. Combining the Buying and Selling Expenses. After the per cent of buying expenses and the per cent of selling expenses have been determined, the two per cents are added, and the resulting per cent of the cost is added to the cost of each article to cover all expenses. Although the per cents are merely estimates, determined from the business of former years, firms of long experience use per cents which the business of the past shows to be approximately correct.

Written Work

Complete the following table showing the per cent of buying expenses, the per cent of selling expenses, and the total per cent to be added to the cost of each article purchased.

DEPT.	DIRECT BUYING EXPENSES DOLLARS	SHARE OF OVERHEAD BUYING EXPENSES DOLLARS	TOTAL BUYING EXPENSES DOLLARS	PURCHASES, 1917 DOLLARS	PER CENT OF BUYING EXPENSES	INVENTORY, JAN. 1, 1917 DOLLARS	PURCHASES, 1917 DOLLARS
I	3,500 00	2,400 00	5,900 00	84,285 50	7 %	27,624 85	84,285 50
II	2,112 19	1,354 25		69,328 72		16,582 88	
III	1,565 58	1,327 42		48,216 73		4,532 42	
IV	3,402 51	3,928 36		91,635 82		12,013 16	
V	926 39	712 46		23,412 17		5,969 48	
VI	1,426 12	951 33		39,624 17		2,023 72	

TOTAL COST OF MDSE., 1917 DOLLARS	INVENTORY, DEC. 31, 1917 DOLLARS	COST OF GOODS SOLD, 1917 DOLLARS	DIRECT SELLING EXPENSES DOLLARS	SHARE OF OVERHEAD SELLING EXPENSES DOLLARS	TOTAL SELLING EXPENSES DOLLARS	PER CENT OF TOTAL EXPENSES	TOTAL PER CENT
111,910 35	15,756 50	96,153 85	5,160 00	7,340 00	12,500 00	13 %	20 %
	14,217 23		6,236 48	5,951 56			
	9,532 80		3,829 16	2,653 29			
	14,216 23		10,560 26	7,326 29			
	4,162 39		1,332 21	1,946 29			
	4,328 63		3,929 23	2,788 24			

Save your results for future use.

351. The Cost Book. The cost book is ruled as in the illustration below:

ARTICLE NO.	DEPT. I	DEPT. II	DEPT. III	DEPT. IV	DEPT. V	DEPT. VI
1	$4 68					
2						
3						
4						
5						
6						
7						
8						
9						
10						
11						
12						
13						
14						
15						

The numbers of the departments are stated at the head of the blank. The numbers of the articles in the departments are stated at the left. Each article is given a stock number when it is purchased.

The first figure of the stock number indicates the department; the remaining figures of the stock number indicate the number of the article in the department. For example, the third article purchased for Dept. I is given the number 13, meaning department 1, number 3.

When an article is purchased, the cost clerk adds to the wholesale cost a share of the buying and selling expenses of the department, and enters this total in the proper place in the cost book.

Written Work

You are now to prepare a cost book for articles purchased for six different departments. You will be told the prime cost of each article purchased. The per cent of the prime cost required to cover buying and selling expenses in each department was found in the preceding exercise. Multiply the prime cost by the proper per cent to determine the share of expenses; add this to the prime cost, and enter the total in the cost book.

Example. The prime cost of the first article purchased for Dept. I is $3.90. What cost should be entered in the cost book?

SOLUTION.	$3.90	Prime cost
	.20	To cover buying and selling expenses in Dept. I
	.78	Expenses to be added
	$3.90	Prime cost
	.78	Expenses
	$4.68	Total cost to buy and sell

Prepare a cost book for the following purchases, using the per cent of buying and selling expenses for each department found in the exercise on page 418.

Prime cost of purchases: (The numbers of the articles are stated first; then the prime costs.) The items are given in the order of their purchase from left to right.

31, $22.18; 21, $16.42; 51, $2.40; 11, $3.90; 22, $36.27;
12, 4.25; 23, 45.32; 32, 4.23; 61, 12.90; 52, 3.90;
13, 9.18; 41, 12.46; 62, 18.75; 24, 27.19; 33, 19.27;
42, 9.13; 53, 2.85; 63, 15.30; 25, 36.14; 34, 2.30;
43, 2.56; 14, 2.32; 54, 6.23; 44, 5.80; 26, 27.23;
64, 23.95; 15, 13.50; 35, 4.67; 16, 17.60; 36, 2.13;
45, 4.50; 55, 2.19; 56, 9.29; 17, 13.47; 37, 4.23;
18, 9.28; 57, 4.80; 58, 2.65; 65, 26.80; 19, 4.62;
66, 13.86.

Save your results for future use.

352. Work of the Profit Clerk. In stores where a cost book is kept, it is the duty of the profit clerk to compute the profit or loss on each sale. The profit clerk is provided with a copy of the cost book, and a duplicate of each invoice of goods sold.

He finds the selling price on the invoice, and the cost from the cost book; by subtraction, he is able to compute the profit or loss on each sale.

Written Work

Use the cost book prepared in the preceding written exercises to find the profit or loss on each of the following sales:

1. Invoice No. 2345.

		Selling Price
6	No. 17	$16.45.
2	No. 35	5.80.
8	No. 56	12.00.

2. Invoice No. 2346.

		Selling Price
5	No. 52	$ 4.75.
6	No. 63	19.00.
12	No. 32	5.00.

3. Invoice No. 2347.

12	No. 58	$ 3.25.
10	No. 13	11.00.
8	No. 36	2.70.
24	No. 41	16.00.

Less a cash discount of 1%.

4. Invoice No. 2348.

1	No. 61	$16.30.
15	No. 63	19.00.
12	No. 44	8.00.
20	No. 45	5.00.

5. Invoice No. 2349.

10	No. 26	$33.50.
5	No. 22	50.00.
18	No. 21	22.50.
25	No. 53	3.25.

6. Invoice No. 2350.

20	No. 15	$16.50.
20	No. 13	12.00.
30	No. 12	5.00.

CHAPTER XLV

FACTORY COSTS

During recent years an increasing interest has been shown by manufacturers in the subject of cost records and accounting. While the system of factory cost records is very complex, the fundamental principles are simple.

353. Elements of Cost. Factories usually divide their expenses, or "costs," into four classes, namely, direct labor, direct material, indirect expenses or manufacturing expenses, and selling expenses.

Direct Labor is labor employed directly in the production of an article. This item is found from the pay roll and the time tickets of workmen.

Direct Material costs include all outlays of materials used in the making of an article, entering into and becoming a part of it.

Indirect Expenses, often called **burden** or **overhead expenses**, or **manufacturing expenses** consist of all the other expenses incurred in the manufacture (but not the sale) of the product.

Indirect expenses include such items as rent, taxes on factory, insurance, interest on factory and equipment (some accountants and efficiency experts object to including interest as an expense), heat, light and power, repairs, depreciation. Indirect labor, includes salaries of factory superintendents and foremen; wages of all men employed in the factory at other than direct labor on the product, such as janitors, watchmen, and messengers.

Selling Expenses include all expenses incurred in marketing the product, such as advertising, salaries, and expenses of salesmen.

The various costs are grouped as follows:

Labor + Material = Prime Cost.

Prime Cost + Indirect Expenses = Cost to Make (factory cost).

Factory Cost + Selling Expenses = Total Cost.

Selling Price − Total Cost = Profit.

Example. What are the costs and the profit of a factory, the records of which show the following:

Labor,	$33,286.45
Materials,	$18,973.98
Indirect expenses,	$15,471.25
Selling expenses,	$ 4,726.35
Sales,	$92,487.50

Solution.

Labor	$33,286.45
Materials	18,973.98
Prime cost	$52,260.43
Prime cost	$52,260.43
Indirect expenses	15,471.25
Factory cost	$67,731.68
Sales	$92,487.50
Factory cost	67,731.68
Gross profit on sales	24,755.82
Gross profit on sales	$24.755.82
Selling expenses	4,726.35
Net profit on sales	$20,029.47

354. Finding the Profit or Loss on Each Article. A manufacturer can determine with little difficulty his total costs and his profit, but it is more difficult to determine the exact cost of manufacturing any particular article, and the profit or loss arising from its sale. However, it is very important for the manufacturer to be able to do this, because, while the business as a whole may make a profit, many individual articles may be sold at a loss. It is highly desirable to know what all articles cost.

To find the factory cost of an article, the manufacturer must know:

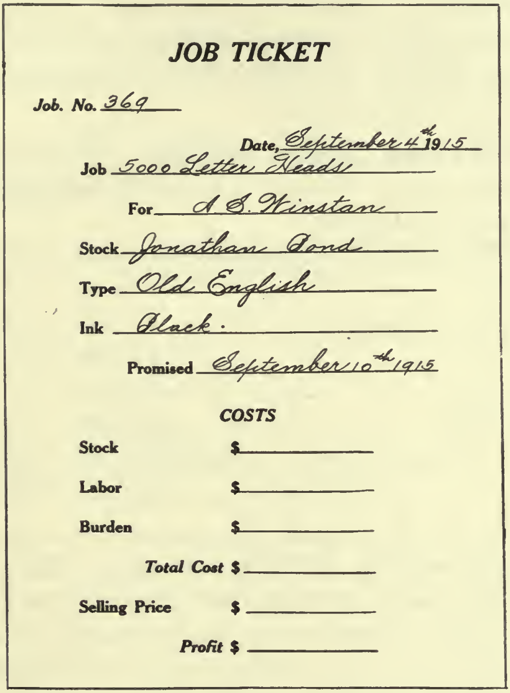
a. The cost of the labor spent upon it;

b. The cost of the materials entering into it;

c. The share of the indirect expenses or burden which should be charged to it.

The records which show these facts, are called "cost records."

355. Cost Records Illustrated. Printing offices usually have well-established cost systems, and we can illustrate the method by following a job through such an office. An order for 5000 letter heads is sent to the printing office. A cost card is made out in the office, similar to the following illustration:



JOB TICKET

Job. No. 369

Date, September 4th 1915

Job 5000 Letter Heads

For A. S. Winstan

Stock Jonathan Bond

Type Old English

Ink Black.

Promised September 10th 1915

COSTS

Stock $

Labor $

Burden $

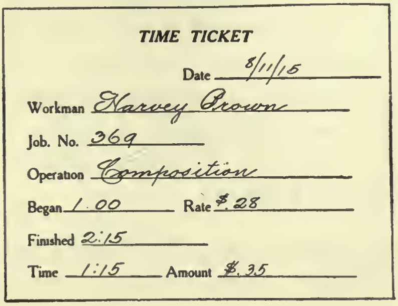
Total Cost $

Selling Price $

Profit $

Finding the Material Cost. When the paper is cut, its cost is computed, and entered on the card.

Finding the Labor Cost. Each workman who is engaged on the job fills out a time ticket showing the number of hours which he spent on it.



TIME TICKET

Date 8/11/15

Workman Harvey Brown

Job. No. 369

Operation Composition

Began 1.00 Rate $.28

Finished 2:15

Time 1:15 Amount $.35

356. Finding the Indirect Expenses. This part of the cost record is the most difficult to obtain. Before any job can be charged with a share of the burden, two things must be determined. They are:

a. The total indirect expenses, overhead or burden.

b. The best method of distributing the expenses among the various articles manufactured.

Computing the Total Indirect Expenses. At the beginning of the year, an estimate, or budget, of probable expenses is prepared. It is similar to the following:

Interest on plant and equipment	$500.00	
Insurance	75.00	
Heat, light, and power	640.00	
Repairs	125.00	
Depreciation	300.00	
Indirect labor	1620.40	
	52)$3260.40	Yearly burden.
	$62.70	Weekly burden.

These items can be foretold with comparative accuracy by referring to the records of the previous years. Many other items may be included, but those stated will illustrate the method.

We have now determined that the overhead expenses for the week are $62.70, and this amount must be distributed over all the jobs completed or worked on during the week.

The various methods for distributing overhead expenses are discussed on page 437.

Written Work

1. What is the weekly burden in a factory with the following annual overhead expenses?

Rent, $420.00; Interest on machinery and other equipment, $90.00; Insurance on equipment, $12.00; Heat and light, $285.00; Power, $600.00; Repairs, $150.00; Depreciation, $25.00; Miscellaneous, $380.00.

2. Find the total burden and the weekly burden in a factory reporting the following:

Building cost, $8000.00; Equipment cost, $16,500.00.

Interest on invested capital at $5\frac{1}{2}\%$.

Taxes: Building valued by assessor at $7500.00, taxed on $\frac{1}{3}$ of its value; Equipment assessed at $5000.00; Tax rate, $2.68.

Insurance: Building insured at 80% of cost; Equipment insured by a $12,000.00 policy. Rate on building and contents, $1.30.

Depreciation: 4% on building; $12\frac{1}{2}\%$ on machinery.

Miscellaneous, $1865.00.

DISTRIBUTING THE OVERHEAD EXPENSES AMONG THE VARIOUS JOBS

Several methods are in use for distributing the burden. The following are the most common:

a. The Direct Labor Cost Method;
b. The Direct Labor Hours Method;
c. The Direct Labor and Materials Cost Method;
d. The Department Machine Rate Method.

357. The Direct Labor Cost Method. When this method is used, the indirect expenses are distributed among the various jobs in the same ratio as the cost of the labor put on each of the jobs.

The computation is made as follows:

Divide the weekly burden by the total labor cost for the week to find what per cent the burden is of the labor cost.

Multiply the labor cost of each job by this per cent to find the burden for each job.

Example. In the printing office with a weekly burden of $62.70, the pay roll for the week ending September 17 was $370.00. The labor cost of job No. 635 was $18.50. What share of the weekly burden should be charged to job No. 635?

SOLUTION.	Labor cost for week	$370.00
	Weekly burden	$62.70

The burden is, therefore, $\frac{62.70}{370.00}$, of the labor, or about 17%.

17% of the labor cost of each job is added as burden.

17% of $18.50 = $3.15, the burden on job No. 635.

Written Work

1. If the annual burden of a factory is $3280, what is the weekly burden?

2. The pay roll of this factory for a week is $735.60. What per cent of the labor cost of each job should be added to cover the burden?

3. The labor cost of job No. 1729 was $27.69. What amount of burden should be included in the "cost to make"?

4. Annual burden, $2150.
Pay roll, week ending May 11, $296.48.
Labor cost of job No. 234, $64.96. What is the burden?
Labor cost of job No. 235, $18.26. What is the burden?

5. Annual burden, $4620.
Labor costs per job, for week ending August 26.

Job No.	1325	$ 27.69
	1326	34.19
	1327	96.41
	1328	117.12
	1329	26.14
	1330	115.86
	1331	76.90
Total pay roll		$494.31

What charge for burden should be added in the cost of each job?

(Find per cent of burden approximate to the nearest even per cent.)

6. The cost of material for each job was as follows:

1325	$19.80	1326	$21.75	1327	$48.69
1328	$59.74	1329	$ 4.90	1330	$48.75
1331	$27.96				

What was the total cost to make each job?

7. The selling expenses have been found, by experience, to be 21 % of the total cost to make. What is the cost to make and sell each job?

8. The selling price is determined by adding 28 % to the cost to make. What is the selling price of each job, and what is the profit?

358. The Direct Labor Hours Method. This method is similar to the direct labor cost method, with the exception that the hours of labor, instead of the cost of labor, are made the basis of distribution.

Example. In the factory with a weekly burden of $62.70, the total number of hours worked by all direct laborers during the week ending October 17 was 482. What burden should be charged to job number 173, on which 32 hours of direct labor were spent?

SOLUTION. Total burden for week,		$62.70
Total direct labor hours,		482

$62.70 ÷ 482 = $.13, hourly burden.

32	Number of hours spent on job number 173
$.13	Hourly burden
$4.16	Burden charged to job number 173

Written Work

1. The following jobs were begun and completed during the week ending September 27.

JOB NO.	HOURS OF LABOR	LABOR COST	MATERIAL COST
2789	93	$28.15	$14.20
2790	104	36.18	15.55
2791	32	10.16	6.12
2796	48	15.28	8.19

The total number of hours of direct labor on all jobs during the week was 623, and the annual burden was $2591.68.

What was the hourly burden rate for the week ending September 27?

What amount of burden should be charged to each job, and what was the total cost to make each job?

2. The following table shows the figures for jobs on which work was done during the week ending September 27, and which were completed during the week ending October 4.

Job No.	Hours of Labor Week Ending Sept. 27	Hours of Labor Week Ending Oct. 4	Labor Cost	Material Cost
2792	19	43	$19.40	$ 8.70
2793	23	69	28.55	13.25
2794	9	46	17.90	6.45
2795	12	52	19.35	7.90
2797	8	95	35.98	14.62

The total number of hours of direct labor for the week ending October 4 was 712. What was the hourly burden rate for work done during the week ending October 4?

What is the amount of burden to be added to the cost of each job in the table above for work done during the week ending September 27?

What is the amount of burden to be added to the cost of each job in the table above for the work done during the week ending October 4?

What is the total cost of each job?

3. 15% of the factory cost is added to the cost of each job as an estimate to cover selling expenses. What is the cost to make and sell each job, 2789 to 2797 inclusive?

4. What profit or loss was realized by selling these jobs at the following prices?

Job No.	Selling Price	Job No.	Selling Price
2789	$63.00	2793	$63.00
2790	80.00	2794	30.00
2791	20.00	2795	40.00
2792	45.00	2796	38.50
		2797	75.00

5. Manufacturers who do not operate a cost system frequently add a certain per cent of the prime cost (material and labor) to cover burden, selling expenses, and profit.

What was the prime cost of each job in Problems 1 and 2?

If 33% were added to the prime cost to determine selling price, what would have been the selling price of each job?

6. The actual cost to make and sell each job was computed by the cost system in Problem 3. Compare these costs with the selling prices determined in Problem 5, and find what profit or loss would have been realized from the sale of each job.

359. Direct Labor and Materials Cost Method. Some manufacturers prefer to distribute the burden among the various products in proportion to the cost of both the labor and the material entering into them. When this method is employed, the process is as follows:

Total labor cost per week (or other unit of time)	$525.80
Total materials cost for the week	264.70
Total labor and materials cost	$790.50

$62.70 (weekly burden) ÷ $790.50 = 7.9 %. Per cent of labor and materials cost to be added to each job as burden.

If the job ticket, or cost sheet, shows that $35.00 worth of material and $53.20 worth of labor have been put into a job, the burden would be computed thus:

$53.20	Labor Cost
35.00	Materials Cost
$88.20	Labor and Materials Cost

7.9 % of $88.20 = $6.97, Burden.

$53.20	Labor Cost
35.00	Materials Cost
6.97	Burden
$95.17	Cost to Make

Written Work

1. If the weekly burden of a factory is $6340, and the cost of the materials and labor put into the product of a week is $923.70, what per cent of the labor and materials cost of each job must be added to cover burden? Approximate result to the nearest half per cent.

2. Labor cost on job No. 884, $16.90; materials cost, $13.25. What is the burden, determined by labor and materials cost method, using the per cent found in the first problem?

3. Complete the following table.

Annual Burden		Weekly Burden		Week's Labor Cost		Week's Materials Cost		% Burden	Labor Cost One Job		Materials Cost One Job		Burden Per Job		Cost to Make Each Job	
$ 3145	90			$215	60	$412	90	Approximate to nearest tenth of a per cent.	$32	15	$ 9	75				
2268	25			186	—	45	25		12	15	4	70				
5496	—			276	25	396	90		19	25	22	70				
12962	—			893	56	592	90		63	—	49	75				

360. Department Machine Rate Method. The preceding methods do not take into consideration the fact that more burden should be charged for some operations than for others.

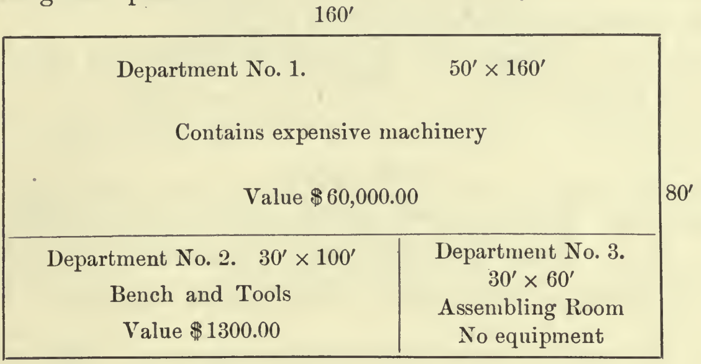
For example, let us suppose that a factory produces two kinds of articles. One is made on a very expensive machine, occupying a large room, and requiring a great deal of power. The machine is delicate, it requires considerable repairing, and will wear out in a few years. The depreciation, therefore, is large. The other article is made at a bench by hand, and with cheap tools.

Suppose the same amount of material and labor were required to make each of these articles. If any of the methods explained in the previous sections were employed, the same amount of burden would be charged to each article. But it is evident that the expense of manufacturing the former is much greater than that of the latter. The items which make up the expense are listed below.

OVERHEAD EXPENSES OF MAKING ARTICLE NO. 1	OVERHEAD EXPENSES OF MAKING ARTICLE NO. 2
Interest on Expensive Machinery.	Interest on Cheap Bench and Tools.
Rent of Large Room.	Rent of Small Room.
Cost of Power.	
Repairs on Machinery.	Repairs (very little).
Depreciation of Machinery.	Depreciation (very little).
Heat and Light.	Heat and Light.
Incidentals.	Incidentals.

The machine rate method is intended to distribute the burden in accordance with the real expenses of manufacture. The factory is divided into departments or "factors," similar processes with about the same expenses being grouped together. The burden is then divided among all departments, as shown by the following illustration.

Suppose that a factory has three departments, as shown by the following floor plan:


160′
Department No. 1. 50′ × 160′
Contains expensive machinery
Value $60,000.00
80′
Department No. 2. 30′ × 100′
Bench and Tools
Value $1300.00
Department No. 3.
30′ × 60′
Assembling Room
No equipment

The burden would be divided among the departments thus:

ITEMS	DEPT. NO. 1	DEPT. NO. 2	DEPT. NO. 3	TOTAL
Rent (floor space 160 × 80)	$1000.00	$375.00	$225.00	$1600.00
Interest	3000.00	65.00		3065.00
Insurance	480.00	10.00	45.00	535.00
Heat and Light	275.00	130.00	60.00	465.00
Power	500.00			500.00
Depreciation	1200.00	13.00		1213.00
Supervision	720.00	250.00	175.00	1145.00
Incidentals	245.00	130.00	85.00	460.00
Total . .	$7420.00	$973.00	$590.00	$8983.00
Per week . . .	$142.69	$18.71	$11.35	$172.76

METHODS OF DISTRIBUTING COSTS AMONG DEPARTMENTS

Rent. The rent is prorated among the three departments according to floor space; thus,

Annual rent of building, $1600. Total floor space, 12,800 sq. ft.

$1600 ÷ 12,800 = $.12½, rent per square foot.

Floor space Dept. 1 is 8000 sq. ft.

8000 × $.12½ = $1000, Rent Dept. 1.

Floor space Dept. 2 is 3000 sq. ft.

3000 × $.12½ = $375, Rent Dept. 2.

Floor space Dept. 3 is 1800 sq. ft.

1800 × $.12½ = $225, Rent Dept. 3.

Interest. 5 % interest on $60,000 = $3000, Int. Dept. 1.

5 % interest on $1300 = $65, Int. Dept. 2.

Insurance. Insurance on the building is prorated among the departments in proportion to floor space.

Insurance on equipment is determined by the value of the equipment.

Heat and Light. Heat is prorated according to floor space.

Light may be prorated by floor space, or a separate meter may show the cost of the light used in each department.

Depreciation. Depreciation on the building is prorated by floor space.

Depreciation on equipment depends upon the value of the equipment.

Finding the Hourly Burden Rate for Each Department. To apply this method of distributing the burden among the various jobs, the total number of hours of labor spent in each department for a week, or other period, must be known. Then the weekly burden in each department is divided by the number of hours of labor each week in the department, to determine the hourly burden rate.

Example. Let us suppose the total hours of labor in each department for a certain week were as follows:

Dept. No. 1, 317; Dept. No. 2, 104; Dept. No. 3, 95.

What was the hourly burden rate in each department?

SOLUTION. Dept. 1. $142.69 (weekly burden) ÷ 317 = $.45, hourly rate.
Dept. 2. $18.72 (weekly burden) ÷ 104 = $.18, hourly rate.
Dept. 3. $11.35 (weekly burden) ÷ 95 = $.12, hourly burden.

Finding the Overhead Expenses and the Total Cost, when the Machine or Department Rate is Used.

Example. The labor cost of an article (made during the week the hourly burden rates of which were determined above) is

$13.40; materials cost, $11.80. The labor hours in the different departments were as follows:

Dept. 1, 28 hours; Dept. 2, 5 hours; Dept. 3, 8 hours.

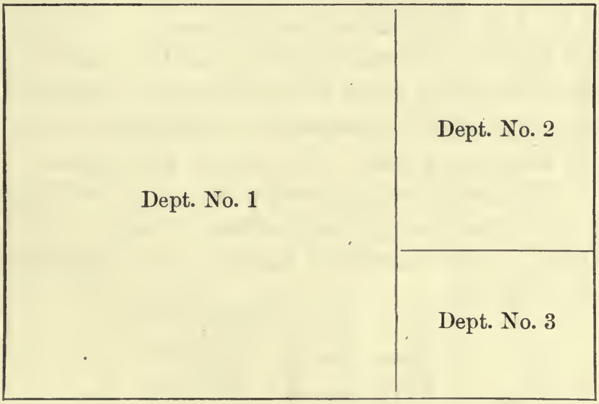
What were the overhead expenses charged to this job, and what was the total cost?

Solution.	Labor cost		$13.40
	Materials cost		11.80
	Burden		
	Dept. 1; 28 hours at $.45	$12.60	
	Dept. 2; 5 hours at .18	.90	
	Dept. 3; 8 hours at .12	.96	
	Total burden		14.46
	Total cost to make		$39.66

Written Work

1. Prepare an estimate of the annual burden of the factory occupying a building with the following floor plan:

Scale $\frac{1}{4}'' = 10'$



Interest on Building. The building cost $15,000.00. $5\frac{1}{2}\%$ interest on this cost is prorated among the departments in proportion to the square feet of floor space occupied by each.

Insurance on Building. The building is insured at $\frac{3}{4}$ of its cost. Rate, $1.65. The total cost of the premium is prorated by floor space.

Taxes on Building. The building is assessed at $\frac{1}{3}$ of its cost, and taxed at $2.90. Tax prorated by floor space.

Depreciation of Building. 4 % annual depreciation on the original value of the building is prorated by floor space.

Interest on Equipment. Dept. No. 1 contains machinery valued at $18,000.00.

Dept. No. 2 contains benches and tools valued at $2,000.00.

Dept. No. 3 is an assembly room and contains no machinery.

Each department is charged $5\frac{1}{2}$ % interest on the value of its equipment.

Insurance on Equipment. A $15,000.00, three-year, insurance policy covering the equipment, cost $3.30 per hundred. One third of the cost of this insurance is charged annually as an expense, and is prorated among the departments in proportion to the value of the equipment.

Taxes on Equipment. The taxes on the equipment, $116.00, are prorated according to the value of the equipment. The tax rate is the same as on the building.

Depreciation. 8 % depreciation is charged on the machinery in Dept. No. 1; 4 % depreciation on equipment in Dept. No. 2.

Heat and Light. The records show that the heat and light bills for the three preceding years were: $625.00; $680.00; $630.00. Find the average annual expense and assume that this will be the expense for the coming year. Prorate by floor space.

Power. The average annual cost of power to run the machinery is $1460.00. Power is required in Dept. 1 only.

Supervision. The salaries of foremen are charged to the departments as follows:

	Dept. No. 1	$865.00
	Dept. No. 2	530.00
	Dept. No. 3	680.00
Incidentals.	Dept. No. 1	$325.90
	Dept. No. 2	280.00
	Dept. No. 3	175.00

2. What is the weekly burden in the factory and in each department?

3. Since the number of hours of labor differs each week, the hourly burden will also differ each week. Complete the following table.

TABLE OF HOURLY BURDEN RATES

WEEK ENDING	DEPARTMENT NO. 1		DEPARTMENT NO. 2		DEPARTMENT NO. 3	
	Labor Hours	Hourly Burden	Labor Hours	Hourly Burden	Labor Hours	Hourly Burden
April 4	460		244		148	
April 11	495		285		163	
April 18	480		260		157	
April 25	395		203		130	
May 2	409		221		142	

Approximate results to the nearest cent.

4. The following table shows the number of labor hours in each of three departments spent on job No. 493, and also the labor cost of the job.

LABOR HOURS AND COST

Order No. 493

DATE		WORKMAN No.	HOURS			RATE	WAGES	
			Department 1	Department 2	Department 3			
April	2	16	9			.36	$3	24
	3	16	7			.36	2	52
		23	9			.29	2	61
	4	16	5			.36	1	80
			30					
	6	23	6			.29	1	74
	7	28		9		.24	2	16
	10	28		6		.24	1	44
		21	8			.25	2	00
	11	28		7		.24	1	68
			14	22				
	13	16	4			.36	1	44
	14	28		5		.24	1	20
	15	21	8			.25	2	00
	16	31			7	.38	2	66
			12	5	7		$26	49

The materials cost of job No. 493 was $13.75.

The burden was computed as follows:

The total number of hours of labor per week in each department was determined; the number of hours of labor in each department was multiplied by the hourly burden rate for the respective weeks. (The rates used in this illustration are not the same as those in the completed table on page 435.)

COST SHEET Order No. 493

WEEK ENDING	DEPT. NO.	HOURS	RATE	AMOUNT		SUMMARY		
April 4	1	30	26.9	$8	07	Labor	$26	49
11	1	14	28.4	3	98	Material	13	75
	2	22	10.6	2	33	Burden	19	28
18	1	12	27.8	3	34	Total	$59	52
	2	5	11.3		57			
	3	7	14.2		99			
				$19	28			

The labor cost is what per cent of the total cost?

The materials cost is what per cent of the total cost?

The burden is what per cent of the total cost?

5. Rule a cost sheet and find the total cost of order No. 494. Use the weekly burden rates found in the table on page 435.

MATERIALS COST, $18.83

LABOR Order No. 494

DATE	WORKMAN NO.	HOURS			RATE
		Dept. I	Dept. II	Dept. III	
April 3	27	6			.35
April 4	27	9			.35
April 4	42		4		.32
April 6	27	8			.35
April 6	42		9		.32
April 8	27	5			.35
April 8	42		3		.32
April 9	27	6			.35
April 9	42		7		.32
April 13	27	4			.35
April 13	42		8		.32
April 15	27	2			.35
April 15	42		9		.32
April 15	61			2	.39
April 16	42		2		.32
April 16	61			7	.39

6. Labor cost is what per cent of total cost to make?
Materials cost is what per cent of total cost to make?
Burden cost is what per cent of total cost to make?

7. If 6 % is added to the cost to make, to cover selling expenses, what is the cost to make and sell?

8. What must the selling price be to make a profit of $16\frac{2}{3}$ % on the cost to make?

CHAPTER XLVI

TABULATIONS FOR THE SALES MANAGER

BUSINESS men who are conducting large enterprises realize the importance of having carefully prepared statistics showing the results of their past activities, in order to judge the wisdom of their policies and to assist them in planning for the future.

361. Duties of the Sales Manager. The sales manager is expected to increase the volume of business. In order to accomplish this, he must know the sales of each clerk, and of each department. When he has this information, he is in a position to compare the efficiency of his clerks. The facts which he will be able to obtain from such tabulations as are illustrated in this chapter will enable him to manage his work more efficiently.

362. Individual Daily Sales Sheet. On the form below there are entered the sales of one day in one department of a store.

If a clerk on some preceding day sold a customer an article which the customer returned, the clerk's profit to the store is thereby decreased. Returned sales are, therefore, entered in the second column, and deducted from the gross sales to determine net sales.

In the last column is entered the number of checks, or sales tickets, made out by each clerk during the day. Record is made of the checks in order to determine the number of sales made by each clerk. The daily sales may have been small, but the clerk may have served a large number of customers.

Of what value would this blank be to the sales manager?

Written Work

Prepare a daily sales sheet from the following:

INDIVIDUAL DAILY SALES SHEET

Section No. 45 = Dress Goods Date 5/2/17 19

Clerk's Number	Gross Sales		Returned Goods		Net Sales		Checks
4501	287	96					113
02	314	39					128
03	136	32	4	92			84
04	315	26	12	25			115
05	503	16					121
06	263	50	7	25			103
07	98	30	2	80			67
08	193	50					123
09	215	67	3	96			89
10	309	50	15	75			131
11	235	80	26	15			98
12	96	45					35
13	103	28	5	60			65
14	128	95					82
15	202	75	3	10			96
16	127	30					60
Total							

363. Salesman's Cumulative Tabulation. This blank is used to record the sales of a traveling salesman who sells the goods of several departments. Each month his sales are entered by departments in the proper column. After the January and February sales are entered, the amounts for the two months are cumulated; that is, added. As each month's sales are entered they are cumulated with the preceding total. Thus, at any time during the year, this blank shows the following facts:

The salesman's sales per month in each department;
His total sales to date in each department;
His total sales each month in all departments;
His total sales in all departments.

Written Work

Prepare a cumulative sales report for Salesman No. 32.

Find the total sales per month.

Cumulate the sales as indicated on the form on page 440.

SALESMAN'S CUMULATIVE TABULATION OF SALES BY DEPARTMENTS

Salesman No. 32

DEPT. No.	JAN.	FEB.	CUM.	MAR.	CUM.	APR.	CUM.	MAY	CUM.	JUNE	CUM.
I	$ 478 60	$ 392 85		$ 429 65		$ 356 00		$ 400 25		$ 351 35	
II	832 40	926 43		845 36		875 63		867 94		925 33	
III	927 39	874 32		898 99		933 40		950 42		895 48	
IV	1246 89	1362 94		1123 25		1150 99		1234 56		1345 67	
V	1023 47	1127 47		1134 57		1126 49		1324 59		1254 32	
VI	247 89	365 85		350 25		375 26		260 50		275 67	
VII	1468 32	1232 74		1334 53		1450 12		1327 68		1227 25	
VIII	784 39	836 39		825 36		794 35		821 33		777 77	
IX	462 37	529 83		531 33		535 35		464 64		525 25	
X	1894 32	1926 39		1899 00		1876 34		1879 32		1923 76	
XI	1362 37	1423 92		1300 25		1234 76		1501 78		1434 76	
XII	2009 93	937 41		1567 32		1655 38		1323 58		1736 87	
XIII	3462 87	2634 77		3047 52		3327 89		2933 89		3000 00	
XIV	347 92	629 89		432 45		567 89		389 23		497 86	
XV	826 27	924 32		859 76		1023 76		991 02		1021 03	

364. Monthly Percentage Comparison of Salesmen. Sales managers consider it very important to have comparisons of their different salesmen. They wish to know not only the amount of the sales made by each, but also what fractional part of the total sales was sold by each. The most convenient way to state these fractions is by percentage.

Written Work

Prepare a form similar to the one indicated in the following illustration:

MONTHLY COMPARISON OF SALESMEN

Salesman's No.	First Week	Second Week	Third Week	Fourth Week	Fifth Week	Total	Per Cent
I	$320.70	$230.49	$563.40	$834.70	$123.70		
II	470.65	870.43	933.60	274.80	85.70		
III	376.80	543.70	625.80	735.43	137.89		
IV	339.40	763.35	347.84	347.96	99.89		
V	276.80	563.90	743.90	876.90	470.96		
VI	723.96	439.65	375.63	648.93	275.93		
VII	374.99	687.93	684.96	532.89	99.64		
VIII	268.37	536.96	648.97	374.88	63.96		
IX	469.89	746.93	387.93	479.69	364.84		
X	479.89	489.89	683.89	874.96	216.80		
XI	376.86	375.94	632.61	276.93	39.96		
XII	546.64	789.94	346.49	832.24	162.27		
XIII	345.49	328.84	769.94	962.63	137.90		
XIV	476.63	374.84	265.89	649.84	202.03		
XV	763.47	239.84	646.63	389 84	62.99		
Totals							

Enter the sales in the proper columns.

Find the total sales made each week by all salesmen. These totals should be entered at the foot of the columns.

Find the total sales made by each salesman during the month. This will require horizontal addition, and the totals should appear in the "Total" column at the right.

Find the grand total sales for the month. (As a means of checking your work, find the grand total in two ways: (*a*) by adding the totals at the foot of the blank, and (*b*) by comparing this result with the total of the "Total" column at the right.

Find what per cent of the grand total sales was made by each salesman. Enter these per cents in the column at the right.

365. Comparisons with Previous Year. Business men are interested in the growth of their sales, profits, etc., and make frequent use of comparative tables which bring out clearly the increase or decrease in the business. In addition to recording each salesman's sales by weeks, months, and cumulatively, and in addition to comparing the records of different salesmen to determine their comparative efficiency as shown by their per cent of sales, the office often prepares a record of each salesman's sales for the current year as compared with his sales of the preceding year, to show the amount of increase or decrease, and also the per cent of increase or decrease.

Written Work

Enter the following information on a properly ruled form. Find the yearly totals, the increase or decrease in dollars, and also in per cents.

Finding the Per Cent of Increase or Decrease. *First express the increase or decrease over the previous year as a fraction, using as the numerator the increase or decrease, and as the denominator, the previous year's sales. Change this common fraction to a per cent.*

Example. What is the per cent of increase in a business that yields, respectively, $3000 and $3600 in two successive years?

SOLUTION. This year's sales for January $3600
Last year's sales for January 3000
Increase 600
Fraction showing increase over last year . . . . . . $\frac{600}{3000}$
Changed to per cent = 20 %.

From this tabulation the sales manager can determine the salesmen whose efficiency is increasing, and can reward them by increased salary.

CHART SHOWING INCREASE OR DECREASE OF SALES

NAME________________

MONTH	SALES LAST YEAR		SALES THIS YEAR		INCREASE		PER CENT INCREASE	DECREASE		PER CENT DECREASE
January	$346	29	$427	95						
February . . .	457	75	515	86						
March	385	86	395	57						
April	416	87	402	75						
May	295	84	312	75						
June	212	67	235	84						
July	416	47	493	75						
August	328	57	319	56						
September . . .	425	65	483	75						
October	513	86	603	52						
November . . .	627	94	673	49						
December . . .	743	85	623	57						
Total										

366. Salesman's Record of Comparative Sales and Profits by Departments. The following form is somewhat similar to the form on page 441. This form, however, compares the *profits* of one year with the *profits* of the preceding year. This traveling salesman sells goods in ten departments. The profits from his sales are computed by the profit clerk, and his total profits by departments are entered on the record.

Written Work

Prepare a record of comparative sales and profits for the sales of E. H. Barlow.

Find the total sales and the total profits for each year.

The profits were what per cent of the sales in each department?

After the per cent of profit in each department for the two years has been determined, the increase or decrease in the per cent of profit can be found by subtraction. Enter increases in black ink, and decreases in red ink.

SALESMAN'S RECORD OF COMPARATIVE SALES BY DEPARTMENTS

Salesman's Name — E. H. Barlow

DEPT.	1917			1918			PER CENT DECREASE INCREASE
	SALES	PROFITS	PER CENT	SALES	PROFITS	PER CENT	
I	$12162.40	$1456.80		$13162.90	$1562.80		
II	3147.90	629.60		3349.80	640.90		
III	14127.65	1732.86		15216.90	1927.90		
IV	3287.96	362.80		3347.80	367.90		
V	5372.47	437.89		5629.90	469.90		
VI	6392.49	326.80		5146.79	294.90		
VII	15271.34	1376.45		15423.66	1389.90		
VIII	896.29	46.86		1989.60	66.80		
IX	1247.89	97.62		1462.96	103.60		
X	13467.47	1263.43		14362.62	1346.90		
Total							

367. Daily Classification of Sales by Departments. The tabulation described in this section shows the daily sales in each department, the total daily sales in the entire store, and the total sales of each department for the week.

WEEKLY CLASSIFICATION OF SALES BY DEPARTMENTS

DEPT.	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY	TOTAL	
I								
II								
III								
IV								
V								
VI								
VII								
VIII								
IX								
X								
Total								
Per Cent								

The per cent column at the bottom shows what per cent of the week's total sales was made each day.

The per cent column at the right shows what per cent of the week's total sales was made in each department.

Written Work

Prepare a form similar to the illustration. Enter the following facts on the form.

Dept.	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
I	$1213.85	$1194.83	$ 949.37	$1384.39	$1029.47	$1593.48
II	1023.75	1094.82	1127.49	1284.29	974.92	1326.83
III	1125.94	1183.27	1248.48	1494.84	827.84	1426.46
IV	923.85	1184.37	1395.47	1029.84	831.15	1285.31
V	2539.75	1823.57	1531.84	1925.29	2041.73	2848.57
VI	1925.85	1482.16	1632.83	1451.61	1348.41	2154.38
VII	1594.85	1238.57	1285.94	923.74	1128.48	1328.94
VIII	1923.84	1528.49	1228.47	1395.47	1463.94	1927.47
IX	2594.48	2049.47	1829.46	1739.84	1832.47	2885.81
X	1523.82	1449.82	1327.46	1554.53	1232.84	1723.49

Find the total sales for each day.

Find the total sales for the week in each department.

Find the total sales made by the store during the week.

On the bottom line, show what per cent of the week's sales was made each day.

In the column at the right, show what per cent of the week's sales was made in each department.

Plot a curve on graphically ruled paper, showing the total sales made in the store each day of the week.

From this form and the graph, the sales manager can determine which days had the lowest sales and he will probably offer extra bargains on those days in order to attract more customers. He will also be able to locate the departments with the smallest per cent of sales, and will take measures to increase the trade in those departments.

368. Yearly Sales by Months in All Departments. The form illustrated in this exercise shows the total sales per month in each department of a store. It furnishes the basis for various comparisons; for illustration, the gross sales by departments can be compared either by months or for the entire year. A graph could easily be prepared from this table, showing the variation of monthly sales in each department.

Written Work

Prepare a table from the following information. Find the annual sales in each department; find the total sales each month for the entire store.

	DEPT. I	DEPT. II	DEPT. III	DEPT. IV	DEPT. V	TOTAL
January .	$1238.95	$3728.94	$5238.64	$2927.46	$6223.58	
February .	1136.95	3927.47	5329.47	2842.14	6128.47	
March . .	1145.98	4023.45	5183.92	2812.51	5923.74	
April . .	1032.84	3925.36	5023.94	2642.23	5414.42	
May . .	947.83	3723.75	4823.46	2523.95	5324.94	
June . .	923.45	3582.45	4213.27	2348.41	5942.38	
July . .	1123.76	3327.74	3928.48	2493.48	5283.85	
August .	946.73	3527.94	3215.13	2493.15	5328.47	
September	982.67	3628.94	3921.14	2532.57	5623.95	
October .	1095.85	3825.93	4428.48	2724.71	5923.42	
November	1123.86	3927.85	4913.73	3124.93	6113.81	
December	1357.94	4129.54	5492.14	3295.58	6395.73	

Plot a curve on graphically ruled paper, showing the variations in monthly sales in Department I.

369. Sales and Returned Goods by Departments. The form illustrated on page 447 shows the annual sales in each department, the goods returned by customers to each department, the net sales, and the per cent of sales returned to each department.

Written Work

Prepare a form similar to the model.

Enter the following facts :

Dept. I, Sales, $27,493.75; Returns, $823.85. Dept. II, Sales, $54,294.74 ; Returns, $924.74. Dept. III, Sales, $82,583.84 ; Returns, $1239.36. Dept. IV, Sales, $107,239.75 ; Returns, $1257.34. Dept. V, Sales, $85,294.63 ; Returns, $923.75. Dept. VI, Sales, $112,437.85 ; Returns, $723.74. Dept. VII, Sales, $82,374.50 ; Returns, $1328.85.

Dept.	Sales		Returned Goods		Net Sale		Per Cent of Sales Returned
Total . . .							

Find the net sales for each department, the per cent of sales returned to each department, the total sales for the year, the total amount of goods returned to all departments during the year, the net sales for the year, and the per cent of sales returned to the store during the year.

Show by a curve plotted on graphically ruled paper, the per cent of sales returned to each department.

370. Sales Classified by Days and Departments. The form on page 448 shows the daily sales for one week in twenty-five selling sections. (Each department may be subdivided into several selling sections ; for example, the men's clothing department into sections for suits, overcoats, hats, etc.)

Written Work

Prepare a classification of sales by days and by departments.

Enter the amounts in the proper column ; find the total sales for each day, and for each section. Also find the grand total sales. Be sure that this grand total checks all additions.

SALES CLASSIFIED BY DAYS AND BY DEPARTMENTS

DEPT.	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY	TOTAL
1	$423.60	$187.47	$279.19	$789.01	$203.94	$101.97	
2	543.54	632.65	443.22	339.75	365.93	234.56	
3	100.10	999.99	203.82	901.09	584.85	475.86	
4	594.32	567.98	213.45	191.12	218.12	345.09	
5	555.33	224.60	100.00	201.35	857.02	905.43	
6	504.60	543.52	421.05	164.30	305.34	465.89	
7	102.93	621.19	178.92	630.95	304.89	985.64	
8	567.23	324.54	858.87	664.88	234.98	876.32	
9	908.23	765.54	510.25	654.21	894.32	564.32	
10	809.32	432.66	141.15	234.65	124.56	784.32	
11	239.08	111.03	400.00	191.31	123.45	621.31	
12	178.92	543.98	753.60	555.33	135.62	152.02	
13	164.30	324.98	827.26	578.98	113.56	105.15	
14	594.32	234.65	252.42	421.05	211.31	202.01	
15	339.75	432.66	323.23	213.45	135.62	615.20	
16	911.26	234.56	789.02	512.12	218.21	201.51	
17	345.67	890.23	456.78	518.29	141.19	520.20	
18	902.34	567.89	589.13	234.13	720.95	111.11	
19	979.15	860.17	762.95	729.84	789.01	33.33	
20	519.79	790.11	825.85	678.11	101.97	754.46	
21	200.20	996.57	908.23	456.34	656.54	644.57	
22	987.87	100.98	819.38	517.83	671.78	135.76	
23	112.34	567.67	450.78	189.58	150.85	200.43	
24	109.32	528.49	246.18	180.81	917.99	774.82	
25	933.47	467.83	239.08	652.29	493.86	496.73	
Total							

371. Daily Sales Compared with Sales of Corresponding Day of Preceding Year. One of the most common methods of noting the growth of a business is to compare the sales of each day with the sales of the corresponding day of the preceding year. It is assumed that trade conditions will be about the same, year after year, at the same time of the year. By comparing the sales of the first Monday in October, 1917, with the sales of the first Monday in October, 1916, a basis of comparison is furnished to show increase or decrease in business. It is customary to enter on the blank the weather conditions for the two days, since the weather would affect the volume of sales.

Written Work

Prepare a blank similar to the model.

Enter the following facts on the form.

DAILY SALES REPORT BY SELLING SECTIONS
Aug. 2, 1917

SECTION NO.	THIS YEAR	LAST YEAR	INCREASE	PER CENT	DECREASE	PER CENT
1	$619.20	$603.60				
2	85.70	90.21				
3	214.00	196.80				
4	716.26	654.30				
5	426.80	397.80				
6	562.70	571.32				
7	627.80	503.60				
8	462.80	403.40				
9	1037.90	862.40				
10	729.60	642.80				
11	327.96	417.84				
12	526.94	327.64				
13	827.83	941.62				
14	987.60	864.95				
15	479.90	532.60				
16	362.85	325.90				
17	762.35	629 89				
18	236.80	329.40				
19	246.89	639.90				
20	427.86	547.77				
21	572.96	563.84				
22	294.84	289.77				
23	276.89	286.89				
24	347.80	329.99				
25	532.87	531.39				
26	274.79	239.80				
Total						

Find the increase or decrease, and the per cent of increase or decrease, in the sales for each selling section, and for the entire department.

372. Form Showing the Per Cent of the Stock on Hand at the Beginning of the Month Sold during the Month. The following form shows what per cent of the stock on hand at the beginning of the month is turned during the month. An inventory is taken at the beginning of the month, the sales of the month are recorded, and the per cent of the inventory sold is computed.

Example. The inventory at the beginning of January showed 540 suits on hand. During the month 360 suits were sold. What per cent of the stock was sold?

SOLUTION. $\frac{360}{540}$, the fraction of the stock sold.
$360 \div 540 = 66\frac{2}{3}\%$.

Written Work

Prepare a blank similar to the model.
Enter the following data.

MONTH	1916			1917				
	STOCK	SALES	PER CENT OF STOCK SOLD	STOCK	SALES	PER CENT OF STOCK SOLD	PER CENT INCREASE	PER CENT DECREASE
Jan.	$1864	$1631	87.5	$2163	$1731	80.0		7.5
Feb.	1632	1437		1973	1562			
March	1689	1307		1865	1437			
April	1432	787		1437	986			
May	1063	836		1237	884			
June	1773	824		1147	835			
July	1089	829		1312	846			
Aug.	1002	786		1562	897			
Sept.	1347	965		1575	979			
Oct.	1689	1087		1734	1137			
Nov.	1947	1476		2074	1562			
Dec.	2137	1694		2263	1734			
Total								

Complete the form by showing what per cent of the stock on hand at the beginning of the month was sold during the month; also show the per cent of increase or decrease the second year.

373. Department Record of Sales, Profits, etc. The following table shows one of the more elaborate forms of store records.

Written Work

Rule a form similar to the illustration. Use the following data and complete the blank. Your ability to prepare this exercise without further explanation will be a good test of your efficiency. Prorate the salaries, commissions, and other expenses, according to sales.

DEPARTMENT SALES, GROSS PROFITS, RETURNED GOODS, NET PROFITS, PER CENT OF NET PROFITS, AND COMPARISON WITH LAST YEAR

DEPT.	SALES	GROSS PROFITS	PER CENT	RETURNED GOODS	PROFIT	PER CENT	PRORATED SHARE OF SALESMAN'S SALARY AND EXPENSES	NET PROFITS	PER CENT NET PROFITS	PER CENT PROFIT LAST YEAR	PER CENT INCREASE	PER CENT DECREASE
1	$ 3465.80	$ 832.46		$134.60	$27.30					7		
2	2247.95	629.22		94.97	21.13					12		
3	1276.96	169.94								4		
4	974.44	202.41								15		
5	326.37	164.69								21		
6	16,374.39	2,234.62		99.84	11.20					10		
7	22,365.24	4,162.94		232.62	49.20					14		
8	16,264.64	3,148.88		194.39	41.29					10		
9	22,239.39	3,366.86		265.24	42.30					12		
10	1,646.62	624.30		42.00	19.00					14		
11	9,347.27	1,689.90		164.69	16.24					8		
12	6,362.94	2,144.64		196.62	62.21					18		
13	976.46	202.03		12.13	2.63					14		
14	3,274.74	968.66		47.31	11.21					12		
15	9,639.99	1,244.19		111.12	12.16					10		
16	12,244.84	2,646.29		199.96	35.19					13		
17	8,348.98	1,294.68		96.25	12.12					11		
18	16,264.42	3,000.99		47.73	10.09					13		
19	18,297.78	2,346.98		169.89	18.94					12		
20	1,362.63	423.37		21.31	7.96					11		

Salesman's total salary and commissions . .	$2,461.00
Salesman's total expenses	1,992.00

MISCELLANEOUS

CHAPTER XLVII

CONSIGNMENTS AND COMMISSIONS

GRAIN, live stock, and many other articles of produce and manufacture are usually marketed through commission merchants or brokers.

374. Illustration and Explanation of Terms. Mr. Williams, who lives in a small town in Minnesota, sends a car of grain to Owen and Bartlett, commission merchants of Chicago. Owen and Bartlett are engaged to sell the grain, pay all charges, deduct their commission, and remit the proceeds to Williams.

Williams is called the **principal**; Owen and Bartlett, the **agents.**

Williams calls the grain a **shipment**; Owen and Bartlett call it a **consignment.** The statement of the transaction furnished to Williams is called an **account sales.** When a commission merchant is engaged to buy produce, the statement of the transaction furnished to his principal is called an **account purchase.**

375. Selling Consignments of Grain. The city produce exchanges provide a ready market for grain at all times. The commission merchants and brokers are members of these exchanges, and act as the agents of farmers and elevator companies in the sale of the grain. The farmer or elevator owner consigns his cars of grain to his broker, who sells the grain, has it unloaded at the elevator designated by the purchaser, receives cash in payment, and accounts to his shipper for the full amount received, minus his commission and the charges which he has paid.

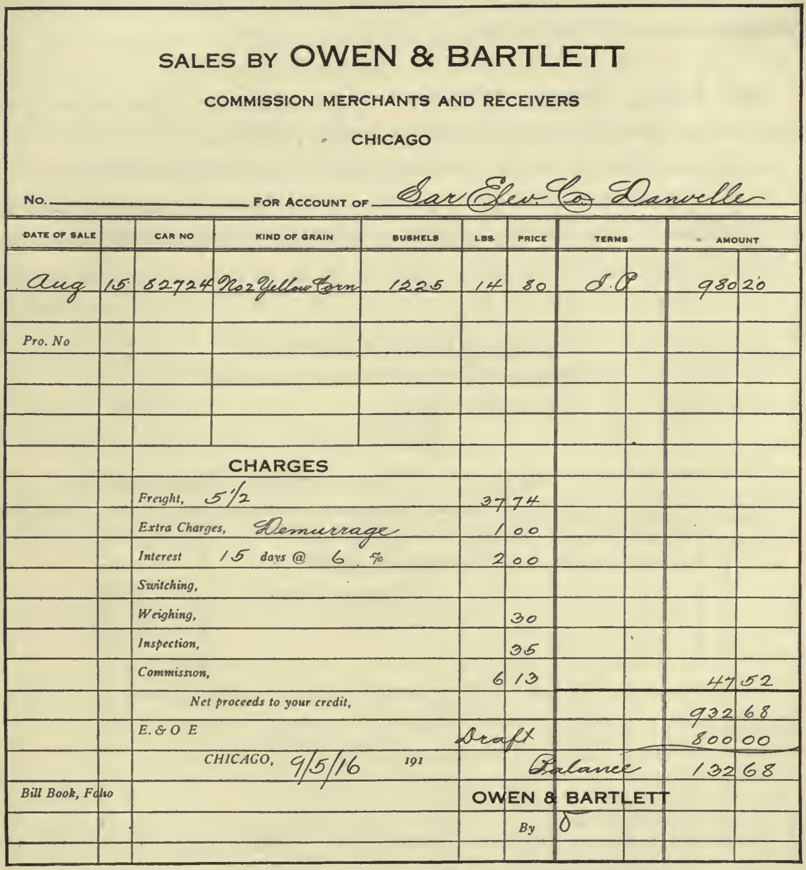
Commission is the charge made by the broker for handling the sale. The following are Chicago rates:

$\frac{1}{2}$ cent per bushel on corn and oats.

1 cent per bushel on wheat, rye, and barley.

Inspection. A nominal fee (usually 35 cents) is charged by the state grain inspection bureau for determining the grade or quality of the grain.

Weighing. A fee is charged by the board of trade weighing bureau for determining the number of pounds of grain in the car.



SALES BY OWEN & BARTLETT

COMMISSION MERCHANTS AND RECEIVERS

CHICAGO

No. ______ FOR ACCOUNT OF Far Elev. Co. Danville

DATE OF SALE		CAR NO	KIND OF GRAIN	BUSHELS	LBS.	PRICE	TERMS	AMOUNT	
Aug	15	82724	No 2 Yellow Corn	1225	14	80	J. P.	980	20
Pro. No									

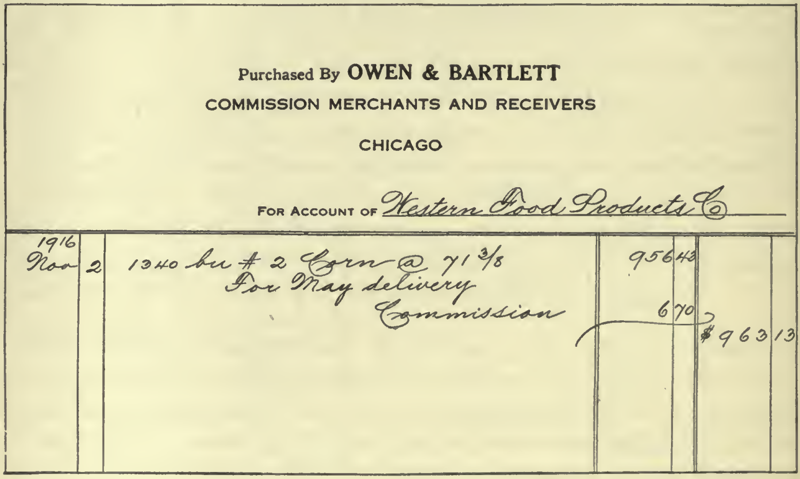
CHARGES				
Freight, 5½	37	74		
Extra Charges, Demurrage	1	00		
Interest 15 days @ 6 %	2	00		
Switching,				
Weighing,		30		
Inspection,		35		
Commission,	6	13	47	52
Net proceeds to your credit,			932	68
E. & O. E.	Draft		800	00
CHICAGO, 9/5/16 191	Balance		132	68
Bill Book, Folio	OWEN & BARTLETT			
	By			

An Account Sales

Drafts. The consignor frequently draws a demand draft on his broker in part payment. The draft does not usually exceed 80% of the estimated value of the grain. The shipper deposits the draft, with bill of lading attached, with his local bank. The local bank sends the draft to its correspondent bank in the city where the broker is located. The city bank presents the draft to the broker, receives payment, and turns over the bill of lading. When the broker renders an account sales of the grain, he deducts the amount of the draft, plus interest at the ruling rate from the date the draft was paid until the date on which he received the money from the sale of the grain.

376. Buying Grain. Brokers buy grain as the agent of millers, refiners, and other manufacturers using grain as a raw material. Purchases are made both for immediate and for future delivery.

In the produce exchanges there is a quotation on grain purchased for immediate delivery, and also on contracts calling for the delivery of grain at some definite future date. These future prices are affected by the prospects of large or small crops, by the demand for grain abroad, by the money market, and by many other factors.



Purchased By **OWEN & BARTLETT**
COMMISSION MERCHANTS AND RECEIVERS
CHICAGO

FOR ACCOUNT OF Western Food Products Co

1916 Nov	2	1340 bu # 2 Corn @ 71 3/8 For May delivery	956	43		
		Commission	6	70	$963	13

An Account Purchase

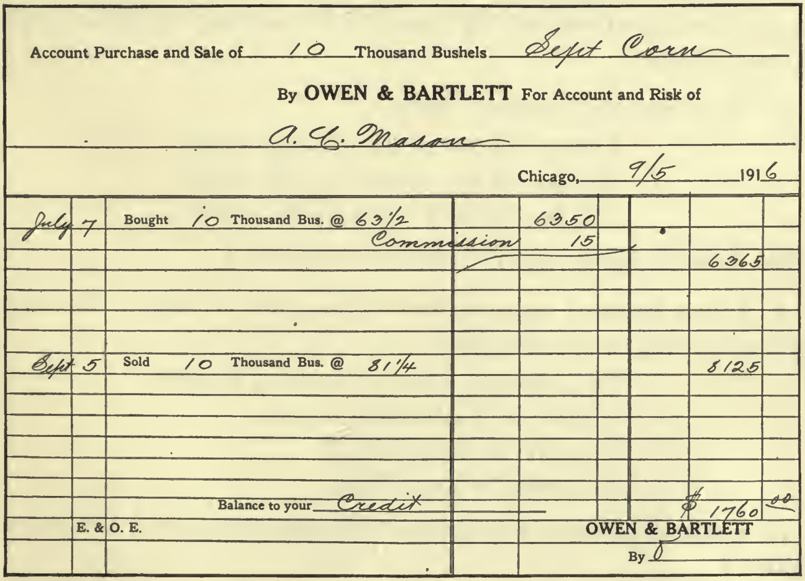
If the purchase is made for immediate delivery, grain is purchased by the broker for his principal, at the market price, and delivery is made as soon as possible. If the grain is purchased for future delivery, it is delivered at the time specified when the contract was made, and charged at the price prevailing for future orders on the day of purchase.

Charges for Buying Grain. The only charge made by brokers for buying grain, is commission. The commission on a purchase is the same as on a sale.

377. Speculating in Futures. Speculators often instruct a broker to buy grain for future delivery, hoping that when the day of delivery arrives, the market price will have risen and a gain may be realized.

On the other hand, a speculator may instruct his broker to sell grain for future delivery, hoping that when the day of delivery arrives, the market price will have dropped. In such a case he can buy the grain at the market price, and deliver it to the purchaser at the contract price, or the purchaser may pay him the difference between the market price and the contract price.

The following account purchase and sale shows a transaction in which a speculator contracted for the purchase of September grain in July, and sold it in September, realizing a profit of $1760.00, after paying his commission.



Account Purchase and Sale of 10 Thousand Bushels Sept Corn

By **OWEN & BARTLETT** For Account and Risk of

A. C. Mason

Chicago, 9/5 1916

July	7	Bought 10 Thousand Bus. @ 63½		6350			
		Commission		15			
							6365
Sept	5	Sold 10 Thousand Bus. @ 81¼					8125
		Balance to your Credit					$1760 00
	E. & O. E.			OWEN & BARTLETT			
				By			

TABLE OF GRAIN WEIGHTS PER BUSHEL

Corn	56 pounds
Wheat	60 pounds
Oats	32 pounds
Barley	48 pounds

Written Work

1. What is the value of 71,980 pounds of corn at 62 cents per bushel?

2. What is the value of 89,720 pounds of wheat at $1.42 per bushel?

3. What is the value of 54,350 pounds of oats at 52 cents per bushel?

4. What is the value of 67,320 pounds of barley at 81 cents per bushel?

5. If a farmer sells the elevator company in his town 514 bushels and 28 pounds of corn at $62\frac{1}{2}$ cents per bushel, how much does the farmer receive?

6. If an elevator company buys

514 bu. 28 lb. of corn at $58\frac{1}{2}$ cents,
312 bu. 14 lb. of corn at $58\frac{3}{4}$ cents,
410 bu. 14 lb. of corn at 59 cents,

what does the grain cost the elevator company?

7. If the elevator company sells this grain through a Chicago broker for $65\frac{1}{2}$ cents, and pays

$6\frac{1}{2}$ cents per hundred pounds, freight,
$\frac{1}{2}$ cent per bushel commission,
25 cents for weighing,
35 cents for inspection,

how much profit does the elevator company realize on the transaction?

8. The Sedan Elevator Company shipped you, as its broker, 1145 bushels and 40 pounds of wheat, which you sold at $1.40. Charges were as follows:

Commission, 1 cent per bushel;
Freight, $8\frac{1}{2}$ cents per hundred pounds;
Weighing, 30 cents;
Inspection, 35 cents.

The elevator company drew a draft for $700, which you paid eleven days before receiving your payment for the grain.

Prepare an account sales for the transaction.

9. A milling concern bought 96,740 bushels, 20 pounds of wheat through a broker at $1.40. They paid freight at the rate of $11\frac{1}{2}$ cents per hundred pounds, a commission of one cent per bushel, and switching charges of $34. How much did the grain cost them, delivered at their mills?

10. The manager of a breakfast food company purchased on April 25, 75,000 bushels of September corn at $61\frac{1}{2}$ cents. On September 5, the price of September corn was $79\frac{1}{4}$ cents. How much did this company save by taking advantage of the lower price of corn in April? Do not consider interest.

CHAPTER XLVIII

LIFE INSURANCE

INSURANCE against loss due to sickness, accident, or death is called **Personal Insurance.**

There are several classes of Personal Insurance of which the following are the most common: Accident Insurance, Health Insurance, Liability Insurance, and Life Insurance. Only Life Insurance will be considered in this chapter.

In consideration of certain payments, called *Premiums*, a Life Insurance Company agrees to pay, at the death of the insured, or at some other time stipulated in the contract, called the *Policy*, a stated sum of money to the person designated by the insured. The person to whom payment is to be made is called the *Beneficiary*. There are many kinds of policies, and policies of the same name often differ in certain details.

While it will be impossible to present the subject of Life Insurance in great detail in this chapter, the fundamental principles of the various types of policies will be discussed.

378. Principal Kinds of Life Insurance Policies. The principal kinds of policies are:

a. Ordinary Life policy.
b. Limited Life policy.
c. Endowment policy.
d. Term policy.

These differ in three important particulars.

a. The number of premiums paid by the insured.
b. The amount of each premium.
c. The time when payment is made by the company.

COMPARISON OF SEVERAL KINDS OF POLICIES

KIND OF POLICY	NUMBER OF YEARS PREMIUMS ARE PAID	TIME WHEN PAYMENT IS MADE BY THE COMPANY
Ordinary Life . . .	During life of insured [In some companies, if dividends are allowed to accumulate with the company, this period may be shortened.]	At death of insured
*Limited Life —		
20-payment life . .	20 years	At death of insured
10-payment life . .	10 years	At death of insured
*Endowment Policy —		
20-year endowment	20 years	At death of insured payment made to beneficiary; or at expiration of 20 years payment made to insured if still living
10-year endowment	10 years	At death of insured or at expiration of 10 years
20-payment, 30-year endowment . .	20 years	At death of insured or at expiration of 30 years
*Term Policy —		
20 years	20 years	At death of insured if he dies within 20 years. If he lives beyond this term, no payment is made

---

* The policies listed under this heading are illustrative only. Policies are issued for other periods than those mentioned.

379. Insurance Rates. Insurance rates are usually quoted at a certain price per $1000 of insurance. The rate depends upon the age of the insured when he "takes out the policy," and upon the kind of policy. The premiums are payable in advance.

The desirability of insuring at as early an age as practicable, is shown by a study of the comparative premiums at different ages.

The following table shows the premium rates per thousand on various kinds of policies for various ages.

ANNUAL PREMIUM ON DIFFERENT KINDS OF INSURANCE PER $1000.00

AGE	ORDINARY LIFE	20-PAY. LIFE	10-PAY. LIFE	10-YEAR ENDOWMENT	20-YEAR ENDOWMENT
15	$ 17.40	$27.34	$ 44.62	$100.60	$47.79
20	19.21	29.39	47.85	101.57	48.48
25	21.49	31.83	51.67	102.73	49.33
30	24.38	34.76	56.18	104.14	50.43
35	28.11	38.34	61.53	105.87	51.91
40	33.01	42.79	67.90	108.07	54.06
45	39.55	48.52	75.57	111.03	57.34
50	48.48	56.17	84.99	115.28	62.55
55	60.72	66.69	96.66	121.48	70.81
60	77.69	81.60	111.47	130.76	83.82
65	101.48		131.13	145.08	

Written Work

From the preceding table find the annual premium required for:

1. An ordinary life policy for $4000, taken by a man 20 years of age.

2. A ten-payment life policy for $3000, taken by a man 25 years of age.

3. A ten-year endowment policy for $5000, taken by a man 35 years of age.

4. A twenty-year endowment policy for $5000, taken by a man 25 years of age.

5. A man 25 years of age took out a twenty-year endowment policy for $5000, and died after making eight payments. How much less would the combined premiums have been on an ordinary life policy?

6. A man on his 30th birthday took out a twenty-payment life policy for $3000, and 5 years later he took out an ordinary life policy for $5000. He died after making fifteen payments on his first policy. By how much did the insurance received by his beneficiary exceed the premiums paid (not considering accumulations)?

380. Dividends. Policies written on the "participating plan" provide that a portion of the annual profits of the company shall be shared by the holders of the policies. These annual payments of a share of the profits are called "dividends." Since the profits of the company vary from year to year, no specific amount of dividend is guaranteed to the policyholder. Policyholders usually receive no dividend the first year. Dividends may, at the option of the insured, be

a. Paid to him in cash;

b. Applied to the payment of his premium;

c. Left with the company and allowed to accumulate at compound interest;

d. Left with the company to increase the amount of insurance carried;

e. Left with the company in order to decrease the number of payments of premiums.

The following table shows the effect of the dividends in reducing the annual premiums.

Annual Cash Dividends and Net Cost of Insurance on Policies of $1000 — Age 35

Ordinary Life Policy Issued in 1908. Annual Premium, $27.30				20-Premium Life Policy Issued in 1908. Annual Premium, $36.00		20-Year Endowment Policy Issued in 1908. Annual Premium, $50.80	
Year Ending	Dividend at End of Year		Cost for Each Year	Dividend at End of Year	Cost for Each Year	Dividend at End of Year	Cost for Each Year
1908	1	$3 00	$24 30	$3 30	$32 70	$3 80	$47 00
1909	2	3 10	24 20	3 45	32 55	4 05	46 75
1910	3	3 20	24 10	3 60	32 40	4 35	46 45
1911	4	4 10	23 20	4 40	31 60	4 95	45 85
1912	5	4 25	23 05	4 60	31 40	5 25	45 55
1913	6	4 70	22 60	5 30	30 70	6 60	44 20
1914	7	5 25	22 05	6 05	29 95	7 70	43 10
1915	8	5 80	21 50	6 75	29 25	8 65	42 15
1916	9	6 30	21 00	7 30	28 70	9 30	41 50
1917	10	6 50	20 80	7 55	28 45	9 65	41 15

381. Lapses. If the annual premium is not paid when due, the policy lapses. However, most companies allow one month's grace, during which time the overdue premium may be paid with interest. If death occurs during this period of grace, the amount of the overdue premium and the accrued interest are deducted in making settlement with the beneficiary.

If the premium remains unpaid after the expiration of the period of grace, the policy may be reinstated, but the policyholder must furnish satisfactory evidence of health and pay all back premiums with accrued interest.

382. Cash Surrender, Loan, and Paid-up Insurance. After a policy has been in force for a certain number of years (usually two or three), many companies offer the insured the following privileges:

a. Borrowing money from the company;

b. Surrendering the policy for a cash payment;

c. Receiving a "paid-up" policy which gives a fixed amount of insurance during the remainder of life without further payment of premiums.

d. Being insured for the face of the policy for a fixed number of years and months.

For example, after an ordinary life policy for $1000.00, taken at the age of 35, in a certain company, has been in force 8 years, the insured can —

a. Borrow $108.00 from the company on the security of the policy.

b. Surrender the policy and receive $108.00 in cash.

c Cease the payment of premiums and be insured for the remainder of his life for $223.00.

d. Cease the payment of premiums and be insured for $1000.00 for 10 years and 9 months.

383. Modes of Settlement. Upon proof of the death of the insured (or, in case of an endowment policy, at the expiration of the endowment period), the policy becomes payable by the company. Settlement may be made in a variety of ways, the most common of which are:

a. Payment of cash to the beneficiary;

b. Annual payment of interest during the life of the beneficiary, and payment of the face of the policy at the death of the beneficiary;

c. Payment of equal annual installments for the number of years specified to the beneficiary;

d. Payment of equal annual installments for a period of twenty years, and for as many years thereafter as the beneficiary shall live. The amount of each annual payment depends upon the age of the beneficiary at the death of the insured.

Oral Work

1. Explain the reason for the difference in the premiums of the different kinds of policies.
2. Why is it often wise to take insurance at an early age?
3. What is participating insurance?

Written Work

Refer to the table of annual premiums on different kinds of insurance and find the annual premium on the following:

1. $2000, ordinary life policy, taken at the age of 35.
2. $5000, 20-year endowment policy, taken at the age of 35.
3. $3000, 10-payment life policy, taken at the age of 35.
4. $6000, 20-payment life policy, taken at the age of 35.
5. $10,000, 10-year endowment policy, taken at the age of 35.
6. On his 25th birthday Mr. Anderson took out a 20-payment life policy for $3000; on his 30th birthday he took out a 20-year endowment policy for $5000; on his 35th birthday he took out an ordinary life policy for $7000. He died at the age of 38 years 7 months. How much more did his beneficiary receive (dividends excepted) than he paid the company? (Use the table on page 460.)

CHAPTER XLIX

FARM RECORDS

384. The Cash Book. One of the most important records for a farmer is the cash book, in which he can tabulate the various sources of his income and the amounts of his various expenditures.

The following are three of the forms which may be used for this purpose:

A

			Receipts		Expenditures	
1917						
Oct.	1	Balance	123	75		
	1	15 bu. apples @ .70	10	50		
	1	12 doz. eggs @ .30	3	60		
	3	Groceries			5	95
	5	Lumber			26	25

B

Receipts							Expenditures						
1917							1915						
Oct.	1	Balance			123	75							
	1	15 bu. apples @ .70	10	50			Oct.	3	Groceries	5	95		
	1	12 doz. eggs @ .30	3	60				5	Lumber	26	25		

C

Receipts

1917		Item	Dairy		Poultry		Crops		General	
May	1	2 cows to Owen	125	00						
	2	10 doz. eggs @ .28			2	80				
	3	4 T. hay					60	00		

EXPENDITURES

1915		ITEM	DAIRY	POULTRY	CROPS	HOUSEHOLD	GENERAL
May	1	500 # bran	8 00				
	2	10 bu. seed oats			10 00		
	3	Groceries				3 60	
	4	Oyster shell		1 50			
	5	Cement					8 00

Written Work

Rule a form for a cash book similar to the last illustration.

Include an extra column in both "Receipts" and "Expenditures" labeled "Stock" and enter the following facts:

March 1, Balance, $280.00.
1, Sold for cash 1 carload hogs, $1080.00.
1, Bought 2 cows, $45.00 and $58.00.
2, Bought 1 set double work harness, $21.00.
3, Bought 1 work horse, $160.00.
4, Sold 30 doz. eggs at 25 cents.
5, Sold 8 T. timothy hay at $14.00 per ton.
6, Sold 1 wagon load oats to elevator, 95 bu., at 46 cents.
6, Bought groceries, $7.95.
8, Sold one second-hand riding plow, $15.00.
9, Bought 20 sacks cement at $.50.
10, Bought 90 bu. seed oats at 48 cents.
11, Bought 500# middling for hogs at $30.00 per ton.
12, Bought 2 riding plows at $55.00 each.
15, Received milk check for 30 da., $26.47.
16, Bought groceries, $6.83.
17, Bought wheat screening for chickens, $2.50.
20, Sold 2 steer calves at $9.00 and $12.00.
22, Sold 18 young roosters, 57 # at 14¢ per pound.
24, Rented a three-horse team to a neighbor for $2\frac{1}{2}$ da. at $3.75 per day.
26, Bought 10 T. rock phosphate at $3.00 per ton.

28, Sold 28 doz. eggs at 27 cents.

28, Bought 1 pair shoes, $2.85; suit of clothes, $11.00.

Find the total receipts and expenditures for each column, and the balance at the end of the month.

385. Farm Profits. The annual increase or decrease of the farmer's wealth may be determined by taking an inventory each year, and comparing it with the inventory of the preceding year. The inventory should be classified to show the capital invested in land, building, live stock, machinery, and other property. An inventory, taken from Farmers' Bulletin 511, U. S. Department of Agriculture, is reproduced on pages 467 and 468.

386. What is Farm Profit? The annual increase shown by the two inventories is not the farmer's net gain for the year. The real annual profit is found by the following method:

Increase shown by inventories, plus living supplied by farm to family, plus interest paid on indebtedness, minus interest on capital invested, minus wages of farmer and family, equals actual net profit.

Example. Let us suppose the inventories show an increase for the year of $2000. The farm has also supplied a residence and provisions for the farmer's family. If the house rent is estimated at $120 and the supplies are valued at $900, the total, $1020, should be credited to the farm. Adding $80 interest paid on the farm mortgage, we have the following:

$2000	Increase of inventory
1020	Household expenses
80	Interest paid
$3100	Produced by farm

The farmer has $10,000 invested in the farm. If this money were loaned at 5 % interest, it would produce $500 interest annually. $500 interest must therefore be subtracted.

If the farmer had worked for some one else, he might have earned wages of, say, $700. This amount must also be subtracted in order to determine real profit.

Thus, we have

$3100 – $500 (interest) – $700 (wages) = $1900, profit.

*Sample farm inventory: Farm of*______

PROPERTY	APRIL 1, 1916				APRIL 1, 1917			
	No.	Rate	Valuation		No.	Rate	Valuation	
REAL ESTATE								
Farm of 180 acres (155 tillable), including buildings (dwelling $1,600, barns $1,800, other buildings $600), fences, and other improvements				$13,500.00				$13,500.00
LIVE STOCK								
Dairy cattle:								
Cows, dry and in milk	24	$50.00	$ 1,200.00		26	$50.00	$ 1,300.00	
Bull	1		50.00		1		45.00	
Calves	6	14.00	84.00		8	15.00	120.00	
Two-year-olds	4	28.00	112.00		6	20.00	120.00	
Total value of dairy cattle				1,446.00				1,585.00
Hogs:								
Brood sows	2	22.00	44.00		2	21.00	42.00	
Pigs	8	4.00	32.00		6	3.00	18.00	
Total value of hogs				76.00				60.00
Horses:								
Horse, Jim, 7 years old	1		200.00		1		180.00	
Team, Nell and Bess, 5 and 6 years old	1		425.00		1		425.00	
Team, Jack and Prince, 6 and 7 years old	1		400.00		1		400.00	
Colt, 1 year old	1		75.00		1		145.00	
Total value of horses				1,100.00				1,150.00
Poultry:								
Hens	160	.60	96.00		125	.60	75.00	
Roosters	5	1.00	5.00		4	1.00	4.00	
Turkeys	2	3.00	6.00		3	3.00	9.00	
Total value of poultry				107.00				88.00
Total value of live stock				2,729.00				2,883.00
MACHINERY AND TOOLS								
Grain binder	1		90.00		1		82.00	
Sulky plows	2	45.00	90.00		2	41.00	82.00	
Disk harrow	2	28.00	56.00		2	25.00	50.00	
Mower	1		35.00		1		30.00	
Hay rake	1		20.00		1		19.00	
(List all items of farm machines, wagons, harness, and small tools.)								
Total investment in machinery and tools (not all listed here)				475.00				461.00

Sample farm inventory: Farm of ——— —(Continued)

PROPERTY	APRIL 1, 1916				APRIL 1, 1917			
	No.	Rate	Valuation		No.	Rate	Valuation	
FEED AND SUPPLIES								
Farm products:								
Corn............bushels..	80	.60	48.00		125	.60	75.00	
Oats...............do....	200	.42	84.00		90	.50	45.00	
Potatoes...........do....	40	.75	30.00		80	.60	48.00	
Hay, timothy.......tons..	10	16.00	160.00		20	15.00	300.00	
Hay, mixed.........do....	5	12.00	60.00		4	12.00	48.00	
Silage.............do....	40	4.00	160.00		40	4.00	160.00	
Bran...............do....	0½	....	15.00		....	....	....	
Mixed feed.........do....	1	....	31.00		2½	30.00	75.00	
Seed oats.........bushels..	30	.80	24.00		35	.80	28.00	
Seed potatoes.......do....	45	.80	36.00		50	1.00	50.00	
Seed corn..........do....	3	2.00	6.00		3	2.00	6.00	
Cement..............sacks..	4	.50	2.00		....	....	....	
Twine...............pounds..	20	.10	2.00		10	.10	1.00	
Total value of feed and supplies..............	....	....	....	658.00	....	....	....	836,00
BILLS RECEIVABLE								
J. A. Brown, hay........tons..	2	13.00	26.00					
R. S. Jones, potatoes..bushels..	40	.50	20.00					
Total............	....	....	....	46.00				
CASH								
On hand....................	....	....	90.00		....	....	210.00	
In bank....................	....	....	580.00		....	....	1,938.00	
Total..................	....	....	....	670.00	....	....	....	2,148.00
BILLS PAYABLE								
Farm mortgage...............	....	....	....	2,000.00	....	....	....	1,500.00
SUMMARY								
Real estate..................	....	....	13,500.00		....	....	13,500.00	
Live stock..................	....	....	2,729.00		....	....	2,883.00	
Machinery and tools..........	....	....	475.00		....	....	461.00	
Feed and supplies............	....	....	658.00		....	....	836.00	
Bills receivable...............	....	....	46.00		....	....	....	
Cash on hand and in bank......	....	....	670.00		....	....	2,148.00	
Total investment........	....	....	....	18,078.00	....	....	....	19,828.00
Bills payable............	....	....	....	2,000.00	....	....	....	1,500.00
Net worth...............	....	....	....	16,078.00	....	....	....	18,328.00
Increase in inventory $2,250.								

Real profit may be determined by the preparation of a statement similar to the following:

Item	March 1, 1917		March 1, 1918	
Resources				
Real estate	$15,000		$15,000	
Live stock	3,160		3,590	
Machinery and tools	530		575	
Feed and supplies	860		735	
Cash	140		275	
Bills receivable			50	
Accounts receivable	93		126	
Total resources	19,783		20,351	
Liabilities				
Accounts payable	135	60	215	20
Mortgage	1,000		1,000	
Total liabilities	1,135	60	1,215	20
Present worth	18,647	40	19,135	80
Increase in net worth			488	40
Add				
Interest paid on mortgage 5 % on $1000			50	
Personal and household expenses paid in cash			750	
Supplies furnished by farm for household			363	
Rent of farmhouse			120	
Gross farm gain			1,771	40
Deduct				
Interest on net investment at 5 %	932	37		
Labor of owner and family (estimated)	600			
Total deduction			1,532	37
True net gain			239	03

Written Work

From the following facts prepare a statement similar to the preceding illustration.

Condition of farmer's affairs on March 1, 1917.

He owns a farm of 160 acres, worth $225 per acre, on which he has given a mortgage of $6000, bearing 5 % interest. Cash on hand and in bank, $345.85. His live stock is worth $5280; his poultry, $286; his machinery, $1250; unsold crops, $900. He has just sold a car of hogs to a local stock buyer, who owes him $975 for them. He also has on hand fertilizer, lumber, and other supplies worth $387.50. He owes sundry accounts, amounting to $236.25.

During the year between March 1, 1917, and March 1, 1918, he paid the interest on his mortgage, and also paid $1000 on the principal of the mortgage. He paid by cash and by butter, eggs, and other produce, for household expenses, $596.50. He estimated the rent of his house at $15 per month; the produce of the farm consumed by the family, $645; and his wages, $600. He paid taxes and insurance, $275. Compute interest on net investment at the beginning of the year at 5%.

On March 1, 1915, he had, cash $167.20; live stock, $6140; poultry, $270; machinery, same as previous year, less 10% depreciation, plus $165 worth of new machinery; crops on hand, $650; miscellaneous supplies, $295. He owes personal accounts amounting to $193.70 and the unpaid balance of the mortgage.

387. Finding the Profits by Crops. In order to determine the profit from any particular crop, the farmer must know:

a. The value of the crop produced;

b. The cost of producing the crop, including seed, fertilizer, labor, and a share of the taxes, interest, and general expenses.

388. Determining the Expenses. The cost of seed and fertilizer may be determined with comparative ease, because measurable quantities are put on each field.

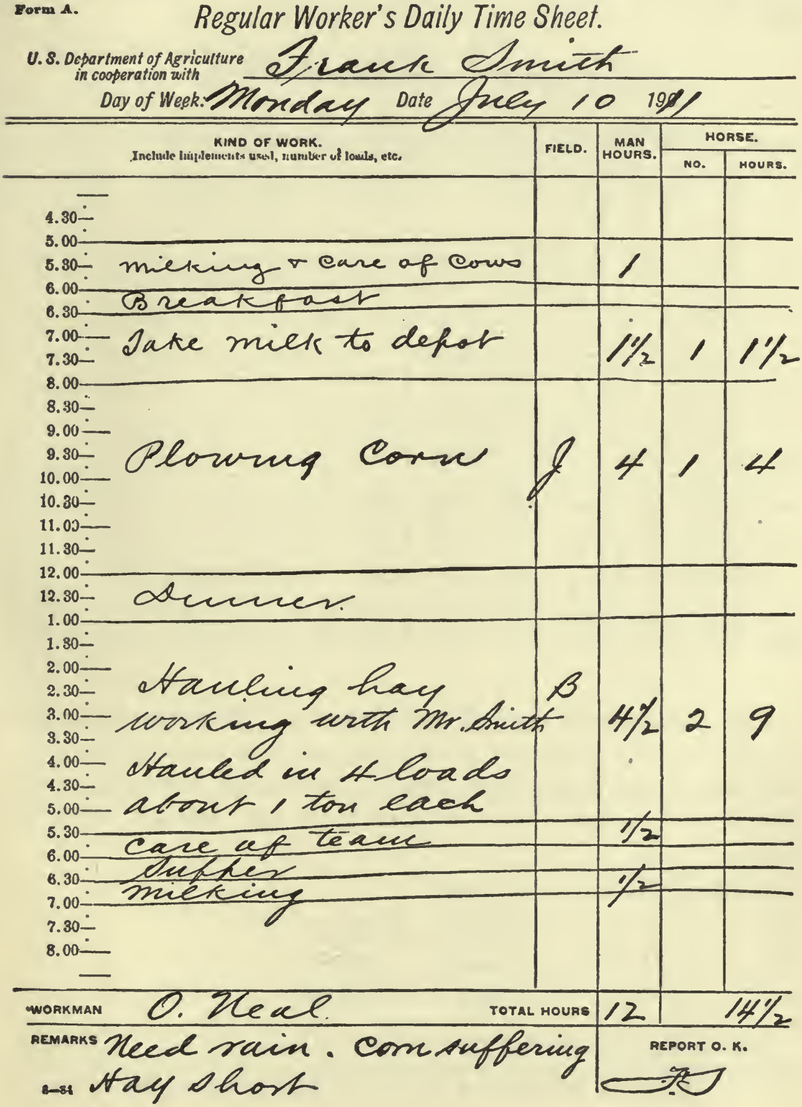
Taxes, interest, and general expenses are prorated among the different crops in the ratio of the land occupied.

Example. A farmer who has 40 acres of hay, 60 acres of corn, and 30 acres of oats under cultivation, finds that his total interest, taxes, and expenses are $1200. How should this be prorated?

SOLUTION. $\$1200 \div 130 = \$9.23$ Charge per acre.
$40 \times \$9.23 = \$369.20$ Share for hay crop.
$60 \times \$9.23 = \$553.80$ Share for corn crop.
$30 \times \$9.23 = \$276.90$ Share for oat crop.

Labor of men and horses must be determined by records of the actual labor spent on each crop.

Accurate labor records may be obtained by a daily time memorandum similar to the following illustration:



Form A.

Regular Worker's Daily Time Sheet.

U. S. Department of Agriculture in cooperation with Frank Smith

Day of Week: Monday Date July 10 1911

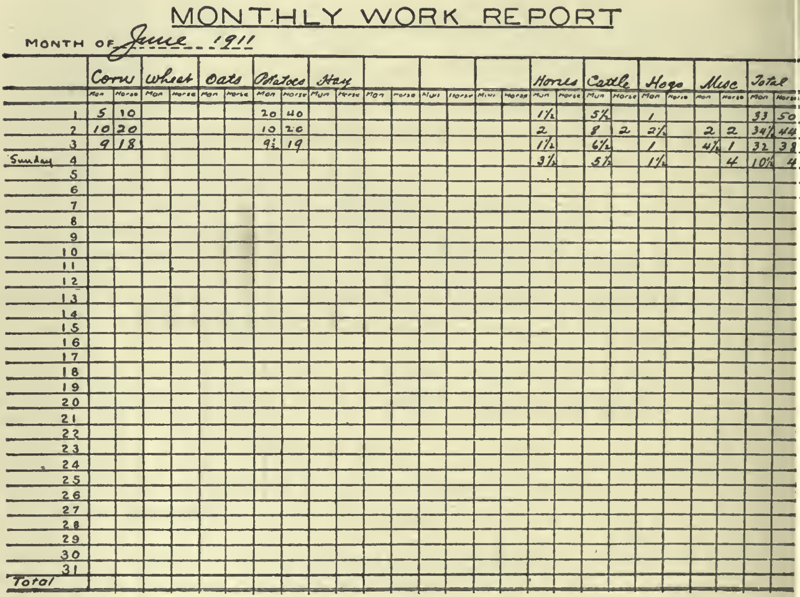
	KIND OF WORK. Include implements used, number of loads, etc.	FIELD.	MAN HOURS.	HORSE. NO.	HORSE. HOURS.
4.30					
5.00					
5.30	milking & care of cows		1		
6.00					
6.30	Breakfast				
7.00	Take milk to depot		1½	1	1½
7.30					
8.00					
8.30					
9.00					
9.30	Plowing Corn	J	4	1	4
10.00					
10.30					
11.00					
11.30					
12.00					
12.30	Dinner.				
1.00					
1.30					
2.00					
2.30	Hauling hay	B			
3.00	working with Mr. Smith		4½	2	9
3.30					
4.00	Hauled in 4 loads				
4.30					
5.00	about 1 ton each				
5.30	care of team		½		
6.00	Supper				
6.30	milking		½		
7.00					
7.30					
8.00					
WORKMAN O. Neal.		TOTAL HOURS	12		14½

REMARKS Need rain. Corn suffering
Hay short

REPORT O. K.

8-34

From the foregoing record a summary may be made on a blank form like the following illustration. At the end of the month the column totals will show the number of man hours and horse hours of labor expended on each of the farm's crops or other industries.



MONTHLY WORK REPORT

MONTH OF June 1911

	Corn		Wheat		Oats		Potatoes		Hay								Horses		Cattle		Hogs		Misc		Total	
	Man	Horse	Man	Horse	Man	Horse	Man	Horse	Man	Horse	Man	Horse	Man	Horse	Man	Horse	Man	Horse	Man	Horse	Man	Horse	Man	Horse	Man	Horse
1	5	10					20	40									1½		5½		1				33	50
2	10	20					10	20									2		8	2	2½		2	2	34½	44
3	9	18					9½	19									1½		6½		1		4½	1	32	38
Sunday 4																	3½		5½		1½			4	10½	4
5																										
6																										
7																										
8																										
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29																										
30																										
31																										
Total																										

389. Finding the Cost of Man Labor and Horse Labor per Hour At the end of a month or other period of time, the total number o man hours and horse hours can be determined from the records The cost per hour may be found by the method illustrate below:

Cost of man labor per year

Wages,	$ 525.00
Board,	100.00
Total,	$ 625.00

Hours worked during year, 2500.

$ 625.00 ÷ 2500 = $ .25, labor cost per man hour.

Cost of a horse hour is determined in a similar way.

Total value of 2 horses, $600.00	
Int. on $600.00 at 5%,	$30.00
Feed and expense,	550.00
Total cost per year,	$580.00

Total number of horse labor hours, 6500.

$580.00 ÷ 6500 = 8.9 cents, labor cost per horse hour.

After the labor cost records have been made, the profit or loss from the crop may be determined by the preparation of a table similar to the following:

ACCOUNT WITH A CROP OF CORN IN FIELD B (36.48 ACRES) ON AN IOWA FARM, 1910[1]

ITEMS OF STATEMENT	DATE	TOTAL			PER ACRE			Per bushel
		Man hours[2]	Horse hours[2]	Cost	Man hours[2]	Horse hours[2]	Cost	
Plowing, fall of 1909 (14-inch gang)	Mar. 25 to Apr. 2	85½	342	$46.48	2.34	9.38	$1.279	—
Disking	Apr. 7 to 29	90¼	361	49.28	2.47	9.89	1.351	—
Harrowing	Apr. 29 to May 4	25½	51	8.56	.70	1.40	.235	—
Planting (with planter)	Apr. 30 to May 5	30½	61	10.25	.84	1.67	.281	—
Harrowing (after planting)	May 10 to 14	33¾	72½	11.86	.92	1.99	.325	—
Cultivating (first time)	May 27 to 30	58	116	19.49	1.59	3.18	.534	—
Cultivating (second time)	June 3 to 6	54½	109	18.31	1.49	2.99	.502	—
Cultivating (third time)	June 14 to 18	51½	103	17.31	1.41	2.82	.474	—
Cultivating (fourth time)	June 23 to July 5	57	114	19.15	1.56	3.12	.525	—
Picking seed corn	Sept. 27 to Oct. 7	59	—	7.44	1.62	—	.204	—
Husking (from standing stalks)	Nov. 2 to 22	305½	611	102.65	8.38	16.75	2.814	—
Total labor cost	—	851	1,940½	310.98	23.32	53.19	8.524	—
Manure charge	—	—	—	124.91	—	—	3.424	—
Seed, 5½ bushels, at $5	—	—	—	27.50	—	—	.754	—
General expense	—	—	—	18.24	—	—	.500	—
Equipment	—	—	—	23.27	—	—	.638	—
Taxes	—	—	—	25.53	—	—	.700	—
Interest (rent)	—	—	—	255.35	—	—	7.000	—
Total cost	—	—	—	785.78	—	—	21.54	—
Summary:								
Income[3]	—	—	—	1,127.36	—	—	30.90	$0.512
Cost	—	—	—	785.78	—	—	21.54	.357
Profit	—	—	—	331.58	—	—	9.36	.155

---

[1] Previous crop: Timothy for seed, 1909.

[2] Rates per hour: Man hours, 12.6 cents; horse hours, 10.5 cents.

[3] Yield: 2200 bushels of grain, at 50 cents (average, 60.3 bushels per acre), $1100; stalks, $27.36; total, $1127.36.

The labor records show the number of horse hours and man hours spent on each crop. After determining the hourly rate, it is necessary to multiply as follows:

The number of man hours by the rate per man hour; the number of horse hours by the rate per horse hour; to determine the total labor cost of each.

The costs and returns of each crop may then be summarized, and the gain computed, as illustrated in the following table:

Corn Crop on South 40

Item		40 Acres	Per Acre
Seed		$ 24.00	.60
Fertilizer		70.00	1.75
Taxes		28.30	.7075
Interest		450.00	11.25
Expense		40.00	1.00
Man labor, 991 hours at 11 cents		109.01	2.725
Horse labor, 2286 hours at 10 cents		228.60	5.715
Total cost		949.91	23.7475
Yield, 2552 bu. corn at $.48	$1224.96		
Stalks, 75¢ per acre	30.00		
Total returns		1254.96	31.374
		949.91	23.7475
Gain		305.05	7.6265

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Written Work

1. Find the labor cost to be charged against the corn crop, in the production of which 136 man hours and 312 horse hours of labor were expended. The total number of hours of man labor for the year was 4825, and the annual cost was $868.50; total number of hours of horse labor, 4312; total cost of horse labor for the year, $388.08.

2. Use the following facts to prepare a table similar to the model above:

CORN CROP ON 20 ACRES

Yield: 10 bushels of seed corn worth $3.50 per bushel.

Corn crop harvested, 67 bushels per acre, worth 54 cents per bushel.

Stalks were estimated at a value of 60 cents per acre.

Costs: $3\frac{1}{2}$ bushels of seed at $4.50 per bushel.

Fertilizer, $38.00.

Taxes: The farm was worth $175.00 per acre, and was taxed on $\frac{1}{5}$ the real value at a rate of $1.65. The farm contained 160 acres, and the taxes were prorated among the various fields in proportion to their acreage.

Interest: 4% on the value of the land.

Expense: $31.25.

Labor: 560 man hours at 19 cents.
1348 horse hours at 9 cents.

APPENDIX

DENOMINATE NUMBERS

SURVEYORS' LONG MEASURE

7.92 inches = 1 link (lk.)
25 links = 1 rod
4 rods, or 100 links = 1 chain (ch.)
80 chains = 1 mile

SURVEYORS' SQUARE MEASURE

625 square links = 1 square rod
10 square rods = 1 square chain
16 square chains = 1 acre
640 acres = 1 square mile
36 square miles = 1 township

The following units of measure are used by sailors:

6 feet = 1 fathom (Used for measuring depths at sea.)
120 fathoms = 1 cable length (Used for measuring depths at sea.)
About 1.15 statute miles = 1 knot, or 1 nautical mile, or 6080.27 ft.

CIRCULAR OR ANGULAR MEASURE

60 seconds (60″) = 1 minute (′)
60 minutes = 1 degree (1°)
90 degrees = 1 right angle
360 degrees = 1 circumference

TROY WEIGHT

24 grains (gr.) = 1 pennyweight (pwt. or dwt.)
20 pennyweights = 1 ounce (oz.)
12 ounces = 1 pound (lb.)

APOTHECARIES' WEIGHT

20 grains (gr.) = 1 scruple (sc. or ℈)
3 scruples = 1 dram (dr. or ʒ)
8 drams = 1 ounce (oz. or ℥)
12 ounces = 1 pound (lb.)

Apothecaries' weight is used by physicians and druggists. Troy weight is used in the measurement of precious metals.

COMPARISON OF TROY AND AVOIRDUPOIS WEIGHTS

1 pound Troy = 5760 grains
1 pound avoirdupois = 7000 grains
1 ounce Troy = 437½ grains
1 ounce avoirdupois = 480 grains

The term *carat* has two meanings:

In weighing precious stones, a carat usually means 3.2 Troy grains.

In expressing the purity of gold, 24 carats means pure gold; 18 carats means $\frac{18}{24}$ pure gold, and $\frac{6}{24}$ alloy.

PAPER MEASURE

24 sheets = 1 quire (qr.)
20 quires = 1 ream (rm.)

Although a ream contains 480 sheets, 500 sheets are usually sold as a ream.

STANDARD UNITS OF WEIGHT

1 barrel flour	weighs 196 pounds
1 barrel salt	weighs 280 pounds
1 barrel pork or beef	weighs 200 pounds
1 keg of nails	weighs 100 pounds

STANDARD BUSHELS IN MANY STATES

1 bushel shelled corn	weighs 56 pounds
1 bushel ear corn	weighs 70 pounds
1 bushel wheat	weighs 60 pounds
1 bushel barley	weighs 48 pounds
1 bushel rye	weighs 56 pounds
1 bushel oats	weighs 32 pounds

TABLE OF ABBREVIATIONS USED IN BUSINESS

(The singular form is commonly used for both the singular and plural.)

A.	acre	ans.	answer
acct. or a/c.	account	Apr.	April
agt.	agent	Aug.	August
amt.	amount	av.	average

TABLE OF ABBREVIATIONS USED IN BUSINESS — (*continued*)

bg.	bag; bags
bal.	balance
bbl. or brl.	barrel
bdl.	bundle
bkt.	basket
bl.	bale
B/L	bill of lading
bot.	bought
bu.	bushel
bx.	box
C	one hundred
cd.	cord; card
cg.	centigram
ch.	chain; chest
chg.	charge
c.i.f.	carriage and insurance free
ck.	check
cm.	centimeter
cml.	commercial
c/o	care of
Co.	company; county
c.o.d.	collect on delivery
coll.	collection
com.	commission
consg't.	consignment
cr.	creditor; credit; crate
cs.	case
csk.	cask
c., ct.	cent; centime
cu. ft.	cubic foot
cu. in.	cubic inch
cu. yd.	cubic yard
cwt.	hundredweight
d.	pence
da.	day
Dec.	December
dep't	department
dft.	draft
disc.	discount
do.	ditto (the same)
doz.; dz.	dozen; dozens
Dr.	debit; debtor; doctor
E.	East
ea.	each
e.g.	for example
e.o.e.	errors and omissions excepted
etc.	and so forth
ex.	example; express
exch.	exchange
exp.	expense
f.	franc
far.	farthing
Feb.	February
fir.	firkins
f.o.b.	free on board
frt.	freight
ft.	foot
gal.	gallon
gi.	gill
gr.	grain
gro.	gross
gr. gro.	great gross
guar.	guaranty; guarantee
hf.	half
hf. cht.	half chest
hhd.	hogshead
hr.	hour
i.e.	that is
in.	inch; inches
ins.	insurance
inst.	instant; the present month
int.	interest
I.; inv.	invoice
inv't	inventory
Jan.	January
kg.	keg
km.	kilometer
l.	link
lb.	pound
l.p.	list price
ltd.	limited
M	one thousand
m.	mill; meter
Mar.	March
mdse.	merchandise

TABLE OF ABBREVIATIONS USED IN BUSINESS — (*continued*)

Messrs.	*Messieurs;* Gentlemen; Sirs
mi.	mile
min.	minute
mo.	month
mortg.	mortgage
Mr.	Mister
Mrs.	Mistress
N.	North
no.	number
Nov.	November
Oct.	October
O.K.	all correct
oz.	ounce
p.	page
pay't	payment
pc.	piece
pd.	paid
per	by; by the
per cent	per centum; by the hundred
pfd.	preferred
pk.	peck; pecks
pkg.	package
pp.	pages
pr.	pair
prox.	the following month
pt.	pint
pwt.	pennyweight
qr.	quire
qt.	quart
rd.	rod
rec'd	received

rec't	receipt
rm.	ream
Rm. (or M.)	Reichsmark; Mark
s.	shilling; shillings
S.	south; sales
sec.	second
sec'y	secretary
Sept.	September
set.	settlement
ship.	shipment
shipt.	shipped
sig.	signed; signature
sq. ch.	square chain
sq. ft.	square foot
sq. mi.	square mile
sq. rd.	square rod
sq. yd.	square yard
stk.	stock
sund.	sundries
T.	ton
tb.	tub
Tp.; Twp.	township; townships
tr.; trans.	transfer
treas.	treasurer; treasury
ult.	last month
via	by way of
viz.	namely; to wit
vol.	volume
wk.	week
wt.	weight; weigh
yd.	yard
yr.	year

TABLE OF SYMBOLS

a/c	account
a/s	account sales
+	addition
( )	aggregation
&	and
...	and so on
@	at; each; to

c/o	care of
¢	cent
√	check mark; correct
°	degree
÷	division
$	dollar; dollars
=	equal; equals

TABLE OF SYMBOLS — *(continued)*

′	foot; feet; minutes
$^1, ^2, ^3$	fourths (written as exponents, thus, $3^1 = 3\frac{1}{4}$)
>	greater than
C	hundred
″	inch; inches; seconds
<	less than
×	multiplication
#	if written before figures, means number; if written after figures, means pounds
o/d	on demand
%	per cent
£	pounds sterling
:	ratio
∵	since
−	subtraction
∴	therefore
M	thousand

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